

# Stress analysis of the membrane structure in the shape of cone

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**Abstract.** The paper is focused on the static analysis of the cone, which is one of the basic shapes used for membrane structures. These structures are specific by their ability to transmit only tensile forces. Pressure forces in the membrane construction may cause unwanted wrinkling of the used textile materials. When designing membranes, one of the variable parameters can be the height of cone and the radius of the circular profile at the top of the cone, which affects the resulting stress and internal forces. This paper aims to analyze the reliability of the cone in terms of the extreme stress and internal forces while maintaining the same ground plan.

## 1 Introduction

Membrane structures are very specific types of structures. They can be designed as roofing for temporary and permanent buildings [1]. Main advantages of these structures are very low weight, aesthetic airiness, high shape variability, large spans, and also the aesthetics of the structures themselves.

Due to their specific properties and increasing popularity, there is a great scope for research in this area. Very investigated area is the optimization of initial shape of membrane structures [2, 3] and also numerical modelling concerning of the behaviour of the membrane structures under the load [4].

The aim of this article was to analyze the reliability of one of the basic shapes of membrane structures, cone. The analysis was performed in terms of extreme stresses and to assess the suitability of its use. For analysis was chosen the software RFEM which allows find initial shape of membrane and rope structures using the additional module, RF Form-Finding. For finding optimal shapes of membrane structures, RFEM use method by U. R. Brightzer and E. Ramma, Updated Reference Strategy (URS).

The static analysis of membrane structures with information about the form-finding process and the URS method, which is used for finding optimal shapes of structures, are discussed in Chapter 2. In Chapter 3 can be found a description of the cone, which is one of the basic shapes of membranes. Parameters of analyzed membrane structure in the shape of cone and results are presented in Chapter 4.

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## 2 Static analysis of membrane structures

For static analysis, geometrical non - linearity needs to be taken into account. The resulting deflections are many times greater than the thickness of the materials and calculations cannot be performed based on The Theory of Small Deformation. The principle of superposition is no longer valid, and the structure has to be loaded over the whole range, not as individual load cases.

It is necessary to apply pre-stressing into the construction which is used for achieving a suitable shape. Pre-stressing prevents inversion of the curvature and after that, the structure can be loaded.

The spatial effect of pre-stressing is given by the double curvature. The basic shapes of membranes can be divided into hyperbolic paraboloid, cone, and saddle. In this study, the shape of cone was analyzed.

### 2.1 Form – Finding process

The process of searching for the equilibrium shape of membrane structures is called Form-Finding [5, 6, 7]. Form-Finding finds an optimal shape of construction with respect to the given stress distribution, which acts in the deformed membrane. It is necessary to find an optimal shape with respect to the pre-specified stress field.

Based on the Form-Finding process, the initial pre-stress is determined. Pre-stressing is the basic loading state. It activates the structure's anchor structure, stiffness, and ability to carry the load. The value of the pre-stressing is typically in the range of 0.5 – 2 kN/m<sup>2</sup>. However, for larger spans, the pre-stressing value can reach up to 5 kN/m<sup>2</sup> [8].

### 2.2 Updated reference strategy – URS

When analyzing membrane structures, their geometric non-linearity must be taken into account [1, 9, 10]. It is necessary to find an optimal shape with respect to the pre-specified stress field. It is necessary to find the configuration in which the surface Cauchy stress,  $\sigma$  (i.e. the actual stress measured at a given moment on the deformed body) is replaced by the 2<sup>nd</sup> Piola – Kirchhof stress,  $S$ , which is related to the undeformed body. Because these two stresses are artificially connected by a tangential mesh deformation, there is no singularity when using  $S$ . If there is no deformation,  $\sigma$  and  $S$  are identical. Consider a field of tension that acts in the plane tangential to the surface in equilibrium. The stress field is thus the total stress resulting from the initial prestress and the deformation of the textile membrane without fixed boundaries. At this point, it is not necessary to specify a particular material because the stress field is considered as given. It is necessary to determine the geometry of the surface that allows this tension to be in equilibrium. When using this method, the equilibrium state of the membrane is defined by the principle of virtual work as follows:

$$\delta_w = t \int_a \sigma : \frac{\partial(\delta_x)}{\partial(x)} da = t \int_a \sigma : \partial u_x da, \quad (1)$$

where  $a$  is the minimal surface,  $\sigma$  is the Cauchy stress tensor, which is applied to the surface in equilibrium;  $\partial u_x$  is the derivative of the virtual displacement with respect to the geometry of the current surface; and  $t$  is the thickness of the membrane, which is relatively thin and assumed to be constant during deformation; the Poisson's coefficient of the thickness is neglected. To express  $\partial u_x$ , the deformation gradient,  $F$ , is introduced into the calculation. It includes the volume and shape changes of the deformed body and defines the actual shape of area  $a$  by the deformed shape of a reference configuration of area  $A$ . After substituting

the equation for the deformation gradient into the equation for the expression of virtual work with subsequent modifications, the final expressions lead to:

$$\begin{aligned} \delta_w &= t \int_A \det F (F \cdot F^{-1} \cdot \sigma \cdot F^{-T}) : \delta F dA = \\ &= t \int_A (F \cdot (\det F \cdot F^{-1} \cdot \sigma \cdot F^{-T})) : \delta F dA = t \int_A (F \cdot S) : \delta F dA = 0, \end{aligned} \quad (2)$$

where  $S$  is the 2<sup>nd</sup> Piola-Kirchhoff stress, which replaced the surface Cauchy stress,  $\sigma$ . However, the  $S$  elements are related to the original configuration and the  $\sigma$  elements are related to the current configuration. If the current and original configurations are the same, then  $S$  and  $\sigma$  will be identical.

The basics of this method have been published in detail, for example, in [11].

### 3 Membrane structure in the shape of cone

This form of membrane structure is constructed with rigid or flexible base and circular high point (see Fig. 1, 2). Cone can be asymmetric, symmetric, with more than one high or low point.

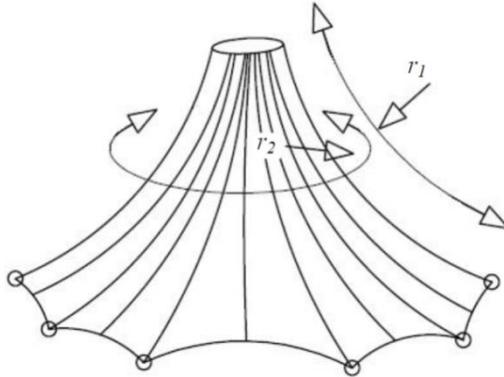


Fig. 1. Scheme of double curvature of cone.



Fig. 2. Membrane structure in the shape of cone in Ostrava.

## 4 Analysis of membrane in the shape of cone

### 4.1 Analyzed models

The membrane structure in shape of cone with a 5x5m square plan was analysed. This shape was modeled with pin supported columns. The rigidity of construction was provided by frame corners using steel beams and system of rods as can be seen in Fig. 3.



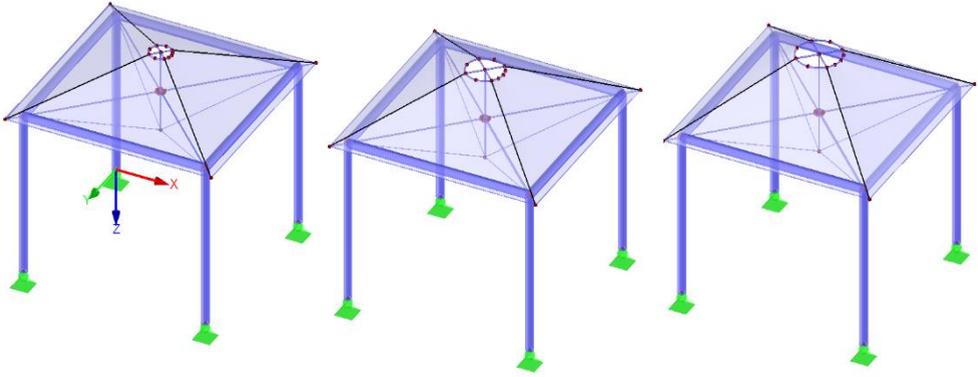
**Fig. 3.** Frame corner and system of rods to ensure rigidity of the structure.

As a textile material, PVC was chosen with a modulus of elasticity of 1000 MPa. In practice, when designing textile membrane structures, material characteristics are taken directly from specific manufacturers.

During the design, the required ground plan for roofing is usually given. Therefore, the height of the cone was analysed as a variable parameter – 1 m, 1.25 m, 1.5 m. (see Fig. 4).

For analysis were chosen two options of structures with different height:

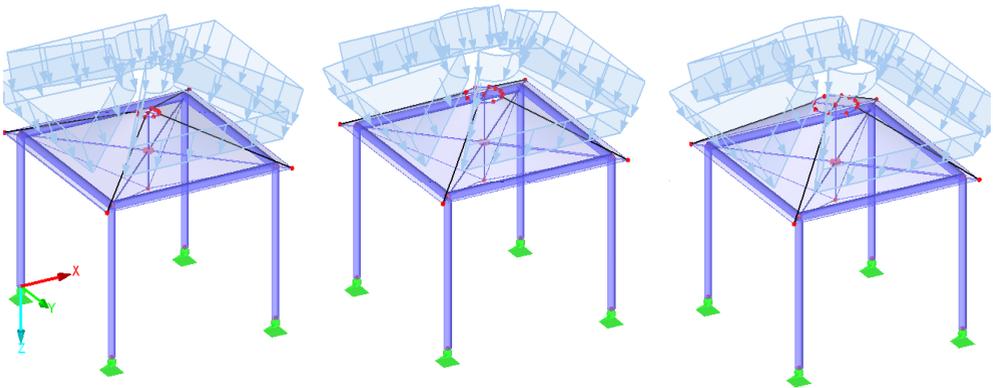
- The structures were modeled as one area without any pre-stressed rods inside the fabric,
- the structures were modeled with 4 pre-stressed diagonal ropes into the fabric.



**Fig. 4.** Analyzed models of membrane structure.

When the membrane structures were modeled, the Form – Finding process by RFEM software was applied to find an optimal shape.

After finding the optimal shape, the load was applied to structures. The self-load of the structures is a small part of the load and has been neglected. The subject of analysis was loading by wind as can be seen on Fig. 5.



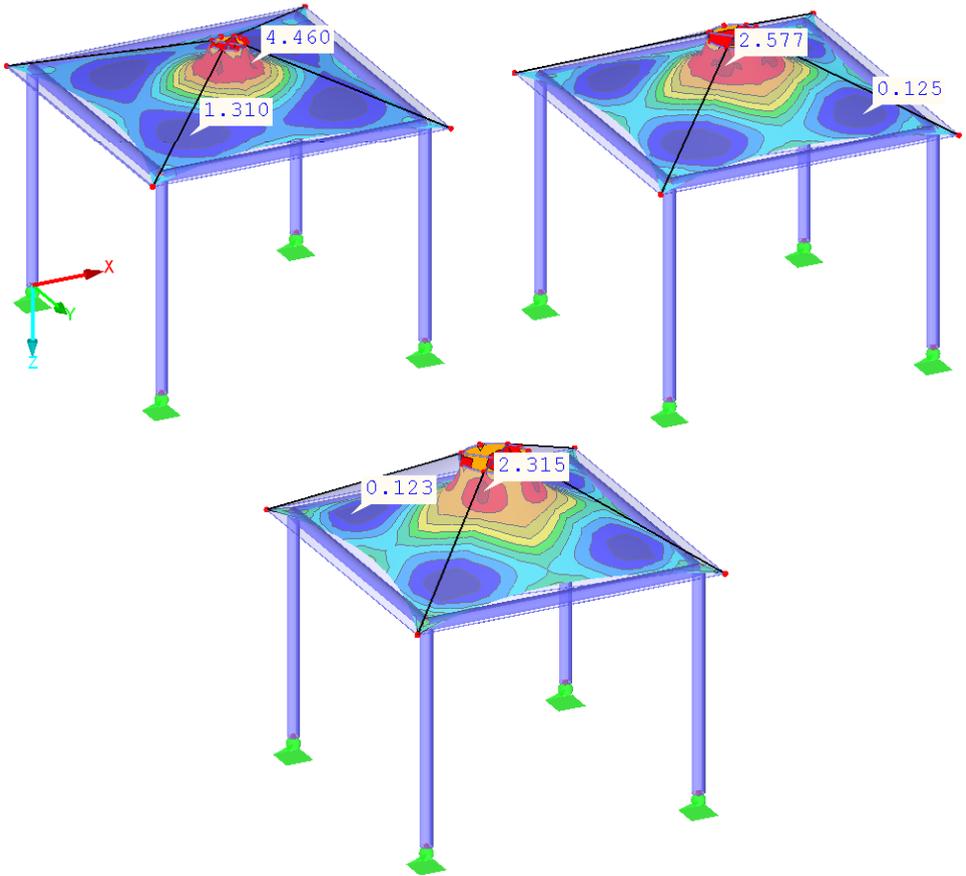
**Fig. 5.** Loaded membrane structures.

## 4.2 Results

### 4.2.1 Models without the pre – stressed ropes inside

The first problem occurred in the Form – Finding process. RFEM could not find the optimal shape. The solution was increasing the radius of the circular profile on the top of the structure. After finding an optimal radius of the circular profile on the top of the structure, the RFEM was able to find the optimal shape.

In Fig. 6 can be seen the resulting values of normal stress  $\sigma_x$  [MPa].



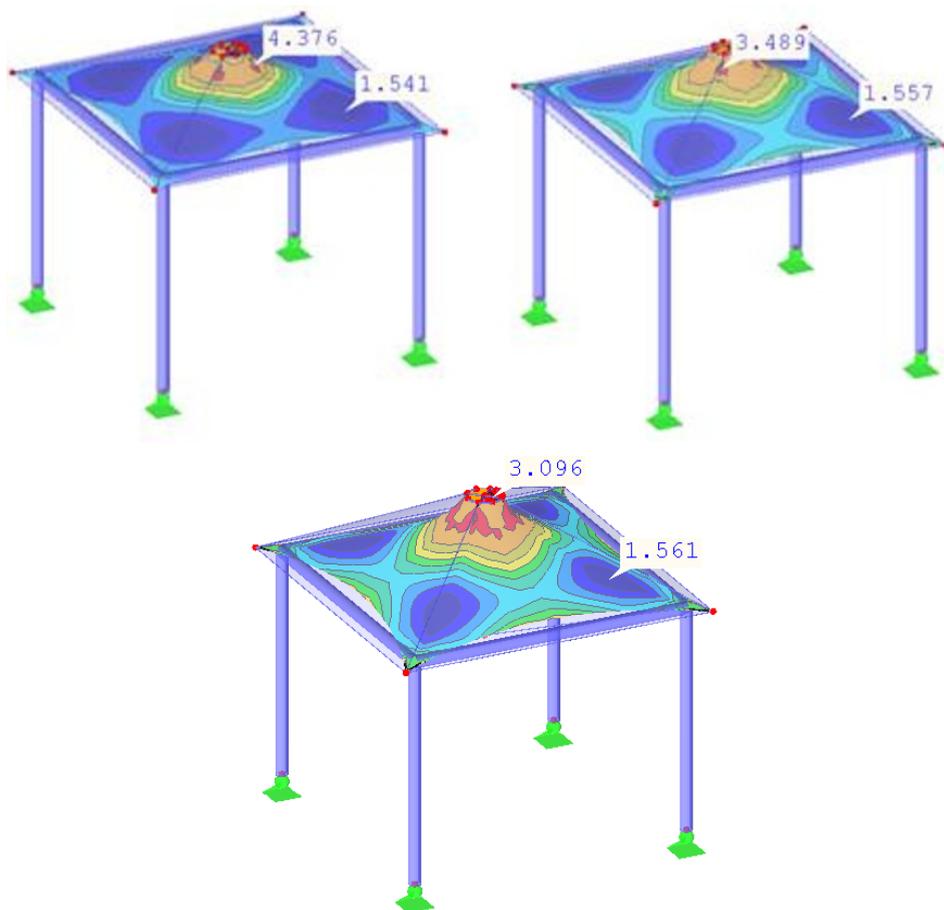
**Fig. 6.** Resulting normal stress for membrane structures without added pre-stressed ropes inside the fabric.

#### 4.2.2 Models with the pre – stressed ropes inside

In the case of structures with added pre – stressed ropes inside the fabric, RFEM was able to reach the equilibrium shape.

In Fig. 7 can be seen the resulting values of normal stress  $\sigma_x$  [MPa].

It was not needed to increase the radius of circular profile on the top of structure. On the other side, the resulting values of normal stress were higher than in the first option without the pre - stressed ropes.



**Fig. 7.** Resulting normal stress for structures with added pre-stressed ropes inside the fabric.

**Table 1.** Comparing the results of normal stress.

Height of cone	Normal stress $\sigma_x$ [MPa]			
	Min		Max	
	Without added pre-stressed ropes	With added pre-stressed ropes	Without added pre-stressed ropes	With added pre-stressed ropes
1.00 m	1.300	1.541	4.460	4.376
1.25 m	0.125	1.557	2.577	3.489
1.50 m	0.123	1.561	2.315	3.096

### 4.2.3 Comparing

For both options, the biggest values of stress occurred on the top of structures and the higher the cone was, the stress was gradually decreasing, see Tab. 1.

The membrane structures with added pre – stressed ropes did not require increasing the circular profile on the top to reach the optimal shape. However, the resulting values of stress were higher compared to the membrane structure without added pre – stressed ropes.

## 5 Conclusions

In this paper, it was analyzed one of the basic types of membrane structures in terms of stresses, the shape of cone. All analyzed models had the same ground plan but different height of cone.

The biggest value of stress occurred on the top of the membrane for every analysed model and was gradually decreasing downward. In comparing all analysed heights of cone, the flatter the structures were, the higher stress occurred in the textile membrane.

The solution for the reduction of stress in the textile material could be increasing the height of cone to get lower values of the stress. However, it has been also found that with the increasing height, it's more difficult to find an optimal shape without adding pre-stressed ropes inside the fabric. In the case of pre-stressed ropes only in edges of the membrane, it was also needed to increase the radius the circular profile on the top. If not, the process of finding an optimal shape could not be reached.

The next option, with added pre-stressed ropes into the textile material, did not require increasing the radius of the circular profile on the top of the cone to reach the optimal shape of the membrane structure.

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