

# STUDY OF THE HERSHEL-QUINCKE RESONATOR

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**Abstract.** In this paper, the transmission loss of a Herschel-Quincke resonator is investigated. An analytical model of such a resonator is considered. The finite element modeling of the resonator has also been carried out. It is shown that the resonance peaks of the transmission loss spectrum in the analytical model are shifted relative to the results of numerical calculations, as a result of which it is necessary to introduce corrections for the length of the resonator tubes into the analytical model. The amendments made it possible to correct the results of analytical calculations, ensuring their reliability. The dependence of the resonator bandwidth as a function of its geometric parameters is investigated.

## 1 Introduction

A Herschel-Quincke pipe is a parallel connection of two pipes that have arbitrary lengths and constant, although not necessarily equal, cross-sectional areas. Studies of this configuration originate from the experiments of Herschel and Quincke in the 19th century, and their further development was carried out by Stewart [1-4]. Extensive studies of the characteristics of the Herschel-Quincke resonator have been carried out by the Selamet [5-6]. In practice, however, relatively little attention has been paid to this configuration, since its bandwidth was often quite narrow, especially when compared with similar characteristics of other types of acoustic elements, such as, for example, a Helmholtz resonator

The purpose of this study is to establish the influence of the geometric parameters of the Herschel-Quincke resonator on its acoustic efficiency, in particular on its bandwidth, and to ensure the identity of the results of calculating the transmission loss of the resonator using analytical and computational models.

## 2 Mathematical model of a Hershel-Quincke resonator

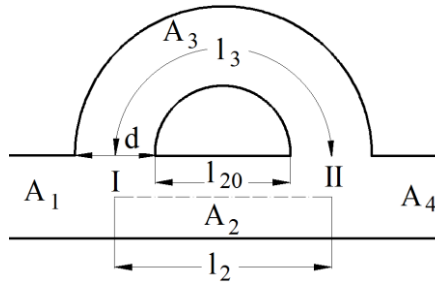
The configuration of the Hershel-Quincke resonator is shown in Figure 1, where the following designations are introduced:  $A_i$  – cross-section areas,  $l_2$  and  $l_3$  – are average lengths, respectively, of a part of the main pipe and branch. In particular, for round pipes with a diameter  $d$  the average length  $l_2 = l_{20} + d$ .

As the acoustic efficiency of the resonator, it is advisable to use transmission losses, determined through the ratio of the amplitudes of the sound waves incident on the resonator  $C_1^+$  and transmitted  $C_4^+$  through the resonator:

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$$TL = 10 \lg \left| C_1^+ / C_4^+ \right|^2 \tag{1}$$



**Fig. 1.** The configuration of the Hershel-Quincke resonator.

From consideration of the conditions for equality of pressures and volumetric velocities in the input *I* and output *II* sections of the resonator, expression (1) for transmission losses can, as shown in [5, 6], be reduced to the form:

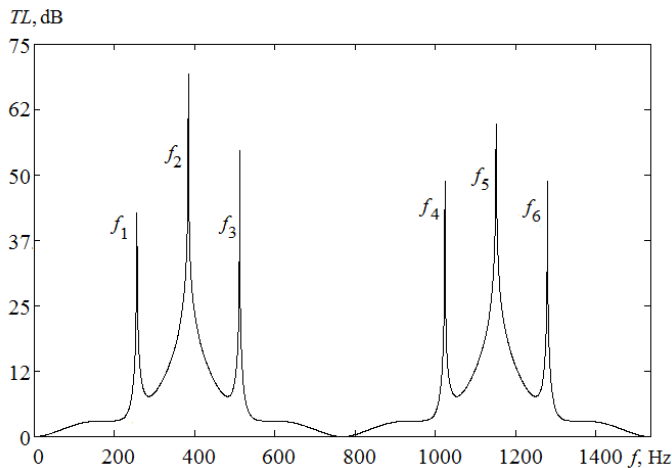
$$TL = 10 \lg \left| \frac{A_2 \alpha_2 + A_3 \alpha_3}{A_1} - \frac{(A_1 - A_2 \phi_2 - A_3 \phi_3)}{4 A_1 (A_2 \alpha_2 + A_3 \alpha_3)} \right|^2 \tag{2}$$

The following notation is introduced here

$$\alpha_j = i / (2 \sin kl_j); \quad \phi_j = i \operatorname{ctg} kl_j; \quad j = 2, 3 \tag{3}$$

where  $k = 2\pi f/c$  is the wave number;  $f$  is the frequency;  $c$  is the sound speed in air.

Calculated with in accordance with (2) and (3) the transmission losses for a resonator with geometric parameters  $A_1 = A_2 = A_3 = 1.85 \cdot 10^{-3} \text{ m}^2$ ,  $l_2 = 0.447 \text{ m}$ ,  $l_3 = 0.894 \text{ m}$ , which corresponds  $l_3/l_2 = 2$ , are shown in Figure 2.



**Fig. 2.** Analytical assessment of the transmission losses a Hershel-Quincke resonator.

The resonance peaks in Figure 2, when the transmission losses of the resonator tend to infinity, correspond to the frequencies:  $f_1 = 256 \text{ Hz}$ ,  $f_2 = 384 \text{ Hz}$ ,  $f_3 = 512 \text{ Hz}$ ,  $f_4 = 1024 \text{ Hz}$ ,

$f_5 = 1151$  Hz,  $f_6 = 1279$  Hz. As follows from formula (2), these frequencies correspond to the fulfillment of the condition

$$A_2\alpha_2 + A_3\alpha_3 = 0, \tag{4}$$

at which the first term and the denominator of the second are zeroed under the logarithm in (2). Taking into account (3), this condition is reduced to the form:

$$\frac{\sin kl_2}{\sin kl_3} = -\frac{A_2}{A_3}. \tag{5}$$

As the analysis of the condition (5) shows, there are two types of the resonances *I* and *II*, which for case  $A_2 = A_3$  and under the assumption  $l_3 > l_2$  are determined by the relations

$$k(l_3 - l_2) = (2n - 1)\pi, \quad n = 1, 2, \dots \tag{6}$$

$$k(l_3 + l_2) = 2m\pi, \quad m = 1, 2, \dots \tag{7}$$

Which in terms of frequency, can be rewritten as:

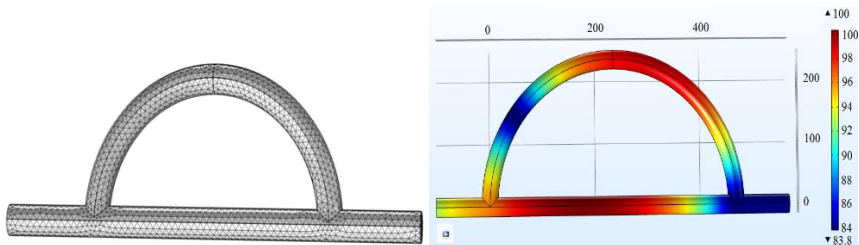
$$f_{I,n} = (n - \frac{1}{2})c / (l_3 - l_2), \quad n = 1, 2, \dots \tag{8}$$

$$f_{II,n} = mc / (l_3 + l_2), \quad m = 1, 2, \dots \tag{9}$$

In this case, in accordance with (8), (9) in the above example, the frequencies  $f_2, f_5$  and  $f_6$  correspond to type I resonances; frequencies  $f_1, f_3$  and  $f_4$  correspond to type II resonances.

### 3 Finite element modeling

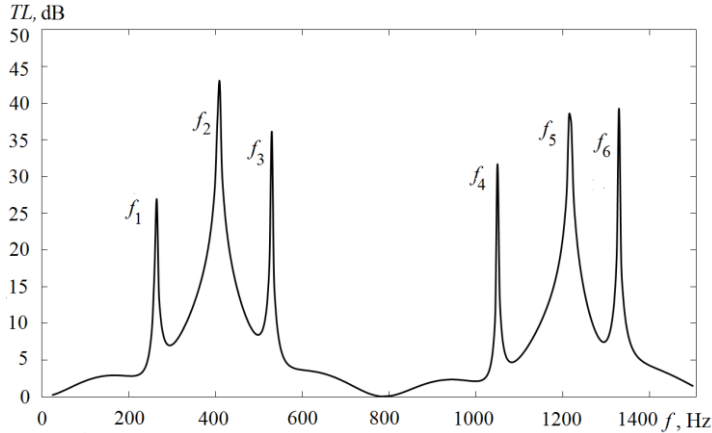
It should be noted that today numerical methods are increasingly used in calculating mufflers and other areas of applied acoustics [7-12]. To analyze the acoustic characteristics of the Herschel-Quincke pipe, a numerical calculation method based on finite element modeling was applied. Partitioning into finite elements (Figure 3, a) and subsequent calculation was carried out using the COMSOL Multiphysics software package. At the same time, to ensure high accuracy of numerical calculations [7], the maximum size of elements was  $\Delta l = 1$  mm and the number of elements was  $N = 100000$ , which has a significant effect on the accuracy of the results of calculations. A typical picture of the distribution of sound pressure levels in a resonator at a frequency of 300 Hz is shown in Figure 3, b.



**Fig. 3.** Numerical calculation of the Herschel-Quincke resonator.

Using the presented finite element model, the transmission losses of the Herschel-Quincke resonator with circular tubes with the same geometric parameters as the analytical

model of the resonator considered above were calculated. The diameter of the pipes is  $d = 48.6$  mm. The calculation results are shown in Figure 4.



**Fig. 4.** Numerical estimate of the transmission losses of a Hershel-Quincke resonator.

A comparison of the graphs in Figure 2 and Figure 4 shows that the shape of the transmission curve in the analytical and numerical models is almost identical, but there is a slight shift of the resonance peaks. The numerical model gives slightly larger values of the frequencies of the resonance peaks:  $f_1 = 265$  Hz,  $f_2 = 410$  Hz,  $f_3 = 530$  Hz,  $f_4 = 1050$  Hz,  $f_5 = 1215$  Hz,  $f_6 = 1330$  Hz.

To eliminate this discrepancy, it is necessary to introduce corrections  $\Delta_2$  and  $\Delta_3$  for the lengths  $l_2$  and  $l_3$ , thus passing to the equivalent lengths:

$$l_{2e} = l_2 - \Delta_2 \tag{10}$$

$$l_{3e} = l_3 - \Delta_3 \tag{11}$$

As a result, comparing the values of the resonance frequencies on the transmission loss curves before and after the correction of the resonator tube lengths with the results of numerical calculations, it was found that the accuracy of the analytical model increased by an order of magnitude as a result of its correction, and with a decrease in the length ratio  $l_3/l_2$ , the calculation accuracy increases, which reflected table 1, where  $\delta$ ,  $\delta_\Delta$  are the relative errors, respectively, before and after adjusting the pipe lengths in the analytical model.

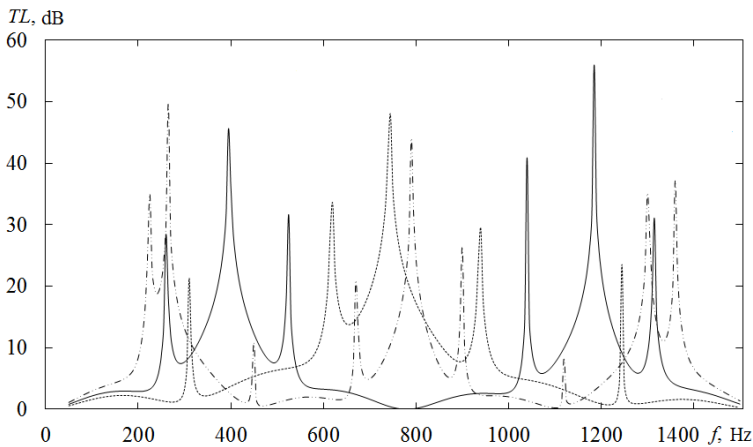
**Table 1.** Dependence of correction  $\Delta_2$  and  $\Delta_3$  on the ratio of pipe lengths at  $d = 48.6$  mm

$l_3/l_2$	$\Delta_2$ , mm	$\Delta_3$ , mm	$\delta$ , %	$\delta_\Delta$ , %
1.5	9.0	33.6	10.7	0.81
2.0	9.0	37.8	9.9	1.22
2.5	8.5	32.0	8.8	1.45

Note that, as follows from the Table 1 of the date, for the used frame of reference for the lengths of the resonator tubes, the correction  $\Delta_3$  turns out to be much larger than the correction  $\Delta_2$ .

Using numerical calculations, the transmission losses of the Herschel-Quincke resonator were determined for those presented in Table 1 of the ratio of the lengths of its pipes  $l_3/l_2$ , if

the diameters of the pipes are equal. As shown in Figure 5, the value  $l_3/l_2$  has a significant effect on the shape of the resonator transfer loss curve, while, at first glance, these changes do not show any regularity.



**Fig. 5.** The transmission loss of the Hershel-Quincke resonator as a function of from attitude  $l_3/l_2$ : (.....) – 1.5; (—) – 2.0; (- · -) – 2.5.

#### 4 Selecting parameters of the Hershel-Quincke resonator

The initial data when choosing the parameters of the resonator is the frequency  $f_0$ , to which the resonator is tuned, the band width of the resonator  $\Delta f$  in the vicinity of the frequency  $f_0$  and the damping level  $\Delta L$ , equal to the minimum value of the resonator transmission loss  $TL_{min}$  in this frequency band.

We assume that the frequency  $f_0$  should be equal to the frequency  $f_2$  from type I, so by (8) we will have:

$$f_0 = f_2 = c/[2(l_3 - l_2)]. \tag{12}$$

The damping bandwidth corresponding to a single resonance peak in the transmission loss graph is very narrow (Figure 5). But it can be expanded if near the frequency  $f_0 = f_2$  there is one more resonant frequency  $f_1$  of the type II resonances. Moreover, according to (9),

$$f_1 = c/(l_3 + l_2). \tag{13}$$

Also, as follows from (12) and (13), the relation  $f_1 < f_2$  always holds. It should be also be borne in mind that the next resonant frequency for the second type resonance  $f_3 = 2f_1$ . Therefore, when the condition  $f_2 = (3/2)f_1$  is satisfied, the frequencies  $f_1$  and  $f_3$  will be located symmetrically for the frequency  $f_2$ . This case is observed, for example, on the transmission loss curve with the length ratio  $l_3/l_2=2$  in the vicinity of the center frequency  $f_0 = 400$  Hz (Figure 5). Also, although in this case, the attenuation bandwidth is large, exceeds an octave, the mini-mum  $TL_{min}$  value in this band turns out to be small, less than 10 dB.

If we now bring the frequencies  $f_1$  and  $f_2$ , closer together, then the damping band in the vicinity of these frequencies will decrease  $\Delta f$ , and the damping level  $\Delta L$  will increase. Introducing the designation  $h = f_2/f_1$ , from (12) and (13) we obtain expressions that determine the length of the resonator tubes

$$l_2 = \frac{c}{4f_2}(2h-1); \tag{14}$$

$$l_3 = \frac{c}{4f_2}(2h+1), \tag{15}$$

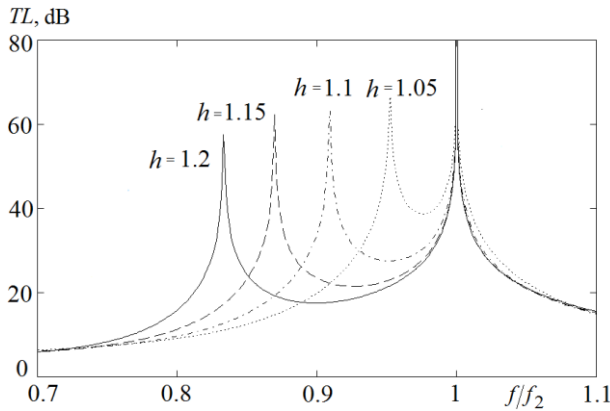
in which these lengths are inversely proportional to the frequency  $f_2 = f_0$ .

Hence it follows that the ratio of the lengths of the resonator tubes

$$l_3/l_2 = (2h+1)/(2h-1), \tag{16}$$

so that for  $h \rightarrow 1$  we get  $l_3/l_2 \rightarrow 3$ , and for  $h \rightarrow 1,5$  we get  $l_3/l_2 \rightarrow 2$

Using formulas (14) and (15), for a given parameter  $h$ , the corresponding the tube lengths  $l_3$  and  $l_2$  were determined, and then were calculated for the obtained configuration of the resonator its transmission loss. The resulting graphs of transmission losses for various values of the parameter  $h$  are shown in Figure 6.



**Fig. 6.** Resonator transmission loss dependence Hershel-Quincke on parameter  $h$ .

For the same values of the parameter  $h$  as in Figure 6, the relative resonator bandwidth  $\Delta f/f_2$  and the minimum transmission loss  $TL_{min}$  in this band were determined (Table 2). As expected, the values of  $\Delta f/f_2$  and  $TL_{min}$  in this band were determined (Table 2). As expected, the values of  $\Delta f/f_2$  and TL are inversely are inversely related.

**Table 2.** The influence of parameter  $h$  on transmission loss and bandwidth.

$h = f_2/f_1$	$TL_{min}, \text{dB}$	$\Delta f/f_2$
1.05	38.8	0.07
1.10	27.6	0.13
1.15	21.5	0.20
1.2	17.6	0.27

Thus, the parameters of the Herschel-Quincke resonator, namely the tube lengths  $l_2$  and  $l_3$ , can be chosen in such a way as to obtain acceptable values of the attenuation bandwidth and transmission loss in this frequency band.

## 5 Conclusion

The analytical and numerical models of the Herschel-Quincke resonator give coinciding results, only when corrections  $\Delta_2$  and  $\Delta_3$  are introduced into the analytical model for the length of the resonator tubes, which will correct the position of the resonance peaks and increase the accuracy of calculations in the analytical model by an order of magnitude.

Even a slight change in the geometric parameters of a Herschel-Quincke resonator, such as the ratio of the lengths  $l_3/l_2$  and the tube diameter  $d$ , can have a significant effect on its acoustic efficiency. It is necessary to choose such a configuration of the resonator so that the required transmission losses are in the given frequency range. The paper presents an algorithm for solving this problem.

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