Predication of life cycle cost of equipment base on unbiased grey Markov models

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Abstract. Life cycle cost(LCC) is an important content of equipment integrated logistics support. While the LCC includes the whole life cycle of equipment from development, production, service and maintenance to retirement, in order to effectively manage and control the LCC and better develop integrated logistics support, it is necessary to analyze and predict it. The unbiased grey markov model(UGMM) was introduced into the LCC prediction in the paper, in order to check model accuracy, the posterior difference method(PDM) was used, also the influence by the number of state intervals in UGMM on the prediction accuracy is analyzed and studied. The result indicate that UGMM can be used to predict the LCC, also have the highest prediction accuracy comparing with unbiased grey model and grey separating model, and in order to ensure the prediction accuracy, the state interval should be divided according to the number of sequence.

1 Introduction

Life cycle cost(LCC) of equipment was first proposed by the U. S. Department of defense in the 1960s, it is defined as the sum of all costs paid for the demonstration, development, production, service, maintenance, support, and retirement of equipment within the life cycle, its main purpose is to reveal the law of the occurrence and development of LCC, so as to take effective methods to control it [1, 2, 4]. The control of LCC is one of the important contents of equipment integrated logistics support(ILS). The purpose of ILS is to minimize the LCC on the basis of ensuring the equipment integrity, so as to maximize the cost-effectiveness ratio of the equipment cost [3, 4]. As in [5], Li Hongtao studied the equipment selection of wind turbine system using the LCC model.

The methods of LCC predication are analogy estimation method, parameter estimation method and engineering evaluation method [1, 2, 4]. Parameter estimation method is commonly used, which is not only suitable for the cost evaluation in the later stage of demonstration and early stage of development, but also can be used to verify the conclusions of other estimation methods [4]. Parameter estimation methods include: grey theory [1, 6],

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principal component analysis [1, 7], support vector machine method[1, 8], neural network method [1, 9]and fuzzy theory [10], etc.

Grey theory and Markov are widely used in all fields [11-15]. Base on previous research results [11, 12], this paper introduces unbiased grey markov model(UGMM) into LCC prediction and the results were checked by posterior difference method(PDM). At the same time, the influence of the number of state interval divisions on the prediction accuracy was studied.

2 Unbiased gray Markov model

2.1 Unbiased grey model

In order to improve the accuracy, the unbiased grey model(UGM)[11, 13, 14] was used. Its principle is:

Assume a observation time sequence of a certain behavior characteristic of the system is:

$$x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}, (n \geq 1) \quad (1)$$

After an accumulation process on sequence $x^{(0)}$, a new sequence can be obtained.

$$x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\}, (n \geq 1) \quad (2)$$

$$\begin{cases} x^{(1)}(1) = x^{(0)}(1) \\ x^{(1)}(2) = x^{(0)}(1) + x^{(0)}(2) \\ \dots \\ x^{(1)}(n) = \sum_{i=1}^n x^{(0)}(i) \end{cases} \quad (3)$$

For the sequence $x^{(1)}$ with grey exponential law, the UGM [10-12] can be established, which is a first-order linear differential equation:

$$dx^{(1)}/dt + Ax^{(1)} = b \quad (4)$$

In order to obtain the parameters b and A , the matrices B and Y_n are constructed:

$$B = \begin{bmatrix} -\frac{1}{2}(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(x^{(1)}(n-1) + x^{(1)}(n)) & 1 \end{bmatrix}, Y_n = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \quad (5)$$

The least square method is used to estimate the values of parameters $\hat{a}$ and $\hat{u}$ of the first order linear differential as in Eq. 6.

$$\begin{bmatrix} \hat{a} & \hat{u} \end{bmatrix}^T = (B^T B)^{-1} B^T Y_n \quad (6)$$

After calculate $\hat{a}$ and $\hat{u}$, the values of parameters b and A of UGM can be obtained by follow:

$$b = \ln \frac{2 - \hat{a}}{2 + \hat{a}}, \quad A = \frac{2\hat{u}}{2 + \hat{a}} \quad (7)$$

Finally, the UGM of the sequence $x^{(0)}$ can be obtained and can be used to predict:

$$\begin{cases} \hat{x}^{(0)}(1) = x^{(0)}(1) \\ \hat{x}^{(0)}(k+1) = A e^{bk}, k=1,2,3,\dots,n-1 \end{cases} \quad (8)$$

2.2 More format Markov model

Form Eq. 8, the residual sequence $z(k)$ will obtain, which:

$$z(k) = \hat{x}^{(0)}(k) - x^{(0)}(k) \quad (9)$$

Taking the residual sequence $z(k)$ as a Markov chain [11, 14-17], according to the maximum and minimum values of the sequence $z(k)$, it can be divided into m state intervals I_m , and the m th state interval is labeled as state E_m , all of the residual of $z(k)$ have their labels. From time k to next time $k+1$, the number of states E_m transferred to E_{m+1} in one step is count, and a $m \times m$ frequency matrix $(n_{ij})_{m \times m}$ is obtained. Through the frequency matrix, the one-step state transition probability matrix can be obtained as follows:

$$R = (p_{ij})_{m \times m} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}, \quad (p_{ij})_{m \times m} = \frac{(n_{ij})_{m \times m}}{n_i} \quad (10)$$

For example, if E_2 is the labeled state of $z(k)$, the frequency of state E_2 transfer to state E_1 in one step is E_{21} , and,

$$E_{21} = n_{21} \quad (11)$$

The frequency of state E_2 transfer to state E_i in one step is H .

$$n_2 = H = \sum_{i=1}^m E_{2i} \quad (12)$$

So the value of p_{21} is:

$$p_{21} = \frac{n_{21}}{n_2} = \frac{E_{21}}{\sum_{i=1}^m E_{2i}} \quad (13)$$

If the initial state of a sequence $z(k)$ which divided according to the state intervals I_m is $F^{(0)}$, after t -step transfer, the state became $F^{(t)}$:

$$F^{(t)} = F^{(0)} \times R^{(t)} (t \geq 1) \quad (14)$$

Thus, the future state of the object is predicted, and the t th step prediction value is:

$$\hat{z}(k) = \sum_{i=1}^m F^{(t)} \times I \quad (15)$$

which I is the intermediate value matrix for dividing the interval. Of course, for different research objects, I can also take different values according to the range of the state interval.

Combined with the above steps, the predicted value is:

$$\hat{x}(k+1) = \hat{x}^{(0)}(k) + \hat{z}(k) \quad (16)$$

2.3 Posterior difference method

In the process of prediction, the PDM [17-20] is used to check the prediction accuracy of model. The principle of PDM is calculated as follows:

Original sequence mean value:

$$\bar{x}^{(0)} = \frac{1}{N} \sum_{i=1}^N x^{(0)}(i) \quad (17)$$

Absolute error mean value:

$$\bar{e} = \frac{1}{N} \sum_{i=1}^N |e(i)|, \text{ which } e(i) = x^{(0)}(i) - \hat{x}(i) \quad (18)$$

Original sequence variance:

$$S_1^2 = \frac{1}{N} \sum_{i=1}^N (x^{(0)}(i) - \bar{x}^{(0)})^2 \quad (19)$$

Absolute error variance:

$$S_2^2 = \frac{1}{N} \sum_{i=1}^N (e^{(0)}(i) - \bar{e}^{(0)})^2 \quad (20)$$

Thus, the posterior difference ratio $C = S_2/S_1$ and small error frequency P are defined to evaluate the model. which:

$$P = P\{|e(i) - \bar{e}| \leq 0.6745S_i\} \quad (21)$$

Generally, the smaller C and the larger P were indicates that the prediction model has a better fitness with the original sequence, and the model accuracy is higher [20], the relationship between C , P and the prediction accuracy of the model is shown in Table 1.

Table 1. Value of C , P and model prediction accuracy level [17].

C	P	Level description
$C < 0.35$	$P > 0.95$	The model prediction accuracy is high.
$C < 0.50$	$P > 0.80$	The model prediction accuracy is acceptable.
$C < 0.45$	$P > 0.70$	The prediction accuracy of the model barely meets the requirements.
$C \geq 0.45$	$P \leq 0.70$	The model prediction accuracy cannot meet the requirements.

3 Example

As in ref. 6, from 1992 to 2001, the maintenance costs of equipment are:

$$X = \{1.15, 1.95, 2.35, 2.70, 3.08, 3.50, 3.89, 4.69, 6.50, 8.19\} \quad (22)$$

which is shown in Table 2.

Table 2. Calculation results of three prediction models.

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	C	P
Cost[million]	1.15	1.95	2.35	2.70	3.08	3.50	3.89	4.69	6.50	8.19	-	-
Grey separating model[6]	1.1815	1.8311	1.9322	2.3374	2.8282	3.4228	4.1434	5.0169	6.0759	7.3600	0.1609	1
UGM	1.150	1.6935	2.0489	2.4790	2.9993	3.6288	4.3904	5.3119	6.4269	7.7758	0.1580	1
UGMM3	1.150	1.5896	2.1246	2.5713	3.0931	3.7225	4.4839	5.4054	6.5204	7.8693	0.1686	1
UGMM4	1.150	1.8055	2.1177	2.5590	3.0713	3.6856	4.4382	5.3578	6.4727	7.8226	0.1539	1
UGMM5	1.150	1.8659	2.1523	2.5590	3.1115	3.7246	4.4772	5.3966	6.5115	7.8611	0.1551	1

NOTE: UGMM3, UGMM4 and UGMM5 represents the unbiased grey Markov model with 3, 4 and 5 state intervals respectively.

3.1 Unbiased grey Model calculation

As in Eq. 2, Eq. 3, the new accumulation sequence is:

$$X_1 = \{1.15, 3.10, 5.45, 8.15, 11.23, 14.73, 18.62, 23.31, 29.81, 38.00\} \quad (23)$$

As in Eq. 5, the matrices B and Y_n can be calculated in follow:

$$B = \begin{bmatrix} -2.125 & 1 \\ -4.275 & 1 \\ -6.800 & 1 \\ -9.690 & 1 \\ -12.980 & 1 \\ -16.675 & 1 \\ -20.965 & 1 \\ -26.560 & 1 \\ -33.905 & 1 \end{bmatrix}, Y_n = \begin{bmatrix} 1.95 \\ 2.35 \\ 2.70 \\ 3.08 \\ 3.50 \\ 3.89 \\ 4.69 \\ 6.50 \\ 8.19 \end{bmatrix} \tag{24}$$

As in Eq. 6 , Eq. 7, the parameters in UGM can be calculated as: $\hat{a}=-0.1900$, $\hat{u}=1.2667$, $b=0.1905$, $A=1.3997$.
Then the UGM is:

$$\begin{cases} \hat{x}^{(0)}(1) = 1.150 \\ \hat{x}^{(0)}(k+1) = 1.3997e^{0.1905k}, k = 1,2,\dots,9 \end{cases} \tag{25}$$

$-\hat{a}$ is less than 0.3, which also indicates that the model can be used as a medium and long-term predication [19], and the prediction results are shown in Table 2.

3.2 Markov model calculation

As in Eq. 9, the residual sequence is:

$$z(k) = \{0,0.2565,0.3011,0.2210,0.0807,-0.1288,-0.5004,-0.6219,0.0731,0.4142\} \tag{26}$$

In the $z(k)$, the maximum value is 0.4142 and the minimum value is -0.6219. According to the range, it can be divided into four states, and the results were be note as UGMM4. The interval of each state which divided is show in Table 3. Of course, it can also be divided into other states.

Table 3. The states and the interval of each state

States	E1	E2	E3	E4
Interval of each state	[-0.6219 -0.3629]	[-0.3629 -0.1039]	[-0.1039 0.1552]	[0.1552 0.4142]

The state division of $z(k)$ is shown in Table 4.

Table 4. States of residual sequence value

Residual value	0	0.2565	0.3011	0.2210	0.0807	-0.1288	-0.5004	-0.6219	0.0731	0.4142
States	E3	E4	E4	E4	E3	E2	E1	E1	E3	E4

According to Table 4, the frequency of transitions from state E1 to state E1 in one step is 1, which is abbreviated as E11=1, and the same reason, E13=1, E12=E14=0, then,

$$p_{11} = \frac{E_{11}}{E_{11} + E_{12} + E_{13} + E_{14}} = \frac{1}{1+0+1+0} = 0.5 \quad (27)$$

As in Eq. 11, Eq. 12 and Eq. 13, the one-step transition matrix and median matrix can be calculated as:

$$R^{(1)} = \begin{bmatrix} 0.5 & 0 & 0.5 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0.3333 & 0 & 0.6667 \\ 0 & 0 & 0.3333 & 0.6667 \end{bmatrix} \quad I = \begin{bmatrix} -0.4924 \\ -0.2334 \\ 0.0257 \\ 0.2847 \end{bmatrix} \quad (28)$$

E. g, the cost of 1993 will be predicted by the cost of 1992. As in Table 4, the state of the residual in 1992 is E3, so the initial state interval $F^{(0)}$ is $[0,0,1,0]$, as in Eq. 14, Eq. 15 and Eq. 28, the residual cost of 1993 is:

$$\hat{z}(2) = S^{(0)}R(1)I_{10} = [0,0,1,0] \times \begin{bmatrix} 0.5 & 0 & 0.5 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0.3333 & 0 & 0.6667 \\ 0 & 0 & 0.3333 & 0.6667 \end{bmatrix} \times \begin{bmatrix} -0.4924 \\ -0.2334 \\ 0.0257 \\ 0.2847 \end{bmatrix} = 0.1120 \quad (29)$$

As in Eq. 9, Eq. 25, Eq. 29 and Table. 2, the cost of 1993 is:

$$\hat{x}(2) = \hat{x}(2) + \hat{z}(2) = 1.6935 + 0.1120 = 1.8055 \quad (30)$$

Fig. 1. Comparison of prediction results of each prediction method.

Similarly, in the same steps, the residuals of costs in various years can be predicted. The comparison between the prediction results is shown in Table. 2, and the diagram is shown in Fig. 1.

3.3 Analysis of state interval division

In order to analyze the influence of state interval division on prediction accuracy, the residual state interval is divided into three and five for comparison.

State was be divided into 3. The interval of each state which divided is show in Table 5.

Table 5. The states and the interval of each state.

States	E1	E2	E3
Interval of each state	[- 0. 6219 - 0. 2766]	[- 0. 2766 0. 0688]	[0. 0688 0. 4142]

The one-step transfer matrix and the intermediate value matrix are calculated as follows:

$$R^{(1)} = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0 & 0.2 & 0.8 \end{bmatrix}, I_{10} = \begin{bmatrix} -0.4492 \\ -0.1039 \\ 0.2415 \end{bmatrix} \quad (31)$$

The predicted results and value of C and P of PDM are shown in Table 2, and the results were be note as UGMM3.

State was be divided into 5. The interval of each state which divided is show in Table 6.

Table 6. The states and the interval of each state.

States	E1	E2	E3	E4	E5
Interval of each state	[- 0. 6219 - 0. 4147]	[- 0. 4147 - 0. 2075]	[- 0. 2075 - 0. 0002]	[-0. 0002 . 2070]	[0. 2070. 4142]

The one-step transfer matrix and the intermediate value matrix are calculated as follows:

$$R^{(1)} = \begin{bmatrix} 0.5 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3333 & 0 & 0.6667 \\ 0 & 0 & 0 & 0.3333 & 0.6667 \end{bmatrix}, I_{10} = \begin{bmatrix} -0.5183 \\ -0.3111 \\ -0.1039 \\ 0.1034 \\ 0.3106 \end{bmatrix} \quad (32)$$

The predicted results and value of C and P of PDM are shown in Table 2, and the results were be note as UGMM5.

3.4 Results

As in Table 1 and Table 2, the value of P are equal to 1, greater than 0. 95. The value of C are 0. 1609, 0. 1580 and 0. 1539, all of them are less than 0. 35, and the C of UGMM have the least value, it indicated that the three models can be used to LCC prediction, and the UGMM has the highest prediction accuracy. Also from Fig. 1, UGMM has the best fitness with true costs.

The calculation results show that the prediction accuracy of UGMM3 and UGMM5 are lower than that of UGMM4. Theoretically, the more the state interval is divided, the more accurately the sequence state transition is described, and the higher the accuracy of the prediction result obtained. However, limited by the number of data in the sample sequence, the more the state interval is divided, the accuracy will decrease.

4 Conclusion

In this paper, the LCC prediction was studied which based on UGMM, results indicated UGMM can be used in LLC prediction, and prediction accuracy is high. Also the prediction result not only can provide important support decision for the integrated logistics support of

equipment, but also has great significance for further improving the cost-effectiveness ratio of equipment costs.

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