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Deformation and forces when compressing spherical particle

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Abstract. Plastic deformation of a spherical particle under the action of a roller is considered. In order to obtain engineering evaluations, a number of assumptions are made. As a result, the relationships for calculating the lengths characterizing the dynamics of the dimensions change of the particle contact surfaces with the roller and the substrate over time are found. After that, the pressure and the compression forces acting on the particle are determined.

1 Introduction

At enterprises in many industries, such as metallurgy, power engineering, chemistry and petrochemistry compression of individual particles and distribution of their material along the substrate surface occurs within a number of technological processes. In this case, as a rule, the material of these particles is in the plastic state.

According to general provisions of plastic deformation theory [1-9] the general stress-strain state of the body is greatly influenced by the inhomogeneity of stresses and deformations, the main causes of which are anisotropy of the material physical properties [10], the contact friction on the deformable surface when interacting with the tool or support, shape of the tool, support. In particular, the unevenness of plastic deformations caused by the contact friction is expressed, firstly, in the appearance of barrel-shaped particles since the material layers are retained by frictional forces near the surface of the tool (roller), support (substrate). The effect of the friction forces decreases as the distance from these surfaces increases [11]. Secondly, the frictional forces are distributed unevenly along the contact surfaces, as a rule, increasing from the periphery to the central sections because when the particle material moves to the periphery it must overcome the resistance of the medium that is in its path. It is obvious that near the free surface the frictional forces are close to zero. Thirdly, the degree of barreling of the particle plastic deformations decreases with the decrease of the friction coefficient. In the limit when in cross-section the frictional forces are small, the lateral surfaces shape will be close to rectilinear.

In the general case, the law of least resistance (the law of the smallest perimeter) takes place, according to it the material primarily moves in the direction of the least resistance of

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frictional forces from the side of the contact surfaces. In other words, the deformable material near the contact surface moves along the shortest paths to the contact surface perimeter, i.e., along the normal to it.

2 Change of particle shape during compression

In addition to the presented qualitative picture of stresses and deformations, it is necessary to determine the forces, material deformations, and the total force acting on the roller (tool) and on the support. The latter will, in turn, allow describing the adhesion of the particle and substrate materials.

First of all, it is necessary to estimate the specific forces and friction forces, their distribution over contact surfaces, and also the change over time to determine the total deforming forces. Let us suppose that after softening of the material the particle on the substrate surface has a shape close to spherical, its diameter d_{p}. In the course of time of its processing (contact with the roller) $\tau_k = \frac{d_{\mathrm{pk}}}{u_m}$ (d_{pk} is the average diameter of the surface of the compressed particle that was in contact with the roller) u_{T} – velocity of the particle displacement relative to the roller, it is possible to distinguish three characteristic stages. The first stage is the beginning of the compression ($\tau \geq 0$), the second stage – the compression at $\tau \sim 0.5\tau_k$, the third stage – the end of the compression ($\tau \leq \tau_k$). The shape of the particle cross-section at the final stage is schematically shown in figure 1.

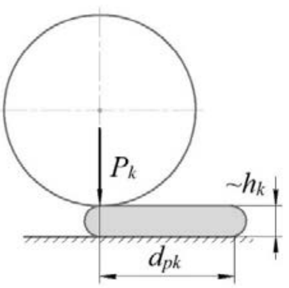

Fig. 1. Compression of a spherical particle using a roller.

It is obvious that in the process of compression the particle shape undergoes significant changes its material is strongly deformed under the influence of both normal and tangential loads.

In order to obtain the engineering (analytical) dependencies for the estimation of the total deforming forces, let us further consider the simplified scheme for deforming the particle assuming that under compression it is located between fixed non-deformable plate (substrate) and another plate parallel to it also non-deformable, moving along the normal for a time τ_{k} (figure 2). The second plate to a certain extent imitates the roller. During the time τ_{k} the upper plate shifts by a distance $d_p - h_k$, hence the average velocity of its movement along the normal is $\bar{v}_k = -\frac{(d_p - h_k)}{\tau_k} = \frac{u_m(h_k - d_p)}{d_{\mathrm{pk}}}$. Accordingly, the height of the deformed particle h_{p} will decrease from d_{p} to h_{k} according to the law

$$h_p = h_p(\tau) = d_p + 2\bar{v}_k\tau\left(1 - \frac{\tau}{2\tau_k}\right). \quad (1)$$

Also, we introduce the angles α_k and β_k calling them the contact angles of the particle material with the surface of the lower and upper plates, $\alpha_k = \alpha_k(\tau)$, $\beta_k = \beta_k(\tau)$.

In addition, let us suppose that in the cross-section the free surface shape of the particle is close to a circle with local radius R_p, $R_p = R_p(\tau) = \frac{h_p}{(\cos\alpha_k + \cos\beta_k)}$. The position of the center O_{p} of the corresponding sector $O_{\mathrm{p}}BD$ is determined by the coordinate $z_O = R_p \cos\alpha_k$.

Thus, the volume V_{p} occupied by the compressed particle consists of the volumes V_{p1}, V_{p2} of two contiguous truncated circular cones with bases *AB*, *CD* and the torus volume V_{p3}.

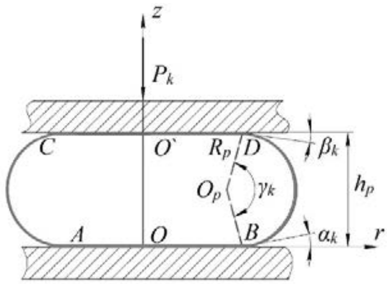

Fig. 2. Schematic representation of the particle shape under compression.

Taking into account that the particle material is incompressible, in the deformation process its volume V_{p} does not change, we write the equation:

$$V_{\mathrm{p1}} + V_{\mathrm{p2}} + V_{\mathrm{p3}} = \frac{4\pi r_p^3}{3}. \quad (2)$$

When the average velocity $\overline{v}_{\mathrm{k}}$ and also angles α_k and β_k are known the relationship (2) establishes the connection between lengths *AO*, *CO*'. In the simplest case, when angles α_k, β_k are close to the average angle $\tilde{\alpha}_k$ the lengths $AO \sim CO' \approx \tilde{l}_{\mathrm{k}}$ are equal to the average value, we get:

$$V_{\mathrm{p3}} \approx 0.8\tilde{\gamma}_k h_p^3 \tilde{l}_k.$$

We obtain the following relationship to find the $\tilde{l}_k$ value:

$$h_p^2\tilde{\gamma}_k\tilde{l}_k + 2.63ctg\tilde{\alpha}_k\left(\tilde{l}_k^3 - \left(\tilde{l}_k - 0.5h_p\right)^3\right) = 5.25r_p^3, \quad (3)$$

where $\tilde{\gamma}_{\mathrm{k}} = 2\arcsin\left(\cos\tilde{\alpha}_{\mathrm{k}}\right)$.

In the case when $\frac{h_p}{\tilde{l}_k} \ll 1$, transforming (3), we find:

$$\tilde{l}_k = \frac{0.77\left(-h_p^2\tilde{\gamma}_k + \sqrt{(h_p^2\tilde{\gamma}_k) + 27.3h_p r_p^3 ctg\tilde{\alpha}_k}\right)}{(h_p ctg\tilde{\alpha}_k)}. \quad (4)$$

Since $\tilde{\alpha}_k$ is quite big because the particle radius r_{p} is very small ($r_{\mathrm{p}} \ll 1$) taking into account the above assumptions from (4) we determine the length $\tilde{l}_{\mathrm{k}}$ characterizing the dynamics of the dimensions change of the particle contact surfaces with the roller and the substrate in the process of compression:

$$\tilde{l}_k = \tilde{l}_k(\tau) \approx \frac{10.5r_p^3}{(h_p^2\tilde{\gamma}_k^2)}. \quad (5)$$

According to formula (5) the $\tilde{l}_k$ value depends on the radius r_{p} of the compressed particle, the distance between the plates h_{p}, and the contact angle $\tilde{\alpha}_k$. Moreover, $\tilde{l}_k$ increases with increasing r_p, decreasing h_{p}, and increasing the angle $\tilde{\alpha}_k$.

If the angle $\tilde{\alpha}_k$ is small, then $R_p \approx 0.5h_p$, angle $\tilde{\gamma}_k \approx \pi$, the equivalent radius of the area of the O_pBD sector $\tilde{r}_p = \frac{R_p}{\sqrt{2}=\frac{0.5h_p}{(\sqrt{2}\,\widetilde{\cos}\,\alpha_k)}}$; $V_{p1} = V_{p2} = 0.5\pi h_p \tilde{l}_k^2$; $V_{p3} \approx 2.4h_p^2\tilde{l}_k$, from (2) we get:

$$\tilde{l}_k = \tilde{l}_k(\tau) \approx -0.4h_p + \sqrt{0.15h_p^2 + \frac{1.33r_p^3}{h_p}}.$$

For small values of $\frac{r_p^3}{h_p^2}$ we get approximately

$$\tilde{l}_k = \tilde{l}_k(\tau) \approx \frac{1.7r_p^3}{h_p^2}. \tag{6}$$

Solutions of (5) and (6) similarly characterize the behavior of the length $\tilde{l}_k$ depending on the basic parameters r_p and h_p. So, if in (5) we put $\tilde{\gamma}_k \sim 2.5$, then $\tilde{l}_k \approx \frac{1.5r_p^3}{h_p^2}$, which, in general, agrees with the formula (6).

In another limiting case, in which the average contact angle $\tilde{\alpha}_k \approx \frac{\pi}{2}$, the friction between materials of particle and the roller (tool), support (substrate) is small, the shape of the compressed particle is close to cylindrical. In this case from (2) it follows:

$$\tilde{l}_k = \tilde{l}_k(\tau) = 1.15k_k r_p\sqrt{\frac{r_p}{h_p}}, \tag{7}$$

where k_k – correction coefficient to take into account the deviations of the compressed particle shape from the cylindrical one, and angle $\tilde{\alpha}_k$ from $\pi/2$.

In particular, for $r_p = 10^{-4}l$ m, $h_p = 0.3r_p$, $k_k = 1.0$ according to (7) the length $l_k \approx 1.83 \cdot 10^{-4}$ m, i.e., it is approximately twice as much as the original radius of the particle.

3 Evaluation of the forces of plastic deformation

Now we shall turn our attention directly to the evaluation of the forces of a particle plastic deformation. There are several methods for determining the specific deforming forces. Let us dwell on the method of calculating the operating stresses [5]. As the simplifying assumptions, we accept that the stress-strain state of the deformable material is axisymmetric; normal stresses in the cross-section do not change.

Taking into account these assumptions, in the deformable body in sector $d\Theta$ using the surfaces located at a distances r and $r + dr$ we select a small element of height $2h = h_p$. Under the condition that the normal σ_r and tangential τ_r stresses with respect to height $2h$ do not change (equal to the averaged values) from the equilibrium condition of forces acting on the element in the projection on the Or axis, we obtain the equation:

$$\frac{d\sigma_r}{dr=\frac{\tau_r}{h}} = 0. \tag{8}$$

Assuming that the tangential friction τ_r is proportional to the normal stresses σ_z, $\tau_r = f_k\sigma_z$ (f_k – friction coefficient), using the plasticity condition $\sigma_r - \sigma_z = \sigma_m$ ($d\sigma_r = d\sigma_z$) from (8) we find:

$$\frac{d\sigma_z}{\sigma_z} = -f_k\frac{dr}{h}. \tag{9}$$

The general solution of equation (9) will be:

$$\sigma_z = C_m \, exp\left(-\frac{f_k r}{h}\right),$$

where the integration constant $C_m = -\sigma_m \, exp\left(\frac{f_k l_{k1}}{h}\right)(l_{k1} = CO')$.

Hence, the normal stresses along the contact surface of the deformed particle are described by the relationship $\sigma_z = \sigma_z(\tau) = -\sigma_m \, exp\left(\frac{2f_k(l_{k1}-r)}{h_p}\right)$. The maximum value of σ_z is reached when $r = 0$ (at the particle axis), $\sigma\sigma_m \, exp\left(\frac{2f_k l_{k1}}{h_p}\right)_{zmax}$; minimum – when $r = l_{k1}$ (at the boundary of the contact surface) $\sigma_{zmin} = -\sigma_m$.

Accordingly, the pressure $p_k = p_k(\tau) = -\sigma_z = \sigma_m \, exp\left(\frac{2f_k(l_{k1}-r)}{h_p}\right)$ will be applied to the upper plate imitating the roller.

The total deforming force

$$P_k = P_k(\tau) = \frac{2\sigma_m}{l_{k1}}\int_o^{l_{k1}} exp\left(\frac{2f_k(l_{k1}-r)}{h_p}\right)dr = \frac{\sigma_m h_p}{f_k l_{k1}}\left(exp\left(\frac{2f_k l_{k1}}{h_p}\right)-1\right). \quad (10)$$

Denoting by f_m the friction coefficient on the substrate surface that is in contact with the particle we similarly find normal stresses on this surface:

$$\sigma_z = \sigma_z(\tau) = \sigma_m \, exp\left(\frac{2f_m(l_{k2}-r)}{h_p}\right)(l_{k2} = AO).$$

The total deforming force acting on the substrate from the side of the particle

$$P_m = P_m(\tau) = \frac{\sigma_m h_p}{f_m l_{k2}}\left(1 - exp\left(\frac{2f_m l_{k2}}{h_p}\right)\right). \quad (11)$$

From the equilibrium condition of the particle in the normal direction it follows:

$$\frac{\left(exp\left(\frac{2f_k l_{k1}}{h_p}\right)-1\right)}{(f_k l_{k1})} = \frac{\left(exp\left(\frac{2f_m l_{k2}}{h_p}\right)-1\right)}{(f_m l_{k2})}.$$

If the quantities $\frac{2f_k f_m}{h_p}, \frac{2f_m l_{k2}}{h_p}$ are small then approximately $\frac{l_{k1}}{l_{k2} \approx \frac{f_m}{f_k}}$. Given that $l_{k1} + l_{k2} \approx 2\tilde{l}_k$ we find the lengths l_{k1}, l_{k2}:

$$l_{k1} = \frac{2\tilde{l}_k \tilde{f}}{(1+\tilde{f})}, \; l_{k2} = \frac{2\tilde{l}_k}{(1+\tilde{f})},$$

where $\tilde{f} = \frac{f_m}{f_k}$ – the ratio of the friction coefficients when moving particles relative to the substrate and the roller.

4 Conclusions

Thus, using the proposed approach based on the equations of mass balance and equilibrium of forces acting on the particle at plastic deformation, we obtained relationships approximately describing the change of its shape, forces at the boundaries of the particle contact with the roller and the substrate during the time of compression. When the substrate characteristics are known, these relationships allow evaluating the indentation of the particle material and the adhesive strength of the compound being formed. We also note that this approach is applicable, in particular, for determining the loads acting on the propulsive agents (wheels, rollers) of vehicles when they run-over a strongly deformable single obstacle.

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