Mathematical modeling of non-steady heat exchange process

in cryogenic coil-wound heat exchangers

***Aleksandr A. Vorob'ev^(a), Dmitriy P. Posanchukov^(a),
Aleksandr A. Kozlov^(a), Aleksey V. Ivanov^(a)***

^(a) Scientific Center of Air Forces "Air Force Academy named after Professor N. E. Zhukovsky and Yu. A. Gagarin"
Voronezh, 394064, Russia, vaiu@mil.ru

ABSTRACT

The paper discusses a dynamic model of coil-wound heat exchanger and its implementation in the MathWorks Simulink™ computer simulation system. As a simulation object was chosen a coil-wound heat exchanger with wire-finned tubes of a commercial low-capacity air separation unit. The methods for obtaining experimental data has been described, the non-steady heat exchange process has been simulated, and the obtained results have been analyzed.

Keywords: heat transfer, mathematical model, coil-wound heat exchanger, air separation unit

INTRODUCTION

Currently, the problem of cryogenic heat exchanger optimization is directly associated with the complexity of developing dynamic models that adequately reflect the nature of thermal processes, over a wide range of changing input parameters and high speed of transition processes inherent in low-capacity cryogenic air separation units. The problem is complicated by the need to account for the influence of change in the thermophysical properties of the flows along the channel on the heat emission, particularly when the heat exchangers operate at the parameters close to the critical state [1, 2]. We should also note that the applicability of the model in many respects depends on the universality of the physical model, the validity of the assumptions, and the methods of calculating the geometric parameters required to determine the dimensions of the heat-transfer surfaces and the hydrodynamic characteristics of the flows.

COIL-WOUND HEAT EXCHANGER DYNAMIC MODEL

Physical model

Modern low-capacity cryogenic air separation units use two- and three-flow coil-wound heat exchangers, which consist of a cylindrical body that accommodates a core with several layers of wire-finned pipes coiled around it. Through the pipes, in forward direction, an airstream flows and cools at a pressure of up to 20 MPa; in opposite direction, along the intertubular space, cooling waste gas moves at a pressure of up to 0.07 MPa. If the heat exchanger is a three-flow

one, the wire-finned pipes also form an additional section, through which the product nitrogen or oxygen counterflows the forward flow when the air separation unit operates in gas conditions. The small inner diameter of the pipes and high pressure allow describing the flows inside them by the ideal displacement model, and the nature of motion and heat emission in such channels are well understood [3].

The nature of the flow in the intertubular space is more complex; however, as in the first case, simplified models are used to describe it. Fig. 1 shows a diagram of a two-flow heat exchanger, in which k pipes with outside diameter d_2 (m) are coiled onto a core with diameter of D_1 (m). In order to evenly distribute the flow through the heat exchanger pipes, they are coiled with preferably the same length l (m). Each of k pipes can be represented as a spring with pitch h (m) coiled onto the core along the mean diameter of the heat-exchanging apparatus, determined by the relation

$$D_{mean} = \frac{D + D_1}{2}, \quad \text{Eq. (1)}$$

where D is the outside diameter of the heat exchanger coiling in m.

Figure 1. Diagram of splitting the coil-wound heat exchanging apparatus into sections

Assuming that flowing through the intertubular space, along the longitudinal axis of the heat exchanger, the waste gas sequentially contacts the pipe surface of each loop in the i -th ($i=1,2 \dots n$) section, and the pipe loops are evenly distributed throughout its volume, the return flow temperature can be considered the same at the moment of contact with the entire surface of the pipe of one loop in each zone. On this assumption, the waste gas flow can be described by the ideal mixing model, and the length of the section will be determined by the coiling h (m). Figure 2 displays a diagram of a heat exchanger section and a diagram of a pipe finning, and table 1 offers expressions for finding the basic geometric parameters that characterize the heat exchanger sections.

Figure 2. Section of a coil-wound heat exchanger with wire-finned pipes
a) heat exchanger element; b) pipe wire-finishing diagram

Parameter	Expression
Total area occupied by pipes in cross-section of heat exchanger F in m	$F = 2k\tau_1\tau_2$
Width of section h in m	$h = \frac{F}{D - D_1}$
Mean loop diameter d_{loop}^{mean} in m	$d_{loop}^{mean} = \sqrt{D_{mean}^2 + (h/2)^2} =$ $= 1/2 \sqrt{2D_{mean}^2 + h^2}$
Mean loop length l_{loop}^{mean} in m	$l_{loop}^{mean} = \pi d_{loop}^{mean}$
Number of partitioning sections n	$n = H/h$
Mean pipe length in heat exchanger l in m	$l = l_{loop}^{mean} \cdot n$
Mean diameter of loop of wire coiled on pipe d_{coil}^{mean} in m	$d_{coil}^{mean} = \sqrt{\left(d_2 + \frac{d_w}{2}\right)^2 + \left(\frac{t_f}{2}\right)^2}$ $= \frac{1}{2} \sqrt{(2d_2 + d_w)^2 + t_f^2}$
Length of pipe fin wire in section l_w in m	$l_w = \pi d_{coil}^{mean} l_{loop}^{mean} / t_w$
Area of outside surface of wire-finned pipe, in section F_o in m ²	$F_o = F_o^p + F_o^s - 2F_{tr} = \pi d_2 l_{loop}^{mean} +$ $+ \pi d_w l_w - 2h_{tr} l_w$
Total area of heat exchange surface in section F_{tot} in m ²	$F_{tot} = kF_o$

Volume occupied by finned pipe in section $V_{f.p.}$ in m^3	$V_{f.p.} = V_p + V_w = l_{loop}^{mean} \frac{\pi d_2^2}{4} + l_w \frac{\pi d_w^2}{4}$
Total volume occupied by finned pipes of heat exchanger in section $V_{f.p.}^{tot}$ in m^3	$V_{f.p.}^{tot} = k V_{f.p.}$
Total volume of section V_{tot} in m^3	$V_{tot} = \pi h / 4 (D^2 - D_1^2)$
Volume of intertubular space in section V_{int} in m^3	$V_{ins} = V_{tot} - V_{f.p.}^{tot}$
Equivalent diameter for intertubular space $d_{2,e}$ in m	$d_{2,e} = 4V_{ins} / F_{tot}$

Table 1. Formulae for calculation of the basic geometric parameters of the heat exchanger

Mathematical model

Taking into account the assumed model, the equation that describes the change of the air flow temperature in the tube side of the i -th section of the heat exchanger will have the form

$$\frac{\partial T_1^{(i)}(l, t)}{\partial t} = -\frac{4G_1}{\pi k d_1^2 \rho_1^{(i)}} \frac{\partial T_1^{(i)}(l, t)}{\partial l} - \frac{4K_1^{(i)}}{d_1 \rho_1^{(i)} c_{p,1}^{(i)}} (T_1^{(i)}(l, t) - T_{wall}^{(i)}(l, t)), \quad \text{Eq. (2)}$$

where G_1 – mass air flow in $kg \times s^{-1}$; $T_1^{(i)}(l, t)$ – air temperature in K; $T_{wall}^{(i)}(l, t)$ – wall center temperature in K; $\rho_1^{(i)}$ – air density in $kg \times m^{-3}$; $c_{p,1}^{(i)}$ – air specific heat in $J \times kg^{-1} \times K^{-1}$; $K_1^{(i)}$ – coefficient of heat transfer from the air flow to the center of the pipe wall in $W \times m^{-2} \times K^{-1}$.

For the waste gas flow, taking into account the ideal mixing for the i -th section of the heat exchanger, we can write

$$\frac{dT_2^{(i)}(t)}{dt} = \frac{G_2}{V_{ins} \rho_2^{(i)}} (T_{2,in}^{(i)}(t) - T_2^{(i)}(t)) + \frac{K_2^{(i)} F_{tot}}{V_{ins} \rho_2^{(i)} c_{p,2}^{(i)}} (T_{wall,mean}^{(i)}(t) - T_2^{(i)}(t)), \quad \text{Eq. (3)}$$

where $\rho_2^{(i)}$ – waste gas density in $kg \times m^{-3}$; $c_{p,2}^{(i)}$ – waste gas specific heat in $J \times kg^{-1} \times K^{-1}$; $T_2^{(i)}(t)$ – waste gas flow temperature in K; $K_2^{(i)}$ – coefficient of heat transfer from the pipe center to the waste gas in $W \times m^{-2} \times K^{-1}$; $T_{wall,mean}^{(i)}(t)$ – mean temperature of the wall center along the length of the pipe in K, determined by the relation

$$T_{wall,mean}^{(i)}(t) = \frac{1}{l_{loop}^{mean}} \int_0^L T_{wall}^{(i)}(l, t) dl, \quad \text{Eq. (4)}$$

The heat transfer coefficients K_1 and K_2 allow for the thermal resistance of the wall, and are determined by the relations

$$K_1 = \frac{1}{\frac{1}{\alpha_1} + \frac{d_1}{2\lambda_{wall}} \ln \frac{d_{mean}}{d_1}}, \quad K_2 = \frac{1}{\frac{d_2}{2\lambda_{wall}} \ln \frac{d_2}{d_{mean}} + \frac{1}{\alpha_2}}, \quad \text{Eq. (5)}$$

where α_1 and α_2 – coefficients of heat transfer from the air flow to the inside surface of the all and from the outside surface to the waste gas, respectively, in $\text{W} \times \text{m}^2 \times \text{K}^{-1}$; λ_{wall} – heat conductivity of the wall in $\text{W} \times \text{m}^{-1} \times \text{K}^{-1}$; d_{mean} – diameter that determines the position of the wall center in m.

The change of the wall element enthalpy over time is equal to the sum of heat flows transferred from the air flowing inside the pipe to the wall center, and from the wall center to the waste gas; hence, we can write

$$F_{wall} \rho_{wall} \Delta c_{wall} \frac{\partial T_{wall}^{(i)}(l, t)}{\partial t} = K_1^{(i)} F_1 (T_1^{(i)}(l, t) - T_{wall}^{(i)}(l, t)) - K_2^{(i)} F_2 (T_{wall}^{(i)}(l, t) - T_2^{(i)}(t)), \quad \text{Eq. (6)}$$

where F_{wall} – area of the wall cross-section in m^2 ; ρ_{wall} – wall material density in $\text{kg} \times \text{m}^{-3}$;

c_{mean} – wall material specific heat in $\text{J} \times \text{kg}^{-1} \times \text{K}^{-1}$; F_2 – area of heat exchange between the waste gas and the high-pressure pipe wall at the element section Δl in m^2 .

The expression for calculation of the pipe wall cross-section, subject to wire finning, has the form

$$F_{wall} = F_p + F_w = \frac{\pi(d_2^2 - d_1^2)}{4} + \frac{\pi d_w^2}{4} \frac{l_w}{l_{loop}^{mean}}, \quad \text{Eq. (7)}$$

The area of heat exchange between the waste gas and the wall F_2 is determined by the relation

$$F_2 = \frac{F_{tot}}{l_{loop}^{mean}}, \quad \text{Eq. (8)}$$

Equations (2), (3) and (6) are a closed system, which allows describing the non-steady process of heat exchange in the coil-wound heat exchanger with wire-finned pipes.

The boundary and initial conditions for the system of equations (2), (3) and (6) will have the form

$$T_1^{(1)}(0, t) = T_{1,in}(t); \quad T_1^{(i)}(0, t) = T_1^{(i-1)}(L, t); \quad i=2, 3 \dots n; \quad \text{Eq. (9)}$$

$$T_1^{(i)}(l, 0) = T_{1,0}^{(i)}(l); \quad T_2^{(i)}(0) = T_{2,0}^{(i)}; \quad T_{wall}^{(i)}(l, 0) = T_{wall,0}^{(i)}(l); \quad i=1, 2 \dots n, \quad \text{Eq. (10)}$$

For solution of the system of equations (2), (3), (6), a model has been developed in MathWorks SimulinkTM computer simulation system, based on the

methods described in [4], and consisting in discretization of the spatial variable by the finite difference method, with replacement of the spatial derivative by the matrix equations. The use of "Repeating Sequence Interpolated" blocks in this model allowed specifying, in the form of arrays, the profiles of change of the temperatures of the air and waste gas flows at the main heat exchanger inlet, as well as the dependences of the change of the waste gas flow through the heat exchanger and the pressure of the forward and return flows, simulating the real change of the said parameters. The thermophysical properties of the waste gas in the sections, and the air along the pipe of the heat exchange apparatus, are calculated at the current temperatures of the flows $T_1^{(i)}(l,t)$ and $T_2^{(i)}(t)$ by the interpolation dependences obtained from the experimental data [5-7].

Methods of obtaining experimental data and comparing them with the simulation results

The experimental data on the dynamics of the temperature change in the forward and return air flows in the coil-wound heat exchanger were obtained on the main heat exchanger of commercial TKDC-100B air separation unit, which provided the real operating conditions and reduced the influence of the ambient heat inflow thanks to the thermal insulation. Fig. 3 displays a simplified diagram of the air cooling unit.

Figure 3. Simplified process flow diagram of the cooling and separation units of TKDS-100V

Ex – expander, HE 1 – main heat exchanger, HE 2 – expander-type heat exchanger, S – subcooler, V – vessel, RC 1 and RC 2 – rectification columns 1 and 2, CV – control valve, SV-4 – shutoff valve, TE – platinum resistance thermometer, FE – measuring diaphragm, PG-2 and PG-6 – pressure gages

For operating control, platinum resistance thermometers C 416 by Heraeus were additionally installed (points T-10 and T-11) that output the measurement information to the control measuring device ITP 2521/2. And the temperature of the waste gas at the main heat exchanger inlet, when valve 3-4 is open, was

determined by sensor T-10, and when the valve is closed – by the formula $T_{10}=T_{11}+\Delta T$, where $\Delta T=2,6$ K was found experimentally after switching the expander off and starting the steady mode. The air and waste gas flow rates were determined using the measuring diaphragm installed on the line for discharging the waste gas into the atmosphere. The flow rate through valve 3-4 was calculated based on the thermal balance conditions after the mixing of the flows. During preparation of the unit for the experiment, the heat exchange apparatuses, pipelines and rectification columns were cooled using the expander down to the temperature of 210...250 K. The operating mode was not started in order to prevent the influence of the change in the waste gas flow rate due to its accumulation in a liquid state in the separating units. After the cooling, the expander was switched off, which allowed performing the required adjustment of the control equipment and switching the unit to the steady operating mode that entails insignificant changes in the flow temperatures over time due to cold generation in a simple throttling cycle. During the unit's operation for 2500 s, the following situations were simulated: at the 241st second, the flow rate of the waste gas through the heat exchanger was changed by closing valve 3-4; at the 628th second, the pressure of the forward flow was decreased from 19.5 MPa to 14 MPa; at the 1552nd second, the pressure of the waste gas was decreased from 0.16 MPa to 0.13 MPa. The comparison of the experimental and calculated data is presented in figure 4.

Air temperature
(experimental
values)

Waste gas
temperature
(experimental
values)

Figure 4. Non-steady heat exchange process simulation results

The graph analysis demonstrates the correspondence between the calculated and experimental values; the deviation of the heat exchanger outlet air temperature does not exceed 4.4%, and the deviation of the heat exchanger outlet waste gas temperature does not exceed 3.7%. The duration of the transition processes, when perturbing the waste gas flow rate at the 241st s and the forward flow pressure at the 1552nd s, as well as the curve shapes, corresponds to the results obtained in the experiment. The pressure change in the intertubular space of the heat exchanger hardly affected the temperature change of both flows, which is consistent with the calculated values and is confirmed by the experimental data. We should note that the deviation between the simulation results and the experimental values for the heat exchanger outlet waste gas temperature, in the interval from the 0th to the 241st s, is caused by the fact that, during the experiment, the heat exchanger outlet waste gas temperature was determined by a sensor at point T-8 in accordance with fig. 3, and, accordingly, within the specified time interval, since shutoff valve 3-4 was fully open, sensor T-8 recorded the gas temperature after mixing of the flows from the main heat exchanger and the expander-type heat exchanger.

CONCLUSIONS

A method of section-wise calculation of a coil-wound heat exchange apparatus is proposed, which allows taking into account the change in the thermophysical properties of flows through a quite simple mathematical tool. Expressions are obtained for determining the basic geometric characteristics of sections of a coil-wound heat exchanger with wire-finned pipes, and a system of equations that describes the non-steady heat exchange process. The comparison of the simulation results against the experimental data obtained from the main heat exchanger of the cooler of the commercial air separation unit, under time-varying input parameters, allows concluding that there is a qualitative conformance between the simulation results and the experiment, and that it is possible to use the proposed model to study dynamic modes and optimize the design of cryogenic coil-wound heat exchangers.

NOMENCLATURE

τ_1	transverse spacing of heat exchanger coiling (m)	τ_2	longitudinal spacing of heat exchanger coiling (m)
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H	heat-exchange coiling length (m)	apparatus	d_w	wire diameter (m)
t_f	fin spacing (m)		F_o^p	area of external surface of one pipe in a loop without finning (m ²)
F_o^s	surface area of fin wire in section (m ²)		F_{tr}	total area of contact of pipe with fin wire in section (m ²)
h_{tr}	trace width at the point of contact of pipe with fin wire, m		V_p	volume occupied by one pipe without finning in section (m ³)
V_w	volume occupied by the pipe's fin wire in section (m ³)			

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