

The modelling of the measuring point reproduction error for optical coordinate measuring machines

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Abstract. Coordinate measuring machines (CMMs) are the most common measuring systems used for assessing the compliance of manufactured components with their geometrical specifications. Optical CMMs are currently gaining popularity as they enable fast and contactless measurement with accuracy almost as high as in case of tactile measurement. During the performing of measurement, the points at which the measurement is conducted are usually not exactly at the intended location, but due to different impacts (error sources), they are in the area surrounding this location. This paper presents a method for modelling the reproduction of point coordinates as probability ellipses geometrically limiting the area in which the actual measurement points may lie. Different mathematical algorithms for the description of these ellipses were examined and a discussion on the selection of the best method is presented in this paper.

1 Introduction

Multisensory machines combine the advantages of different coordinate measuring systems, such as tactile coordinate measuring machines (CMMs) and coordinate microscopes. Therefore, it should come as no surprise that system of this type are gaining popularity both among industry enterprises and research laboratories [1-3]. Research laboratories in particular are interested in the accuracy assessment of measurements performed using multisensory machines. When operating in tactile mode, their accuracy can generally be described in a similar manner to classic CMMs, especially in the case of geometrical error identification and modelling. The real novelty lies in the assessment and modelling of the accuracy of optical measurements and measurements performed as a combination of both modes. One of the major challenges in the field of optical measurements is uncertainty estimation [3]. The requirements of the fourth industrial revolution demands quick and versatile methodology, which should additionally allow automation of the whole process. These requirements are fulfilled by simulative methods of measurement uncertainty estimation, among other methods using the methodology of a so-

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called “virtual machine” which enables multiple simulation of the measurement process [4]. Uncertainty is determined on the basis of the obtained simulated results using simple statistics. The great advantage of such a solution is that the same method can be used for completely different measurement tasks which can be found in GD&T standards because the virtual machine simulates the measurement of each point included in the inspection routine. The biggest difficulty with the application of the virtual machine on a real measuring system is a necessity of the experimental identification of the influence of major errors sources on the measurement result.

The first virtual machine for classic tactile CMM was developed in PTB. It had two main modules responsible for modelling the kinematic errors of the machine and the errors of the probing system. Such a structure was adapted using various solutions that have been developed over the years by different research centres. One of the significant conceptions was developed in the Laboratory of Coordinate Metrology (LCM), it assumes that the accuracy of point coordinate measurement can be described by means of residual error distribution [5, 6], i.e. errors that remain after the application of systematic error compensation methods. Residual errors are determined in certain points covering the measuring volume of the machine, which is comprised of a reference grid of points. The errors can be experimentally determined by, for example, measurements of standard elements (hole plate, ball standards) [4] or using other devices like laser tracking systems [5, 6]. However, because residual errors also accommodate random errors, the results obtained during experiments should not be treated as exact error values but rather as information about the probability with which the reference points can be reproduced by the machine.

Data obtained from experiments are used to check the appropriate probability density function, which enables the performing of simulations, most often using the Monte Carlo method. The basis for proper virtual machine functioning is then the appropriate description of reproduction errors. Depending on the number of axes which are available to the measuring system, the variability of measurement results can be observed in two-dimensional or three-dimensional space. Because the distribution of results can be different for each measuring axis, it can be described for a 2D case as an ellipse (according to [7]) and for a 3D case as an ellipsoid [4]. The virtual machine described in detail in [5] uses ellipsoids of point reproduction probability. A laser tracking system is used for the determination of the position of the retroreflector which is mounted in the place of the CMM's probing system. The machine performs several approaches on each reference point and every time, the laser tracking system measures the obtained position of the reflector. The conducted research showed that t-distribution can be assigned for each reference point, so on the basis of the points obtained from laser tracking system, the mean value of the measured coordinates can be calculated as well as the standard deviations for each axis of a machine. Figure 1 (below) presents an example of the reproduction error distribution for a three-axis coordinate measuring machine.

A similar approach can also be applied for an optical multisensory coordinate measuring system. In such measurements, the coordinates of individual measured points are determined on the basis of digital image processing methods. Most often, appropriate algorithms search for gradient transitions [8-10]. This allows the detection of object outlines in the form of pixel and subpixel contours. Many different factors have an influence on this process such as: type of illumination (backlight, external, coaxial light) and its intensity, usage of autofocus, temperature variability, etc. The impact of these factors on measurement results can be estimated by appropriate experiments based on the measurement of standard elements. A similar procedure can also be used to check reproduction errors. This paper focuses on the comparison of two methods which can be used for the analysis of reproduction errors. The results obtained during the described

research will be useful in the development of the first fully functional virtual model of the multisensory optical machine for estimating the uncertainty of coordinate measurement. The next section describes the experiments that were conducted in order to find distribution of reproduction errors, the description of which would be the basis for the comparison of different methods.

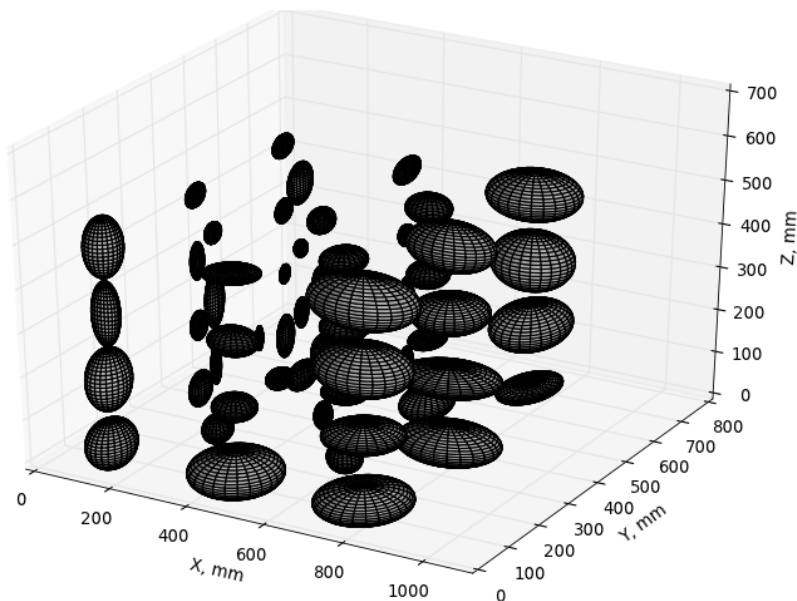


Fig. 1. An example of reproduction error distribution for the three-axis coordinate measuring machine [11].

2 Experiment and Results

The ellipses of point reproduction probability were determined through repeated measurements of the reference object. Measurements were performed with the Zeiss O'Inspect coordinate measuring machine ZEISS O'Inspect 442 at the Laboratory of Coordinate Metrology at Cracow University of Technology (Fig.2A) and with Werth Scope Check S located at NTB Buchs (Fig. 2B).

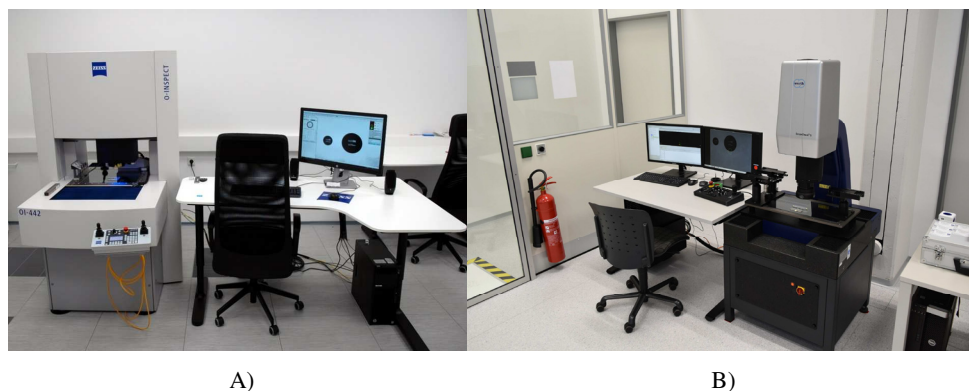


Fig. 2. Measuring systems used in research: A) Zeiss O'Inspect 442 at the Laboratory of Coordinate Metrology, B) Werth Scope Check S located at NTB Buchs.

A glass standard with marked reference circles was used during the experiments (Fig. 3). The circles of 0.254 mm and 0.100 mm diameter were chosen, both characterised by small form deviation (smaller than 0.5 μm).

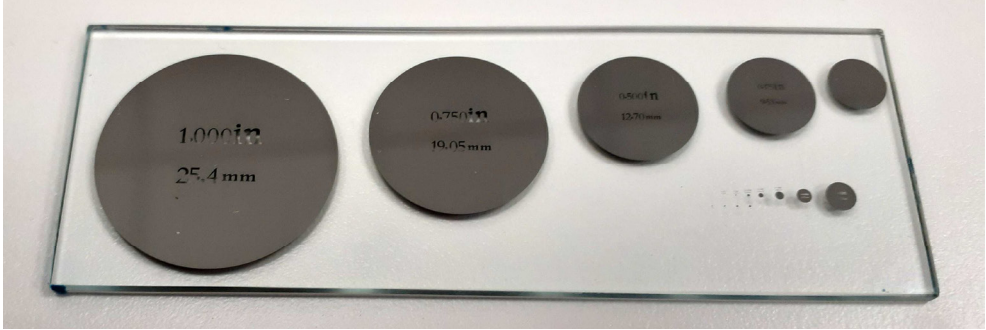


Fig. 3. A glass standard with marked reference circles.

The same procedure was utilised for both machines, including, when possible, usage of the same properties of the optical sensor as backlighting with the intensity set near to half value from the possible range, around 5 x zoom, and with autofocus active for the whole measurement procedure. Ambient conditions were controlled during measurements, and in both cases, it was around 20°C ($\pm 0.2^\circ\text{C}$). The local coordinate system was located in the centre of the measured circle with x and y axes transferred directly from the machine coordinate system. The same coordinate system was used for the whole experimental procedure. The measurement strategy assumed outer circle inspection, performed in 64 evenly distributed points. Each circle was measured 15 times. At each point, the point reproduction probability ellipses were determined according to two methods: using the covariance matrix and using the least squares fitting. Each ellipse can be described by set of parameters: x_0, y_0 – coordinates of ellipse centre; a, b – the semi-major and semi-minor axes of ellipse; α – slope of the ellipse which is the angle between the semi-major axis and the x axis of the assumed coordinate system. The experimental procedure is shown in Fig. 4.

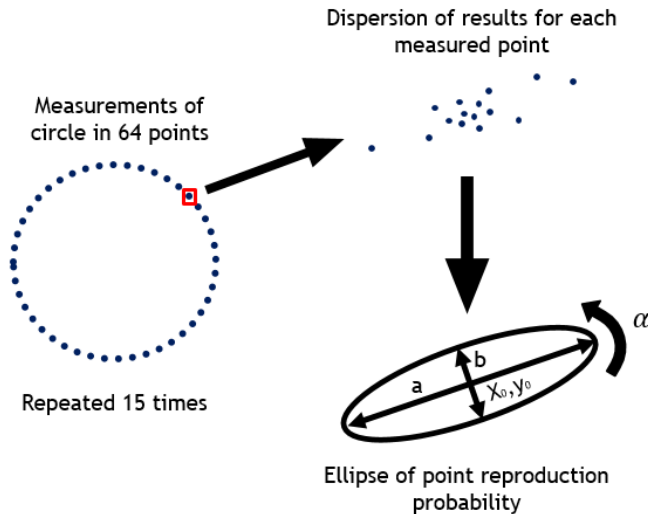


Fig. 4. Experiment procedure – the visible variables are explained above.

Using the same data set obtained from the previously described experiments, it is possible to compare which method is more suitable for the calculation of point reproduction probability ellipses. Both methods are presented below.

Method 1 - Covariance Matrix: The method begins with calculation of the ellipse centre, the coordinates of which equal the average values of the x and y coordinates of all the considered points. Next, the covariance matrix is prepared according to formula (1):

$$K_{XY} = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{YX} & \sigma_Y^2 \end{bmatrix} \tag{1}$$

where: σ_X^2 - variance of first variable x; σ_Y^2 - variance of second variable y; σ_{XY} and σ_{YX} the covariance of two variables, x and y.

Based on the formulated covariance matrix, the eigenvalues λ_1, λ_2 and eigenvectors v_1 and v_2 can be obtained. These are used to calculate the length of the ellipse axes and ellipse slope in relation to the x axis of the applied coordinate system. The ellipse axes are calculated using the following equations:

$$a = 2\sqrt{5.991 * \lambda_1} \tag{2}$$

$$b = 2\sqrt{5.991 * \lambda_2} \tag{3}$$

where: a, b – semi-major and semi-minor axes of ellipse; λ_1, λ_2 – eigenvalues of covariance matrix; 5.991 value corresponding to 95 % confidence level.

Whereas, the ellipse slope is given by the formula:

$$\alpha = \tan^{-1} \frac{v_1(y)}{v_1(x)} \tag{4}$$

where: α - the angle between the semi-major axis of the ellipse and the x axis of the applied coordinate system; $v_1(x), v_1(y)$ – components of largest eigenvector.

The ellipse obtained in the presented way defines the area in which the real coordinates of the measured points lies with 95% probability.

Method 2 - Based on the least square method: The method uses least square fitting in order to find the centre of the ellipse and its slope. However, the least square method cause the fitted ellipse to not be circumscribed on all points but some of them lie outside the outline of the ellipse. Therefore, the fitted ellipse is extended to cover all the considered measured points. This is achieved by extending the semi-major together with the semi-minor axis in such a way that their ratio remains constant.

Figures 5-8 (below) present the results obtained for the two tested machines and the two methods of ellipse determination. The presented results show the distributions of the first point in the inspection routine, which is located at the pole of the measured circle.

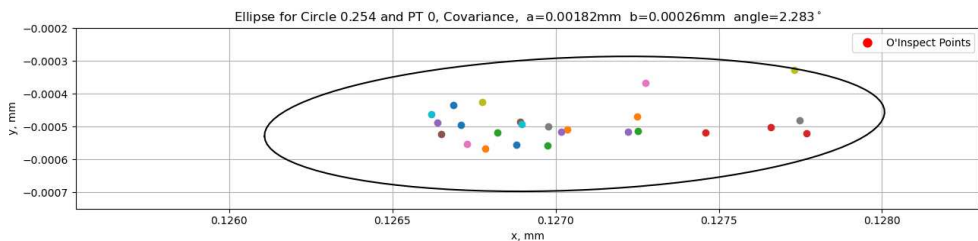


Fig. 5. Reproduction error ellipse with its parameters given at the top of the figure obtained for the O-Inspect machine using Method 1 (covariance matrix).

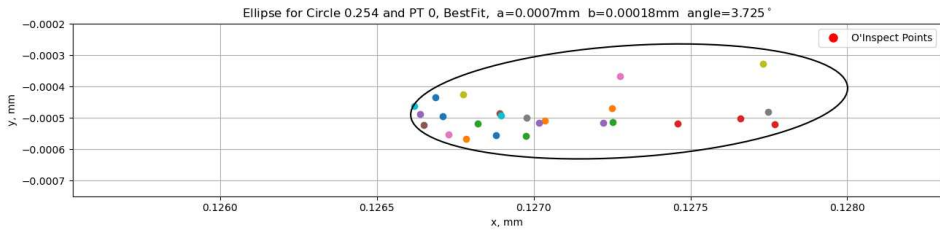


Fig. 6. Reproduction error ellipse with its parameters given at the top of the figure obtained for the O-Inspect machine using Method 2 (least square fitting).

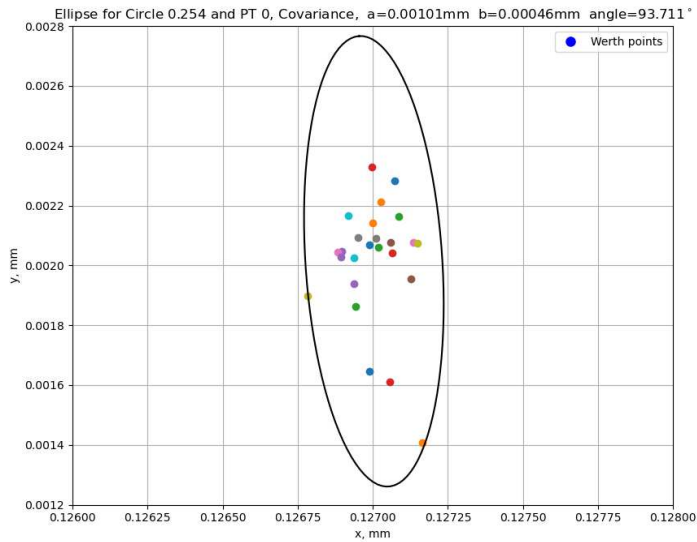


Fig. 7. Reproduction error ellipse with its parameters given at the top of the figure obtained for the Werth machine using Method 1 (covariance matrix).

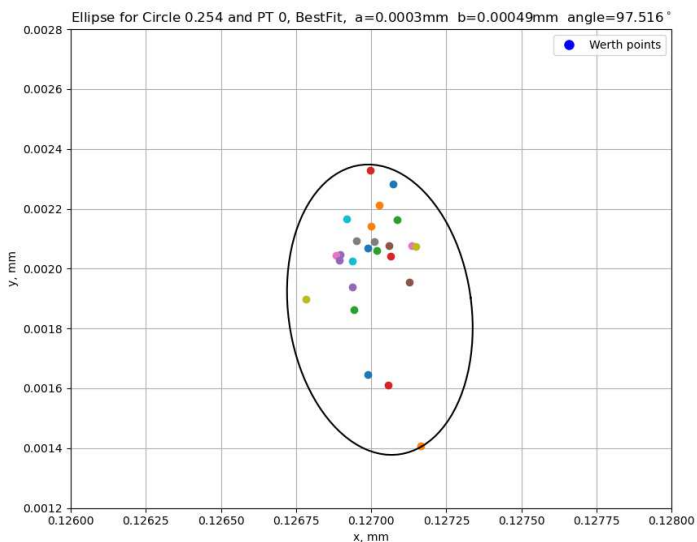


Fig. 8. Reproduction error ellipse with its parameters given at the top of the figure obtained for the Werth machine using Method 2 (least square fitting).

3 Discussion

Although the methodology based on use of the covariance matrix is well-acclaimed and popular for the description of the dispersion of results, research shows that it may give overestimated values. For both machines, the method based on the covariance matrix gives a significantly larger area of ellipses than the second method. It is caused by much higher values of the lengths of the semi-major axis a obtained using the covariance matrix (the length of semi-minor axis b is similar for both methods). Given that in both cases the ellipses cover all points, the smaller area should minimise the risk of overestimating the uncertainty. The problem of the second method is the shift of the ellipse centre in relation to the average of the obtained coordinates. This results in a higher probability of getting points that do not lie in the immediate vicinity of most of the measured points. The ellipse slopes don't significantly differ regardless of the utilised method.

The conducted research has shown that the distribution of reproduction errors significantly differs for each machine and they have to be studied individually. This indicates that a quick and simple method for determining the distribution of reproduction errors is needed (such as the method proposed in this article). Of course, the variability of factors defining the optical properties of the measuring system that remained constant during the research should also be taken into account. Determining their impact on parameters describing the ellipses of point reproduction errors is an appropriate direction for further research.

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