

# Virtual pole method applied at the profiling of the gear shaped cutter tool for generating a K-type bore

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**Abstract.** The “virtual pole” method allows profiling of the cutting tools that generate by rolling. This mode of generation is advantageous in terms of the accuracy of the geometric shape obtained. The mentioned method combines the analytical accuracy of profiling with a simplified calculation mode, compared to the established algorithms. The paper presents an application made on the basis of the “virtual pole” method, for the profiling of the gear shaped tool, designed to generate the bore of a bush with a K-profile. Based on this methodology, in house software was developed for the type of profile mentioned above. The program allows the numerical and graphical representation of the profile to be generated, the contact curve and the profile of the generating tool.

## 1 Introduction

The processing of the bores profiled by generating, by the rolling method, has the advantage of a good precision of the shape of the surface to be obtained. For this type of generating, the primary peripheral surface of the generating tool and the generated surface are reciprocally enveloping surfaces. This determines that the surface to be obtained is generated by a large number of cuts and, moreover, between the contour of this surface and the profile of the generating tool, to be a relative sliding.

For these two reasons, the real surface obtained is less likely to be affected by the shape errors of the generating surface. One of the classic methods of obtaining the profiled bores is the processing with gear shaped cutter tools. In the case of these tools, a problem of major importance is the profiling of the cutting edge of the tool, so that it will obtain the desired shape for the piece profile [1].

The paper presents an application of the „virtual pole” method [2] made for the profiling of the gear shaped cutter tool, designed to generate the profiled bore of a bush with K-profile. Also known as polygonal sections, the K-profiles of the shafts and bores have the advantage of being self-centering, having no areas with tension concentrators and ensuring an easy assembly and disassembly [3].

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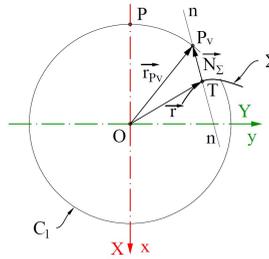
Huang et. al [4] states that this type of assembly ensures a lower processing cost with 40÷50% compared to the splined shaft assembly and also a lower vibration and noise level compared to the key assembly.

Polygonal profiles can have a wide variety of shapes, given their number of lobes, but those with three lobes (P3G) and four lobes (P4G) are standardized [3, 5].

## 2 The applying algorithm for virtual pole method

The “virtual pole” method, widely presented in the papers [2, 6], is a variant of the Willis method, according to which: “the profiles that transmit rotational movement between two parallel axes admit, at the contact point, a common normal, which passes through the gearing pole”. The “virtual pole” method is based on the observation that, in the case of tools that generate by rolling, the enwrapping condition can be defined by identifying a „virtual pole” of gearing. This virtual pole is actually the intersection point between the normal taken to the generated profile, in the current point and the centre associated with the piece.

Figure 1 shows the reference systems:  $xy$  is the fixed reference system;  $XY$  - the mobile system, joined with the profile to be generated,  $\Sigma$ ;  $C_l$  - the centre associated with the piece.



**Fig. 1.** Identifying the virtual pole

In the current point  $T$ , belonging to the profile to be generated,  $\Sigma$ , the normal  $n-n$  has been taken. It intersects the centre,  $C_l$  in the point  $P_v$ , representing the virtual pole.

If it is admitted that the profile to be generated,  $\Sigma$ , has the parametric equations:

$$\Sigma: \begin{cases} X = X(u); \\ Y = Y(u), \end{cases} \quad (1)$$

with  $u$ , independent variable parameter, then the position vector of the current point,  $T$  has the form:

$$\vec{r} = X(u) \cdot \vec{i} + Y(u) \cdot \vec{j}. \quad (2)$$

In this point, the normal vector will have the form:

$$\vec{N}_\Sigma = \lambda \cdot (\dot{Y}_u \cdot \vec{i} - \dot{X}_u \cdot \vec{j}), \quad (3)$$

$\lambda$  - being a scalar dimension which represents the distance between the current point,  $T$  and the virtual pole,  $P_v$ .

The position vector of the point  $P_v$  is obtained by summing the vectors  $\vec{r}$  and  $\vec{N}_\Sigma$  [2], obtaining the form:

$$\vec{r}_{P_v} = \vec{r} + \vec{N}_\Sigma = [X(u) + \lambda \cdot \dot{Y}_u] \cdot \vec{i} + [Y(u) + \lambda \cdot \dot{X}_u] \cdot \vec{j}. \quad (4)$$

The size of the distance  $\lambda$  is determined from the belonging condition of the point  $P_v$  to the centre  $C_l$ .

The centre equations are:

$$C_I: \begin{cases} X = -R_{rp} \cdot \cos \varphi_I; \\ Y = R_{rp} \cdot \sin \varphi_I, \end{cases} \quad (5)$$

and thus the belonging condition becomes:

$$\begin{cases} -R_{rp} \cdot \cos \varphi_I = X(u) + \lambda \cdot \dot{Y}_u; \\ R_{rp} \cdot \sin \varphi_I = Y(u) + \lambda \cdot \dot{X}_u. \end{cases} \quad (6)$$

The parameter  $\lambda$  can be eliminated from the system of equations (6):

$$\left[ -R_{rp} \cdot \cos \varphi_I - X(u) \right] \cdot \dot{X}_u - \left[ R_{rp} \cdot \sin \varphi_I - Y(u) \right] \cdot \dot{Y}_u = 0, \quad (7)$$

representing the enveloping condition, meaning the connection between the independent parameters  $u$  and  $\varphi_I$ .

Once known the specific value  $\varphi_I = \varphi_{Iu}$ , for which the point  $P_v$  coincides with the gearing pole, the position of the  $T$  point, in its absolute movement, can be determined:

$$\mathbf{x} = \boldsymbol{\omega}_3^T(\varphi_{Iu}) \cdot \mathbf{X}(u), \quad (8)$$

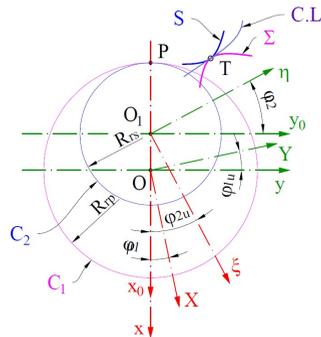
meaning the position which it occupies on the contact curve, in the rolling process:

$$T_F \begin{cases} x(u, \varphi_{Iu}) = X(u) \cdot \cos \varphi_{Iu} - Y(u) \cdot \sin \varphi_{Iu}; \\ y(u, \varphi_{Iu}) = X(u) \cdot \sin \varphi_{Iu} + Y(u) \cdot \cos \varphi_{Iu}. \end{cases} \quad (9)$$

Considering the rolling condition, namely the condition that the two centrodes roll without sliding:

$$R_{rp} \cdot \varphi_1 = R_{rs} \cdot \varphi_2 \Rightarrow \varphi_2 = \left( \frac{R_{rp}}{R_{rs}} \right) \cdot \varphi_1 = i \cdot \varphi_1, \quad (10)$$

it can be determine the position of the tool when the piece rotates with the angle  $\varphi_{Iu}$ , see also figure 2.



**Fig. 2.** Correlation between the movements of the piece and the tool

The absolute movement of the tool is:

$$\boldsymbol{\xi} = \boldsymbol{\omega}_3^T(\varphi_2) \cdot \mathbf{x}_o = \boldsymbol{\omega}_3^T(i \cdot \varphi_1) \cdot \mathbf{x}_o, \quad (11)$$

and the correlation between  $x_o$  and  $x$  systems is given by:

$$\mathbf{x}_o = \mathbf{x} - \mathbf{A}; \quad \mathbf{A} = (A_{12} \quad 0)^T; \quad A_{12} = R_{rp} - R_{rs}. \quad (12)$$

From the equations (9), in the fixed reference system, the  $T_F$ -point coordinates are obtained - be these  $x_{T_F}$  and  $y_{T_F}$ .

If the coordinate transformation (12) is taken into consideration, the  $T_F$ -point coordinates can be determined in the fixed system  $x_0y_0$ , namely  $x_{T_{F0}}$  and  $y_{T_{F0}}$ .

Applying to this point the absolute movement of the tool, given by (11), its coordinates are obtained in the reference system  $\xi\eta$ , associated with the tool:

$$T_S \begin{cases} \xi_{T_S} = x_{TF_0} \cdot \cos(i \cdot \varphi_{1u}) - y_{TF_0} \cdot \sin(i \cdot \varphi_{1u}); \\ \eta_{T_S} = x_{TF_0} \cdot \sin(i \cdot \varphi_{1u}) + y_{TF_0} \cdot \cos(i \cdot \varphi_{1u}). \end{cases} \quad (13)$$

When the current point  $T$  go through the whole profile  $\Sigma$ , the  $T_S$ -point completely crosses the profile  $S$ , reciprocally enveloping to profile  $\Sigma$ .

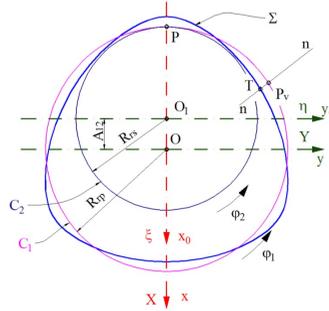
### 2.1 The profiling of the gear shaped cutter tool designed to process a K-type bore

The K-type profile equations are given by:

$$\Sigma: \begin{cases} X(u) = [R - e \cdot \cos(n \cdot u)] \cdot \cos u - n \cdot e \cdot \sin(n \cdot u) \cdot \sin u; \\ Y(u) = [R - e \cdot \cos(n \cdot u)] \cdot \sin u + n \cdot e \cdot \sin(n \cdot u) \cdot \cos u, \end{cases} \quad (14)$$

where  $n$  is the number of lobes, and  $e$  is the eccentricity.

In figure 3 are represented the reference systems, the two centres,  $C_1$  - associated with the piece and  $C_2$  - associated with the tool, the profile  $\Sigma$ , on which the current point  $T$  was considered, as well as the normal to the profile,  $n-n$ .



**Fig. 3.** Reference systems, piece profile and conjugated centres

The reference systems used are:  $xy$  is a fixed reference system, having the origin in the centre of the centre  $C_1$  of radius  $R_p$ ;  $x_0y_0$  - fixed reference system, having the origin in the centre of centre  $C_2$ , of radius  $R_s$ ;  $XY$  - mobile reference system, joined with the  $\Sigma$  profile and which initially has the axes superimposed on the  $xy$  system; the absolute movement parameter of this system is  $\varphi_1$ ;  $\xi\eta$  - mobile reference system, joined with the  $S$  profile (of the tool), initially having the axes superimposed on the  $x_0y_0$  system. The movement parameter of this system is the angle  $\varphi_2$ , related to the angle  $\varphi_1$ , is given by the rolling condition (10).

Taking into account the partial derivatives of the  $\Sigma$  profile equations in relation with the variable  $u$ :

$$\dot{X}_u = [e(1 - n^2) \cdot \cos(n \cdot u) - R] \cdot \sin u, \quad \dot{Y}_u = -[e(1 - n^2) \cdot \cos(n \cdot u) - R] \cdot \cos u \quad (15)$$

According to equation (3), in  $T$ -point, the normal to the profile  $\Sigma$  is:

$$\vec{N}_\Sigma = \lambda \left\{ -[e(1 - n^2) \cos(n \cdot u) - R] \cos u \cdot \vec{i} - [e(1 - n^2) \cos(n \cdot u) - R] \sin u \cdot \vec{j} \right\}. \quad (16)$$

The position vector of the  $T$ -point is given by:

$$\vec{r} = \left\{ [R - e \cdot \cos(n \cdot u)] \cdot \cos u - n \cdot e \cdot \sin(n \cdot u) \cdot \sin u \right\} \cdot \vec{i} + \left\{ [R - e \cdot \cos(n \cdot u)] \cdot \sin u + n \cdot e \cdot \cos(n \cdot u) \cdot \cos u \right\} \cdot \vec{j}. \quad (17)$$

As result according to (4), the position vector of the virtual pole is:

$$\vec{r}_{P_v} = \left\{ \left[ R - e \cos(n \cdot u) \right] \cos u - ne \sin(n \cdot u) \sin u - \lambda \left[ e(1 - n^2) \cos(n \cdot u) - R \right] \cos u \right\} \vec{i} + \left\{ \left[ R - e \cos(n \cdot u) \right] \sin u + ne \sin(n \cdot u) \cos u - \lambda \left[ e(1 - n^2) \cos(n \cdot u) - R \right] \sin u \right\} \vec{j}. \quad (18)$$

Intersecting this vector with the  $C_1$  centrede:

$$C_1: \begin{cases} X = -R_{rp} \cdot \cos \varphi_1; \\ Y = R_{rp} \cdot \sin \varphi_1, \end{cases} \quad (19)$$

is obtained:

$$-R_{rp} \cos \varphi_1 = \left[ R - e \cos(nu) \right] \cos u - ne \sin(nu) \sin u - \lambda \left[ e(1 - n^2) \cos(nu) - R \right] \cos u; \quad (20)$$

$$R_{rp} \sin \varphi_1 = \left[ R - e \cos(nu) \right] \sin u + ne \sin(nu) \cos u - \lambda \left[ e(1 - n^2) \cos(nu) - R \right] \sin u.$$

From (20), by removing the  $\lambda$  parameter, is obtained the specific form of the enveloping condition:

$$\varphi_1 = -u - \arcsin \left[ (n \cdot e \cdot \sin(n \cdot u)) / R_{rp} \right]. \quad (21)$$

The value of the angle  $\varphi_{1u}$  for which the condition (21) is accomplished, represents the value of the angular parameter for which the  $P_v$  point coincides with the gearing pole,  $P$ .

For a certain value of parameter  $u$  and a corresponding value  $\varphi_{1u}$ , determined from the enveloping condition (21), the  $T$ -point, belonging to the  $\Sigma$  profile, will have, in the fixed reference system, the coordinates given by the absolute movement of the blank, (8), are:

$$T: \begin{cases} x_{T_F} = X_T \cdot \cos \varphi_{1u} - Y_T \cdot \sin \varphi_{1u}; \\ y_{T_F} = X_T \cdot \sin \varphi_{1u} + Y_T \cdot \cos \varphi_{1u}. \end{cases} \quad (22)$$

**Note:** In this position,  $T$ -point also belongs to the gearing curve, meaning  $T \equiv T_F$ .

While the blank moved with the angular parameter  $\varphi_{1u}$ , the tool rotates with the parameter  $\varphi_{2u}$ , whose value is given by the rolling condition (10).

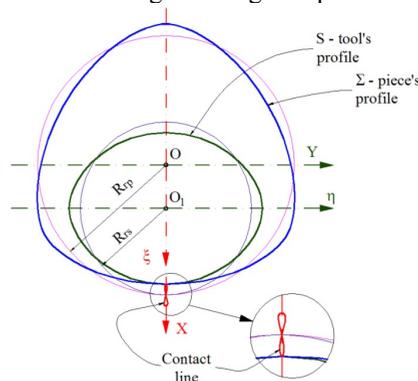
Therefore, the position of the tool reference system is given by its absolute movement, (11), by developing obtaining:

$$T_S: \begin{cases} \xi = x_0(u) \cdot \cos \varphi_2 + y_0(u) \cdot \sin \varphi_2; \\ \eta = -x_0(u) \cdot \sin \varphi_2 + y_0(u) \cdot \cos \varphi_2, \end{cases} \quad (23)$$

For  $u$ , taking values from  $u_{min}$  to  $u_{max}$ , the  $T_S$ -point go through the entire profile of the tool,  $S$ .

## 2.2 Numerical example:

For a profile defined by  $R=60 \text{ mm}$ ;  $e=5 \text{ mm}$ ;  $n=3 \text{ lobes}$  (according to DIN 32711, [7]), the points coordinates on the generating tool profile are presented in table 1, and the profile to be generated, the gearing curve and the generating tool profile are shown in figure 4.



**Fig. 4.** The profile to be generated, the gearing curve and the generating tool profile

**Table 1.** Coordinates of the points on the tool profile and on the gearing curve.

Crt. no.	Tool's profile		Contact line		Crt. no.	Tool's profile		Contact line	
	$\xi$ [mm]	$\eta$ [mm]	x [mm]	y [mm]		$\xi$ [mm]	$\eta$ [mm]	x [mm]	y [mm]
1	0.000	-45.000	55.000	0.000	14	34.323	8.277	64.945	0.321
2	1.838	-44.909	55.306	0.304	15	32.363	16.161	64.763	0.600
3	3.899	-44.597	56.171	0.482	16	29.322	23.302	64.410	0.792
4	6.372	-43.957	57.450	0.458	17	25.502	29.440	63.830	0.849
5	9.379	-42.823	58.949	0.233	18	21.267	34.429	62.979	0.741
6	12.945	-40.990	60.467	-0.116	19	16.978	38.249	61.845	0.476
7	16.978	-38.249	61.845	-0.476	20	12.945	40.990	60.467	0.116
8	21.267	-34.429	62.979	-0.741	21	9.379	42.823	58.949	-0.233
9	25.502	-29.440	63.830	-0.849	22	6.372	43.957	57.450	-0.458
10	29.322	-23.302	64.410	-0.792	23	3.899	44.597	56.171	-0.482
11	32.363	-16.161	64.763	-0.600	24	1.838	44.909	55.306	-0.304
12	34.323	-8.2775	64.945	-0.321	25	0.000	45.000	55.000	0.000
13	35.000	0.000	65.000	0.000					

### 3. Conclusions

The algorithm based on the “*virtual pole*” method represents an improved method for profiling of tools that generate by enwrapping, by the rolling method.

The application presented in the paper demonstrates the accuracy and simplicity of the “*virtual pole*” method and, at the same time, allows verifying the results obtained with this method.

Based on the presented methodology, a calculation program was created, which allows the profiling and graphical representation of the gear shaped cutter tool, designed to process a *K*-type bore.

### References

1. V.G. Teodor, N. Baroiu, F. Susac, *The Synthesis of New Algorithms for CAD Profiling of Cutting Tools* (Lambert Academic Publishing, ISBN 978-613-7-08923-1, 2018)
2. G.A. Costin, V.G. Teodor, N. Oancea, *Virtual Pole Method - Alternative Method for Profiling Tools Which Generate by Enwrapping*. Ann. of “Dunarea de Jos” Univeristy, TMB, **5**, 31 (2019)
3. R. Bhatta, *Comparison of Load Carrying Capacity of Three and Four Lobed Polygonal Shaft and Hub Connection for Constant Grinding Diameter*, Masters Theses, (2016)
4. B. Huang, J. Li, Q.J. Sun, L.Q. Wang, J. Zhejiang Univ. Sci. B, **3**, 039, (2010)
5. E. Ghiță, *Teoria și tehnologia suprafețelor poliforme (Theory and technology of poliform surfaces)*. (Bren Publishing House, Bucharest, 2001)
6. V.G. Teodor, G.A. Costin, *Virtual Pole Method - Alternative Method for Profiling Rack Tools*, Ann. of “Dunarea de Jos” Univeristy, TMB, **5**, 5 (2019)
7. \*, *DIN 32711-1. Welle-Nabe-Verbindung - Polygonprofil P3G - Teil 1: Allgemeines und Geometrie (Shaft to collar connection - Polygon profile P3G - Part 1: Generalities and geometry)*, (2009)