

# Analysis of the geometry and contact density of globoid gearing

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**Abstract.** Globoid worm gears have been widely applied in a range of technological branches in which other types of worm gears are less effective. The main functional indicators which facilitated their popularity include high load capacity and durability, low vibroactivity and small energy losses. As the experience of application and the results of the study of globoid worm gears showed, the level of operational properties of globoid gears is higher than that of the others only if the gearing and technological parameters of their manufacturing are optimally chosen during the design stage. This paper describes a method for estimating the gearing parameters of a globoid gear with an account of the geometry of the elements (the geometry of the teeth of the wheel and the worm thread), namely the calculation of the gap fields in the globoid gearing by means of numerical methods.

## 1 Introduction

The design of worm gears requires consideration of a large number of factors caused by operating conditions, a variety of shapes, sizes and options of transmission design. This is possible only if rational gearing parameters which take account of the manufacturing process are selected at the design stage. As a rule, the solution to the problems arising when designing and manufacturing worm gears is based on operational experience and experimental research. The proposed method of computer modeling allows us to take into account different variants of technological processes when manufacturing globoid gears, which substantially reduces the costs and the timing for choosing gearing parameters.

## 2 Questions of the theory of production and analysis of globoid gears

The approach to calculating the geometry of globoid gearing which is described in the article is based on the principles developed by the Russian school of theory and geometry of worm gears, particularly globoid ones. These principles are reflected in a number of works by Russian scientists [1-16].

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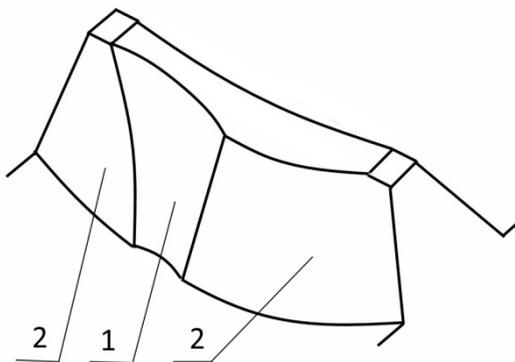
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The known types of worm gears with a globoid worm, which vary depending on the scheme for shaping the worm threads, are given, for example, in the dissertation by Rubin M. A. [6].

Each of these groups of globoid gears has both common and specific features. Let us consider some of these features using the example of a classical globoid gear.

In this gear, the lateral surface of the thread of the globoid worm is formed by the straight generating edge of the tool, which belongs to the plane in which it rotates around the axis of the table of the machine. In this case, the axis of the rotation of the worm also belongs to the same plane (the plane of the rotation of the generating edge). It should be noted that the corresponding calculated values of the machine setting parameters when cutting the worm and forming the teeth of the wheel of the classical globoid gear coincide and are equal to the corresponding calculated values of the parameters of the relative position of the worm and the wheel in the gear.

In the globoid gear, the lateral surface of the tooth of the wheel is formed as being conjugated with the surface of the worm thread. In this case, the generating worm is theoretically an analog of the original (working) gear worm. Fig.1 and Fig. 2 show that in the middle plane of the wheel of the classical gear on the lateral surface of the tooth of the wheel there are two fractures, which are commonly considered as the boundaries of the envelope of the surface 1 and the ones of the scoring surface 2, formed by the input edge of the (forming) generating surface of the worm thread [1].

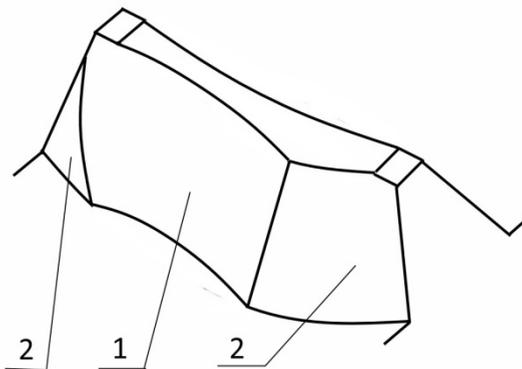


**Fig. 1.** Shape of the teeth of the wheel in a classical globoid gear

In the globoid gear, the instantaneous contact lines of the contacting surfaces of the teeth and threads (lines of zero gaps) are located along the height of the tooth of the wheel. The angles between the tangents to the instantaneous contact lines and the velocity vectors of the relative slip of the surfaces at the points belonging to these lines are close to zero.

As the operational experience has shown, the classical globoid gear acquires (as it wears out) a new, so-called modified form of gearing which has higher performance indicators [1]. This explains why researchers are interested in modified gears. P.S. Zak showed [1] that the envelope tooth surface of a modified globoid gear (Fig. 2) exceeds the dimensions of a similar tooth surface of a classical transmission wheel (Fig.1). It is concluded that for a particular globoid gear the envelope tooth surface of the wheel is not static and varies with wear [1].

Questions of the theory of formation, analysis and synthesis of gearing, as well as those of the production technologies for modified globoid gears are considered in a number of publications [1, 3, 5-10].



**Fig. 2.** Shape of the teeth of the wheel in a modified globoid gear

The need to improve the technological and operational characteristics of globoid gears required the discovery of new technical solutions. In this regard, one should note the introduction of the process of grinding worms and tools corresponding to an increase in the durability and accuracy of gear hob and shavers for cutting wheels, as well as improving the quality of worms of globoid gears [11, 16].

Analysis of the geometry of the globoid gearing and the features of contact between the surfaces of the wheel teeth and the worm threads in the above mentioned publications was based on the calculations of the coordinates of the points of the instantaneous contact lines, the curvature of the surface at the points of these lines, the construction of the contact field, and the determination of a number of other geometric parameters. The sections of the investigated surfaces of the thread of the worm and the tooth of the wheel were constructed. As a secant surface, a plane perpendicular to the generatrix of the thread was chosen. Then, based on these calculations, diagrams of the gaps in the globoid gearing were constructed.

The analysis of the sources shows that when assessing the contact density in the globoid gearing only over contact lines and curvature of surfaces, the researcher faces the following problems.

1. Real neighborhoods of the point of the contact line, where the reduced curvature describes with sufficient accuracy the nature of the abutment of surfaces, are unknown. The gearing phase has a significant effect on the variation of the reduced curvatures of the envelope of the tooth surface and the surface of the worm thread at the points of the instantaneous contact lines.

2. The lateral surface of the tooth of the wheel consists of different parts, which are formed (as indicated above), in different ways. Therefore, it is not known in advance whether the actually found theoretical point of the contact line corresponds to the nominal lateral tooth surface.

A more accurate picture of the adherence of the teeth and the threads of globoid gear makes it possible to obtain a method based on the construction of sections of their lateral surfaces. This method was developed by V.N. Syzrantsev [17, 18] with reference to gears formed by a spiral-disk tool. The method is based on calculating the fields of these gaps and allows estimating the density of the actual contact of surfaces with an account of the given errors in the manufacture and installation of gear elements. The application of a similar approach to the analysis of the geometry of globoid gearing is reflected in the research by Bondarenko A.V., Smolin A.I., Vyatkin A.I. [19, 20], who consider both the features of the formation of globoid gear and the errors in manufacturing and installation.

The development of methods for estimating the geometry of gearing for various types of gears can also be found in a number of recent publications [21-24]. Much research in this

direction was carried out. This paper presents one of the methods for estimating the geometry of globoid gearing.

### 3 Method for estimating the geometry of globoid gearing. General approach to the estimation of the geometry of globoid gearing

The main provisions of this method are presented in the following works [25-27].

First, let us consider a number of basic points that make it possible to understand the essence of the above mentioned approach to the analysis of the geometry of globoid gearing.

In the presence of production and installation errors, the instantaneous gear ratio of globoid gear is an unknown value. The situation is possible when the surfaces are either retracted or embedded.

The actual picture of the adhesion of surfaces can be obtained by screwing one of the elements of globoid gear (wheel or worm) to an angle compensating for the minimum gap or maximum penetration.

In order to determine the nature of the contact in the globoid gear on the surfaces of the teeth of the wheel and the threads of the worm, a set of points is given (Fig.3). Each node is the intersection of the corresponding surface by the cylinder, coaxial to the wheel in the gear, and a plane perpendicular to their wheel axis (Fig. 4, Fig.5). Then, along the arc of the circumference of the secant cylinder, the distance between each pair of given points of the surfaces of the tooth and the thread is calculated:

$$\Delta_{ijn} = R_{ij} [\arcsin(X_{23}/R_{ij}) - \arcsin(X_{13}/R_{ij})], \tag{1}$$

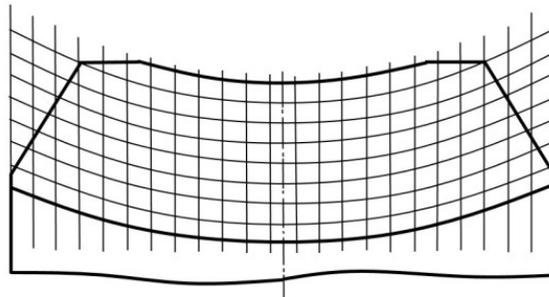
where  $i, j$  are indices indicating the number of the node by height and width of the tooth of the wheel;

$n$  is the number of pairs of teeth and threads in gearing;

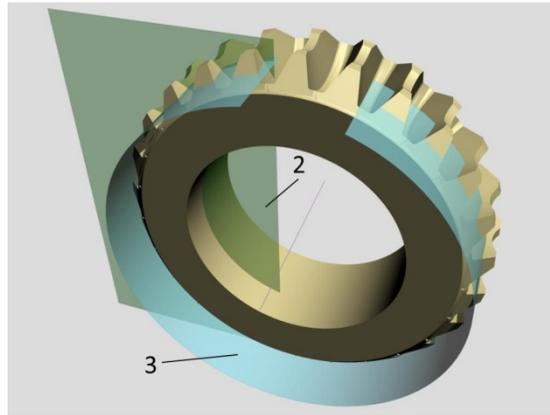
$\Delta_{ijn}$  is the gap ( $\Delta_{ij} > 0$ ) or introduction ( $\Delta_{ij} < 0$ ) of the specified points;

$R_{ij}$  is the radius of the secant cylinder;

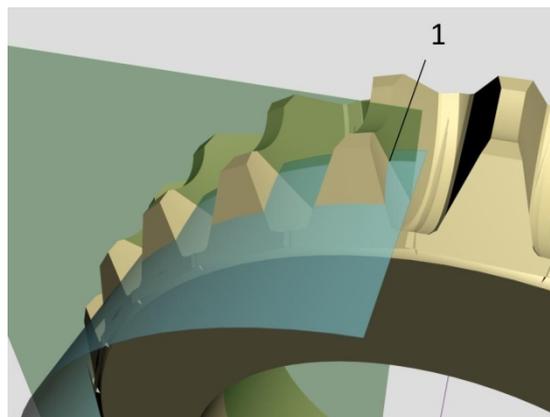
$X_{23}, X_{13}$  are projections of the radius of the vectors of the given points of the tooth and thread surfaces determined with an account of the features of the thread surface formation.



**Fig. 3.** Location of the calculated points on the surface of the tooth of the wheel



**Fig. 4.** Forming surfaces for obtaining the calculated points (2 – the plane perpendicular to their wheel axis, 3 – a cylinder coaxial with the wheel)



**Fig. 5.** Formation of a calculated point for calculating gaps in the gearing. 1 – a calculated point

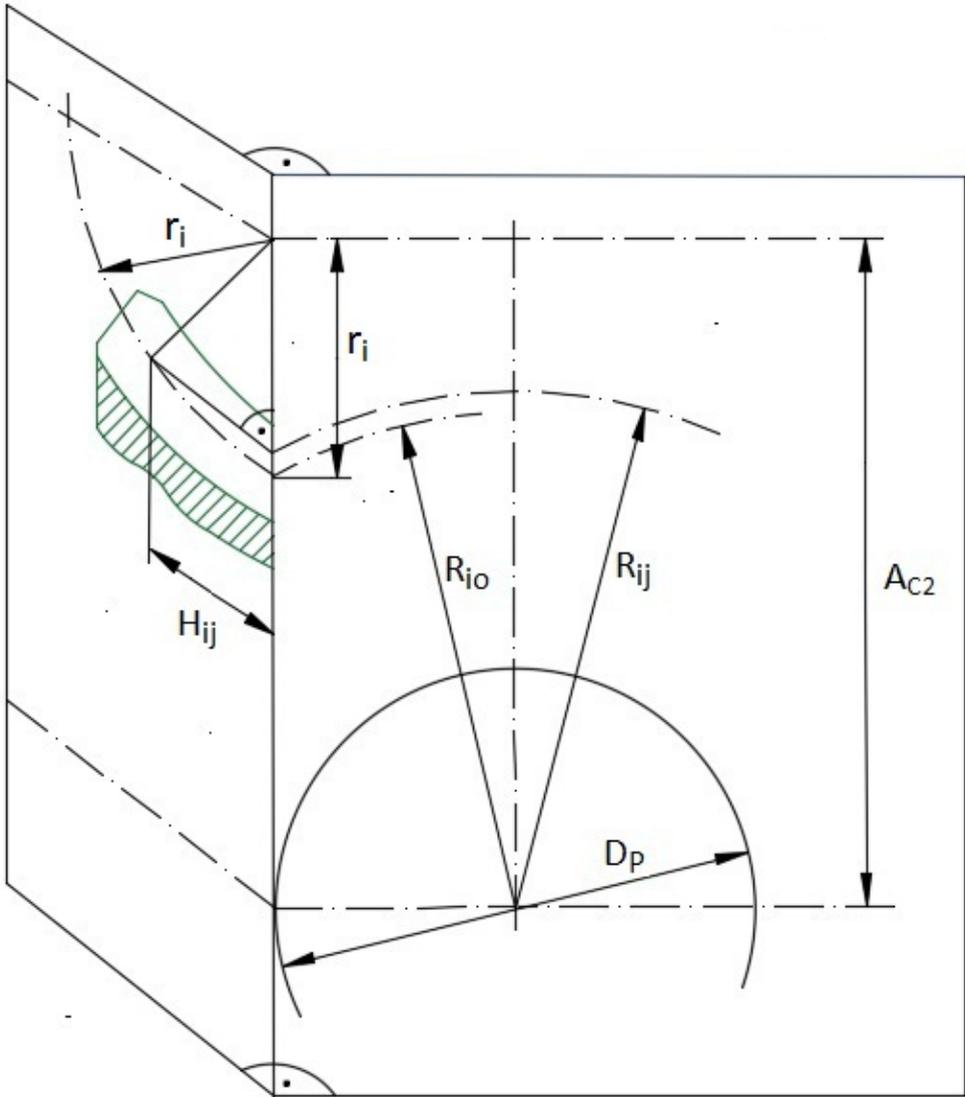
With the chosen method of determining the distances between the given points of the surfaces of the teeth and threads, the minimum clearance or maximum insertion is compensated by the wheel rotation by an angle:

$$\Delta\mu_2 = \min(\Delta_{ijn}/R_{ij}), \tag{2}$$

where  $i=1,I, j=1,J, n=1,N$ ;  $I, J$  are the number of nodes for the height and length of the wheel.

Moreover, from the whole set of given points when determining  $\Delta\mu_2$ , those that lie outside the area bounded by the geometric dimensions of the tooth wheel rim and the worm are excluded. The values of  $\Delta_{ijn}$  at such points were assumed to be equal to the known limiting value.

This technique made it possible to represent more conveniently the field of gaps in the memory of a computer by two-dimensional arrays.



**Fig. 6.** Identification of the interrelation between the parameters  $R_{ij}$  and  $H_{ij}$

Next, taking into account the angle  $\Delta\mu_2$ , which compensates for the minimum gap or maximum penetration, a set of values  $\delta_{ijn}$  was found. It determines the field of the reduced gaps and characterizes the density of the contact of surfaces of the  $n^{\text{th}}$  wheel tooth and the thread of the worm in the fixed phase of the working gearing:

$$\delta_{ijn} = \Delta_{ijn} - \Delta\mu_2 R_{ij}, \quad (3)$$

where  $i=1,I, j=1,J, n=1, \text{const.}$

The position of the nodes on the surfaces of the teeth and threads studied was specified by the parameters  $R_{ij}$  and the related dependence (Fig.6):

$$R_{ij} = \sqrt{(A_{c2} - \sqrt{r_i^2 - H_{ij}^2})^2 + D_p^2/4}, \quad (4)$$

$$r_i = A_{c2} - \sqrt{R_{io}^2 - D_p^2/4}, \quad (5)$$

where  $A_{c2}$  is the interaxial distance in the second machine gearing;

$D_p$  is the diameter of the profile circle;

$R_{io}$  is the radius of the secant coaxial cylinder at  $H_{ij} = 0$  for the  $i^{\text{th}}$  line of the level of the nodes along the length of the wheel tooth.

## 4 Conclusion

The proposed approach makes it possible to evaluate the geometry of the worm pair not only in the static condition, but also to numerically simulate the change in the geometry of globoid gear during the running-in. At the same time, one can take account of not only the geometrical dimensions of the gear elements and the features of the shaping of the wheel teeth and the thread of the worm, but also the specified errors in manufacturing and installation, as well as changes in gaps due to elastic deformations

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