

## **A mathematical modelling of frost growth dynamics on a surface of fin-and-tube air cooler**

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### **ABSTRACT**

A mathematical model was developed to predict the performance characteristics of fin-and-tube air coolers with plane fins, taking into account the temperature and humidity conditions, non-uniform thickness of the frost layer along the heat exchanger core, flow-pressure characteristics of the fans and the refrigerant superheat zone. The model was verified on the experimental data of air cooler performance in frosting conditions obtained by the authors in air temperature range from minus 6 to minus 20 °C, and experimental data from independent researchers. The presented model predicts the cooling capacity and the overall heat transfer coefficient in the evaporator with error not exceeded 15%. It was found that during dry operating conditions of the air cooler, the refrigerant boiling temperature descent by 10 °C leads to a decrease in the overall heat transfer coefficient by 35%. The main reason of that is double decrease of the refrigerant mass velocity and the refrigerant-side heat transfer coefficient by 75%. The process of frost grow leads to an increase in overall thermal resistance in the studied air cooler by 30%, more than double increase in the aerodynamic resistance and decrease in cooling capacity by 15%.

**Keywords:** Frost, Fin-And-Tube Heat Exchanger, Air Cooler, Evaporator, Cooling Capacity.

### **INTRODUCTION**

Currently, refrigeration is one of the most common ways to preserve food products. The operation of air coolers in refrigerators is accompanied by an undesirable frost growth process. The process of frost growth leads to a decrease in the overall heat transfer coefficient by 25-35% and an increase in the aerodynamic resistance of air coolers by 2.5-3 times (Deng, 2003), and the periodic defrost leads to increase of energy consumption by 10-25% (Liu, 2014). Tests and selection of existing air coolers according to the EN328 (2014) standard completed in dry conditions do not allow to evaluate air coolers performance under conditions of frost growth (Machielsen, 1989). A significant number of factors affect the characteristics of air coolers operating in conditions of frost growth: geometric parameters of the fin surface, installation location, temperature and humidity in the refrigerator

(Patil, 2017), which necessitates the development of mathematical models for calculation of air coolers performance.

One of the first numerical models for calculating the performance characteristics of an air cooler under conditions of frost growth was presented in (Sanders, 1974). The author of the model used the assumption of the equal thickness of frost layer on the surface of the fins and tubes. The author notes the need for both the development of more accurate methods for calculating the dynamics of frost thickness growth, as well as experimental studies of the operation of the air cooler for testing and further improving of the model he proposed. Later, in (Silva, 2011), based on experimental studies, it was found that thicker frost layer forms on the tubes of the apparatus than on the surface of the fins, and the main decrease in cooling capacity is associated with the frost layer blockage of the cross section of the air cooler. With a decrease in the pitch of the fins, a thicker and denser frost layer is formed.

An analysis of the accuracy of numerical models of frost layer growth performed in (Breque, 2016) shows that simple semi-empirical models demonstrate good accuracy when using empirical correlations in the permissible temperature range, and complication of models (for example, taking into account the variation of a frost density over the layer) in some cases leads to a decrease in the accuracy of calculations. Many of the existing models for calculating the frost layer growth dynamics cannot be used at low air temperatures.

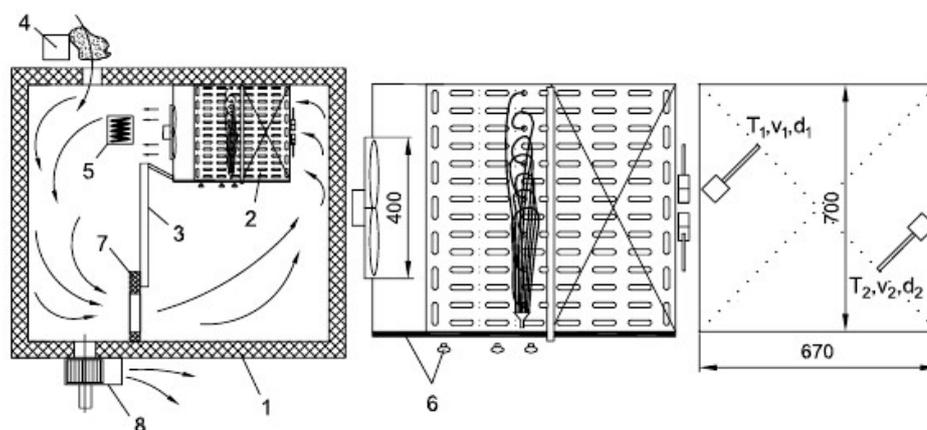
In (Verma, 2002), a team of authors developed a mathematical model of an air cooler with a single-phase liquid coolant. At positive air temperatures above + 5 °C, the (Hayashi, 1977) correlation was used to calculate the density of the frost layer. Due to the limited possibilities of applying this correlation at near-zero and low air temperatures, the frost density in the model was assumed by the authors to be constant and equal to 130 kg / m<sup>3</sup>. The presented model made it possible to achieve a 15% reduction in the energy consumption of the prototype air cooler due to an increase in the heat transfer coefficient through the rational selection of the fin spacing, uniform growth of the frost layer along the core of the air cooler and a decrease in the dynamics of pressure drop growth. The authors note the possibility of approximately doubling the defrosting frequency by using fans with adjustable speed regulator and maintaining a constant volumetric air flow rate through the air cooler core.

Despite significant progress in this area in recent decades, the development of a mathematical model of the air cooler with plate fins and direct boiling of a refrigerant remains highly relevant. When developing the model, the following tasks were taken into account: it should be simple enough and allow the calculating of the air coolers performance characteristics under the conditions of frost growth in a wide air temperature and humidity range for the most common refrigerants (R22, R134a, R404a, R507, R717) based on the value of the coil inlet temperature difference or volumetric displacement of the refrigerant compressor.

## MAIN SECTION

### 1.1 Experiments

In order to verify the developed model, the authors created an experimental apparatus (Fig. 1). In the existing refrigerating chamber 1, equipped with a low-temperature dual-circuit industrial air cooler with direct boiling 2, aerodynamic partitions 3 were installed to ensure air circulation in a closed circuit. The process of frost growth occurred in the second circuit along the air cooler core. The technical characteristics of the air cooler are presented in Table 1. The air in the chamber was humidified by supplying room temperature air from outside of the chamber with a blower 8. It was additionally humidified by an ultrasonic steam generator 4. The air temperature was maintained constant by an electric heater 5 with a maximum power of 4 kW and a PID PWM temperature controller. As a result of their joint work, the air temperature at the inlet to the air cooler could be maintained with an error of not more than  $\pm 0.5$  °C, and air humidity in the range from 40 to 100 percent with an error of up to  $\pm 5\%$ .



**Figure 1.** Experimental setup and the geometric characteristics of the studied air cooler

To observe and measure the frost layer growth, a double glass and video cameras 6 were installed instead of the air cooler tray. Also LED lamps were installed to provide illumination in the internal volume of the air cooler. In a number of experiments, a calibrated restriction 7 was installed to provide a different flow-pressure characteristic of the fan (Table 2).

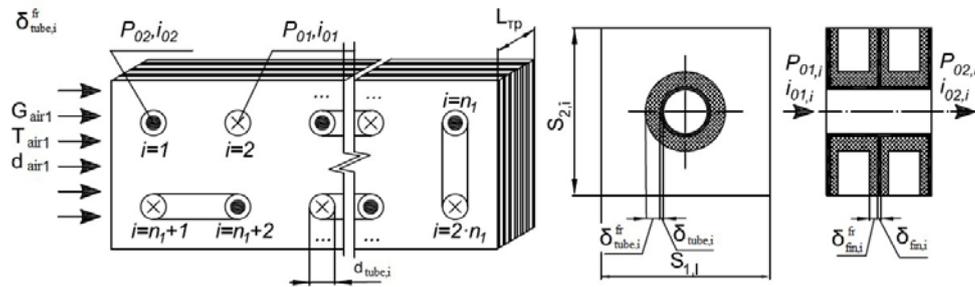
Heat transfer surface area of one circuit, m <sup>2</sup>	33,9
Fin pitch, mm	10
Fin thickness, mm	0,5 (Al)
Longitude and transparent tube spacing, mm	50
Tube diameter and wall thickness, mm	15,5x0,5 (Cu)
Number of rows in height and depth of the circuit (whole apparatus)	14x7 (14x14)

**Table 1.** Technical characteristics of the air cooler

According to the calculation results, the measurement error of the volumetric air flow in the experiments did not exceed 8%, the cooling capacity and the heat transfer coefficient measurement error did not exceed 12% and the frost layer thickness measurement error did not exceed  $\pm 0.25$  mm.

### 1.2 A mathematical model

The studies showed that the distribution of frost was uniform over the width of the air cooler, and as a finite element of the apparatus a separate finned tube equal in length to the width of the air cooler was taken. Moreover, the growth pattern of frost is significantly affected by both the flow directions in the apparatus and the superheating zone of the refrigerant, which accounts for 8 to 25% of the overall heat transfer surface area. The scheme of the air cooler section division into finite elements is shown in Figure 2.



**Figure 2.** The scheme of the air cooler section division into finite elements

The calculation of the heat flux through the outlet tube surface of the finite element can be performed by the correlation:

$$q_{out.tube,i} = \frac{[T_{air,i} - T_{0,i}]}{\frac{1}{\alpha_{fet.air,i}} + \frac{d_{tube,i}}{2\lambda_{tube}} \ln\left(\frac{d_{tube,i}}{d_{tube,i}^{inn}}\right) + \frac{1}{\alpha_{0,i}} \frac{d_{tube,i}}{d_{tube,i}^{inn}}}$$

Eq (1)

Where:  $T_{air,i}$  and  $T_{0,i}$  are the average air temperature and refrigerant temperature, K;  $\alpha_{0,i}$  is the refrigerant-side heat transfer coefficient by (Wattelet, 1994) correlation,  $W \times m^{-2} \times K^{-1}$ ;  $\alpha_{fet.air,i}$  is air side heat transfer coefficient fetched to the outer surface of the tubes,  $W \times m^{-2} \times K^{-1}$ :

$$\alpha_{fet.air,i} = \frac{1}{\pi * d_{tube,i}} * \left( \frac{\alpha_{air,i} \xi_{fr,tube,i} f_{fr,tube,i}}{1 + \alpha_{air,i} \xi_{fr,tube,i} \frac{d_{fr,tube,i}}{2\lambda_{fr,tube,i}} \ln\left(\frac{d_{fr,tube,i}}{d_{tube,i}}\right)} + \frac{1}{1 + \alpha_{air,i} \xi_{fr,fin,i} \frac{\delta_{fr,fin,i}}{\lambda_{fr,fin,i}}} + \frac{R_{collar}}{\alpha_{air,i} \xi_{fr,fin,i} E_{fin,i} f_{fr,fin,i} + f_{collar}} \right)$$

Eq (2)

This coefficient takes into account:  $\alpha_{air,i}$  – the air-side heat transfer coefficient by (Kim, 1998),  $W \times m^{-2} \times K^{-1}$ ; thermal resistance of the frost layer on the fins  $\delta_{fr,fin,i} / \lambda_{fr,fin,i}$  and outlet surface of the tubes  $\delta_{fr,tube,i} / \lambda_{fr,tube,i}$ ,  $W^{-1} \times m^2 \times K$ ; thermal efficiency of the fins  $E_{fin,i}$ ; fins collars contact resistance  $R_{collar} = 1.4 \times 10^{-5}$

$W^{-1} \times m^2 \times K$  according to (Kozhevnikova, 2013), and the change of the heat transfer surface area by frost layer growth. More detailed information on the calculation of temperatures, fins area and heat fluxes values is presented in (Kanavets, 1979).

To calculate the thermal conductivity of the frost layer, the obtained earlier by the authors (Marinyuk, 2017) correlation for frost density less than  $350 \text{ kg/m}^3$  was used:

$$\lambda_{fr} = 0.00285 * \exp(0.0196 * T_{fr} - 2.41) [1 + 0.0134 * \rho_{fr}] \quad \text{Eq (3)}$$

Where:  $T_{fr}$  - average frost temperature, K;  $\rho_{fr}$  - frost layer density,  $\text{kg} \times \text{m}^{-3}$ . For frost layer thickness calculation the semi-empirical correlation proposed by prof. Marinyuk B.T. based on the results of an approximate analytical solution of the Stefan problem was used:

$$\delta_{fr}(\tau) = \sqrt[3]{\frac{6.34 * 10^{-6} * \alpha_{air,i} (d_{air,i} - d_{surf,i}^{fr}(\tau))^2 * \tau^2 (T_{surf,i}^{fr}(\tau) - T_{wall,i})}{Cp_{air,i} [L (d_{air,i} - d_{surf,i}^{fr}(\tau)) + 2Cp_{air} (T_{air,i} - T_{surf,i}^{fr}(\tau))]}]} \quad \text{Eq (4)}$$

Where:  $d_{air,i}$  - absolute moisture content of air, kg/kg;  $Cp_{air}$  – heat capacity of air,  $\text{J} \times \text{kg}^{-1} \times \text{K}^{-1}$ .

The increase of the frost mass on the fin over a period of time  $\Delta\tau$  (to calculate the thickness and mass of frost on the outer surface of the tubes, the corresponding values of their surface temperature should be used) and the frost layer density on the surface of the fins of the finite element:

$$dm_{fin,i}^{fr} = \Delta\tau * q_{fin,i}^{base} \frac{f_{fin,i}^{base} \xi_{fin,i}^{fr} - 1}{f_{fin,i} \xi_{fin,i}^{fr} * L} \quad \text{Eq (5)}$$

$$\rho_{fr,fin,i}(\tau) = \frac{m_{fr,fin,i}(\tau)}{\delta_{fr,fin,i}(\tau)} \quad \text{Eq (6)}$$

Air flow rate at each time step is calculated based on the specified flow-pressure characteristics of the fan, and the total aerodynamic resistance of the air cooler computed as the sum of the resistances of the finite elements by the (Gogolin, 1962) correlation:

$$\Delta P_{air} = \frac{1}{2} \sum_{i=1}^{2n_1} \Delta P_i$$

Eq (7)

$$\Delta P_i = 0.115 * \frac{S_{1,i}}{d_{eff,i}} * M_{air,i}^{1.7}$$

Eq (8)

Where:  $S_{1,i}$  is the longitudinal tubes pitch in the apparatus, m;  $d_{eff,i}$  is the effective diameter of the channel, taking into account the thickness of the frost layer, m;  $M_{air,i}$  is the mass air velocity in the channel, taking into account the frost layer,  $kg \times m^{-2} \times s^{-1}$ .

### 1.3 Results and discussion

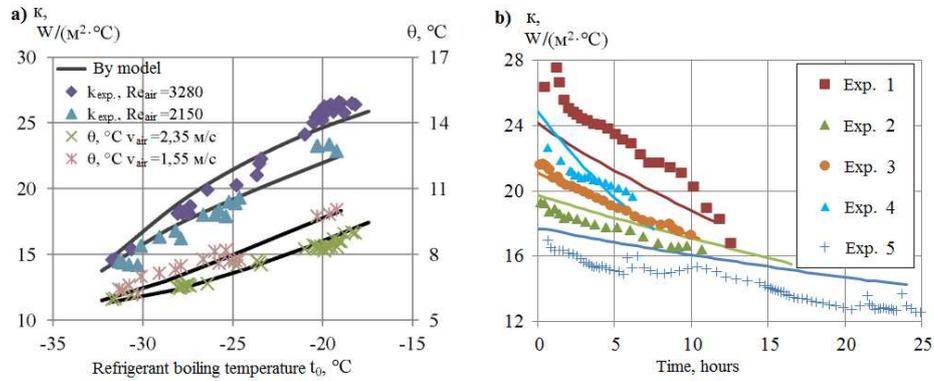
An analysis of the experimental data obtained under “dry” conditions showed that with a refrigerant boiling temperature descent from minus 20 to minus 30 ° C, there is a decrease in the heat transfer coefficient by more than 35% and the temperature difference by 30-40% (Fig. . 3, a). The reason for this is a decrease in the volumetric flow rate of the refrigeration compressor by 32%, a decrease in the mass velocity of the refrigerant by 2 times, and as a consequence the descent of refrigerant-side heat transfer coefficient by 75%. Thus, in the selection and design of air coolers, an important parameter is to ensure a high mass velocity of the refrigerant by selecting the number and length of the apparatus circuits. This is of particular importance in refrigeration systems with frequency and mechanical refrigerant compressors displacement regulators.

№ exp.	Air inlet temperature, t °C	Air inlet relative humidity, φ%	Test duration τ, hr	Air restrictor
1	-12,2	86	12	no
2	-18	98,7	11	no
3	-13,3	95	11	yes
4	-6,2	87,2	7	yes
5	-19	87	30	yes

**Table 2.** Temperature and humidity conditions of the experiments

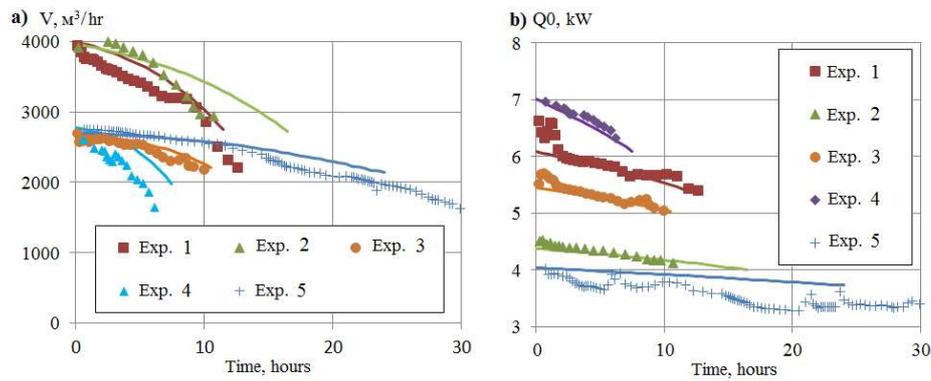
On figures (3, b) and (4, a-b) a comparison of experimental data (overall heat transfer coefficient, air volumetric flow rate and cooling capacity) obtained by the author (points) and calculation results (curves) by the mathematical model for various temperature and humidity conditions is shown. The temperature and humidity conditions of experiments are presented in table 2. The results of calculations according to the model are in good agreement with experimental data.

**Figure 3.**



a) the effect of the refrigerant boiling temperature on the overall heat transfer coefficient and coil temperature difference in a low-temperature air cooler in dry conditions;

b) the influence of temperature and humidity conditions on the overall heat transfer coefficient in the air cooler



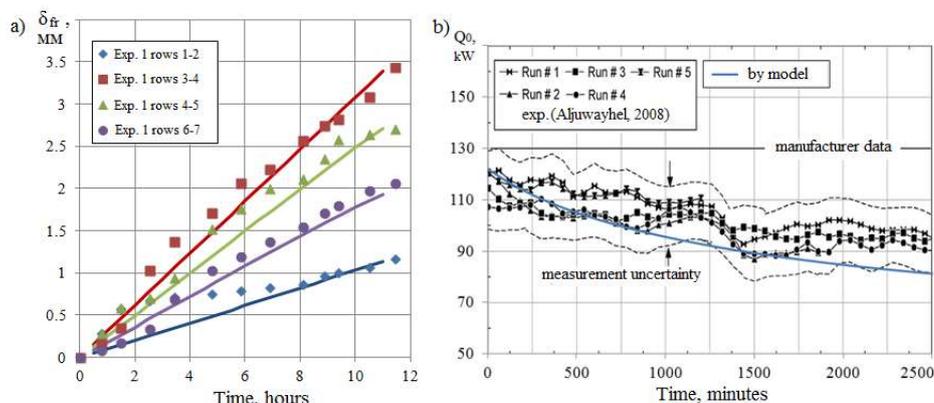
**Figure 4.**

a) the influence of air temperature and humidity on air flow rate descent in the air cooler;

b) the influence of air temperature and humidity on the cooling capacity of the air cooler

Four zones of the air cooler ( the first row (superheating zone), the second row, the middle of the apparatus (3rd and 4th row), the exit from the apparatus (7 row)) were selected to compare the calculation results of frost layer thickness distribution along the air cooler core, by presented model with experimental data (figure 5, a). It was found that low relative air humidity contributes to the most intensive frost layer grow at the outlet of the apparatus.

In order to extra verification of the developed mathematical model, the cooling capacity calculation results were compared with experimental data (Aljuwayhel, 2008), obtained during operation of an industrial low-temperature air cooler installed in an ice cream storage chamber (Figure 5, b). A feature of this refrigeration unit was the use of ammonia as a refrigerant and a CPR-fed liquid overfeed scheme with a constant refrigerant boiling temperature. During 42 hours of operation, the relative humidity in the refrigerator was reduced from 90 to 88%. For almost the entire duration of the experiment, the deviation of the calculated cooling capacity from the experimental data does not exceeded the measurement error. The prediction of the air velocity at the inlet to the apparatus error at the end of the experiment does not exceed 15%.



**Figure 5.**

a) Comparison of the calculation results of the frost layer thickness distribution on the tubes of the air cooler core with experimental data;

b) Comparison of model cooling capacity prediction results with experimental data (Aljuwayhel, 2008)

### CONCLUSIONS

Experimental studies of the air cooler operation at air temperatures from minus 6 to minus 20 ° C, relative humidity from 40 to 100% and various flow-pressure characteristics of the fans were carried out and it was found that:

- in “dry” operating conditions of the air cooler with a refrigerant boiling temperature lowering from minus 20 to minus 30 ° C, the overall heat transfer coefficient decreases by more than 35%. The reason of this is a double decrease in the mass

velocity of the refrigerant in the channels of the heat exchanger and a corresponding decrease in refrigerant-side heat transfer coefficient by 75%.

- the process of frost deposition in the investigated apparatus leads to an increase in thermal resistance by 30%, aerodynamic resistance of the apparatus by more than 2 times, as well as a decrease in cooling capacity by 15%;
- with a decrease in air temperature from minus 6 to minus 20 ° C, the frost growth dynamics decreases by more than 2 times;

A mathematical model of the air cooler taking into account frost formation has been developed, which predicts the variation of cooling capacity, air volume flow, coil temperature difference and overall heat transfer coefficient in the apparatus with an error not exceeding 15% in a wide air humidity and temperature range.

A comparison of the calculation results by presented mathematical model of the air cooler with experimental data confirms its adequacy and the admissibility of its use in design and verification calculations of heat exchangers in frost conditions, as well as for the development of technological recommendations for the operation of refrigerators.

### NOMENCLATURE

$G$	mass rate (kg/s)	$\xi$	wet enhancement coefficient
$T$	temperature (K)	$V$	volumetric air flow ( $\text{m}^3 \times \text{h}^{-1}$ )
$k$	overall heat transfer coefficient ( $\text{W} \times \text{m}^{-2} \times \text{K}^{-1}$ )	$\delta$	thickness, мм
$\alpha$	heat transfer coefficient ( $\text{W} \times \text{m}^{-2} \times \text{K}^{-1}$ )	$\lambda$	conductivity ( $\text{W} \times \text{m}^{-1} \times \text{K}^{-1}$ )
$R$	heat resistance ( $\text{W}^{-1} \times \text{m} \times \text{K}$ )	$d$	diameter, мм
$Q_0$	cooling capacity (W)	$\tau$	time, с
$P$	pressure (Pa)	$q$	specific heat flux ( $\text{W} \times \text{m}^{-2}$ )
$F$	surface area ( $\text{m}^2$ )	$f$	specific surface area ( $\text{m}^2/\text{m}$ )
$L$	энтальпия сублимации водяных паров $L=2834(\text{kJ} \times \text{kg}^{-1})$	$m$	mass (kg)

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