Two-degree-of-freedom manipulator path planning based on zeroing neural network

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Abstract. In this paper, the shortest path problem of manipulator path planning is transformed into a linear programming problem, and solved by zeroing neural network (ZNN). Firstly, the method of constructing the zeroing neural dynamics is given, and the ZNN model is constructed for shortest path problem of manipulator. Then, the Lyapunov method is utilized to prove the stability of the ZNN model. Finally, the ZNN model is applied to the path planning of manipulator to generate an optimal planning path. The simulation results show that the proposed method can effectively realize the optimal path planning of the manipulator.

1 Introduction

In recent years, more and more scholars pay attention to the path planning of manipulator. The so-called path planning of 2-dof manipulator is to find a path of minimum length from the initial point to a given target point in the network. Robot path planning algorithm problems include: traditional algorithms and intelligent algorithms, such as A* algorithm [1], Dijkstra algorithm [2], ant colony algorithm [3-4] and so on. Many scholars have introduced intelligent algorithms such as genetic algorithm [5] and neural network algorithm [6] into robot path planning. Adamu carried out global motion planning algorithm by considering ways to improve the speed, using particle swarm optimization (PSO) technology to converge to the global minimum, and using custom algorithm to generate search space coordinates [7]. Han will improve the three-exchange crossover heuristic operator in the genetic algorithm to produce more optimized descendants to obtain more information and obtain the best shortest total path distance [8].

When the neural network considers different internal and external factors, especially when the nonlinear activation function is added to the network model, the traditional algorithm cannot be used. Excitingly, in this paper, a ZNN method for solving the shortest path problem of a robot arm is proposed. This method expresses the optimal path planning problem as a linear programming problem, and solves the linear programming problem by

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using the ZNN model to solve the shortest path in the moving process of the robot arm. Zhang et al. proposed the ZNN model for the first time and made great progress in stability analysis. ZNN can solve the problem of estimation error convergence effectively and improve the computational efficiency and accuracy of model [9].

Before ending this introductory section, the main contributions of this paper can be described as follows: there are 3 important contributions in this paper. The first contribution is to translate the shortest path problem into a linear programming problem and solve it with ZNN. Secondly, the ZNN model is strictly proved and studied by Lyapunov theory, which proves the stability of the model. Finally, the effectiveness of the algorithm is proved by numerical simulation experiments. Surprisingly, the shortest path problem is solved when applied to the two-degree-of-freedom manipulator.

\section{2 Preliminary theory}

Consider the following time-varying matrix inverse problem

\begin{equation}
C(t)X(t) = d(t), \quad t \in (0, +\infty)
\end{equation}

where, \( t \) represents time, \( C(t) \in R^{n \times n}, d(t) \in R^n \), \( X(t) \) is an unknown matrix. In this paper, the unknown matrix \( X(t) \) is solved by constructing ZNN model.

Build the ZNN model as follows:

Step 1: design the error function

\begin{equation}
e(t) = C(t)X(t) - d(t)
\end{equation}

Step 2: according to Zeroing neural dynamics (ZND) formula, deriving the formula 2, and the relationship between the error function and the activation function is obtained as follows

\begin{equation}
\frac{de(t)}{dt} = -\Gamma F(e(t))
\end{equation}

where, \( F(x) : R^{n \times n} \to R^{n \times n} \) is the activation function in the neural network, which generally satisfies the monotone increasing odd function property, and \( \Gamma \) is a scalar.

The activation function can accelerate the convergence performance of ZNN model. If the selected activation function \( F(\cdot) \) is monotone increasing odd, then the ZNN model is globally convergent.

\section{3 Problem Description}

In this section, the optimization problem and the proposed ZNN model will be described.

Deriving the error function over time yields the following equation:

\begin{equation}
E(x(t),t) = -\gamma F(E(x(t),t))
\end{equation}
The parameters $\gamma > 0$ and design parameters should be in accordance with the allowable range of the computer, $F(x): \mathbb{R}^{\text{nn}} \rightarrow \mathbb{R}^{\text{nn}}$ is the activation function, the following ZNN model calculation formula is obtained:

$$E(x(t),t) = (t)A(t)A^T(t) + x(t)A(t)A^T(t) + x(t)A(t)A^T(t) - A^T(t)$$

(5)

and

$$x(t)A(t)A^T(t) = -\gamma F(E(x(t),t)) - x(t)A(t)A^T(t) - x(t)A(t)A^T(t) + A^T(t)$$

(6)

Add $x(t)$ to both sides, we can get

$$x(t) = x(t)[I - A(t)A^T(t)] - \gamma F(E(x(t),t)) - x(t)A(t)A^T(t) - x(t)A(t)A^T(t) + A^T(t)$$

(7)

Among them, $A(t)$ is the time-varying matrix, $x(t)$ is the state variable of the neural network and $F()$ is the activation function. Using the ZND formula, we can get

$$\dot{E}(t) = -\gamma F(E(t))$$

(8)

For each element in the error matrix $E(t)$, we can get

$$e_{ij}(t) = -\gamma f(e_{ij}(t))$$

(9)

where $e_{ij}(t) \in \mathbb{R}$.

Theorem 1: Assuming that the activation function is a monotonically increasing odd function, the ZNN model satisfies the global convergence. By constructing the Lyapunov equation:

$$L(t) = \frac{\left\|e_{ij}(t)\right\|^2}{2} = \frac{Tr\left(e_{ij}(t)^T e_{ij}(t)\right)}{2}$$

(10)

Then the time derivative satisfies the following equation:

$$\frac{dL(t)}{dt} = \frac{Tr\left(e_{ij}(t)^T e_{ij}(t) + e_{ij}(t)^T e_{ij}(t)\right)}{2}$$

$$= -\gamma Tr\left(e_{ij}(t)^T F(e_{ij}(t)) + e_{ij}(t)^T F(e_{ij}(t))\right) = -\gamma Tr\left(e_{ij}(t)^T F(e_{ij}(t))\right)$$

(11)

By selecting the activation function as a power function:

$$f(x) = x^p \quad (p > 3)$$

(12)

Then equation (12) can be written as:
\[ \frac{dL(t)}{dt} = -\gamma \text{Tr}(e(t)^\top (e(t))^\top) \]

(13)

Because \( f(\cdot) \) is a monotonically increasing function, \( L(t) \) is negative. According to Lyapunov's stability theorem, any element of \( E(t) \) in the error matrix converges globally from an arbitrary initial value to 0, so the error matrix \( E(t) \) as a whole is global from any initial value. Convergence to zero. The theorem is complete.

4 Simulation

In order to verify the feasibility and effectiveness of the proposed algorithm, MATLAB simulation software was used to simulate the experiment. Code was run in MATLAB R2014a environment. Firstly, the design method is compared with the traditional CNN model to prove the superiority. Secondly, the design method is applied to the path planning of the two-degree-of-freedom manipulator to solve the path planning problem of the manipulator.

In the ZNN model, select the parameters \( \gamma = 1000 \). Consider the shortest path overall map shown in Figure 2, which consists of 7 points. Node 1 is the target vertex. The numbers shown in the figure are the values randomly generated by the system. Figure 2 shows the optimal path map for node 1 to node 3, the shortest path of which is the sum of the distances of node 1 - node 5 - node 3. As can be seen from Figure 2, the algorithm solves the problem of optimal target selection in path planning.

![Node Diagram](image)

**Fig. 1.** Path diagram of the system.
Fig. 2. The vertex 3 to vertex 4 of min distance.

Figures 3 and 4 show the distance maps of the two neural networks. Figure 4 show the distance from the remaining vertices to the vertices 1 under CNN (the upper limit of the iteration is 200 times). Figure 3 is the distance map under ZNN. The initial conditions are the same.

Fig. 3. ZNN curve of the distance to vertex 1.

Fig. 4. CNN curve of the distance to vertex 1.
Figures 5 and 6 represent the designed approach and CNN model in the path planning, where the distance between the two vertices to the target vertices (due to the typesetting restrictions, other simulation diagrams are not listed, contact the author if necessary). It can be seen from the figure that with the increase of the number of iterations, the approach is superior to the CNN model regardless of the distance from which vertex to the target point. Figures 7 shows the calculation error $\|e(t)\|_2$ image of the zeroing neural network, it can be seen that the error is getting smaller and smaller, which shows that the designed approach is very good for the accuracy of path planning. The simulation can prove that the approach has more advantages for the traditional path algorithm than the traditional algorithm.

Fig. 5. Compared with the CNN model in X2.

Fig. 6. Compared with the CNN model in X3.

Fig. 7. Error of neural network algorithm.
Next, the proposed method is applied to the two-degree-of-freedom manipulator path planning problem. The mechanical arm model is shown in Figure 12 and is a simple rigid model. Figure 13 shows the shortest path diagram of the two-degree-of-freedom robotic arm moving in the maze, where green is the starting point, red is the target point, black is the obstacle, and white is the space in which the robot arm moves. Through simulation, it can be seen that the proposed method can solve the path planning problem of the robot arm and make it move to the optimal position with high efficiency.

![System model](image)

**Fig. 8.** System model.

![Manipulator path planning simulation results](image)

**Fig. 9.** Manipulator path planning simulation results.

### 6 Conclusion

In this paper, for the highly nonlinear two-degree-of-freedom manipulator system, the path planning of the manipulator is expressed as a linear equation and the corresponding linear equation is solved by the method of the tension neural network. The asymptotic stability of the neural network algorithm is proved by Lyapunov stability theory. In addition, an example of finding the shortest path during the movement of the two-degree-of-freedom manipulator is provided to verify the feasibility and effectiveness of the theoretical results of the designed approach.

In the study of the manipulator path planning, Lyapunov stability was verified by ZNN, but the most basic activation function type was chosen. With the development of mathematical theory in the future, new models should be deduced to solve linear...
optimization problems. All future developments will be accompanied by computational mathematics and robotics for building and developing neural networks. How to establish a nonlinear activation function to accelerate the convergence speed of the zero-return neural network to solve the complex value problem remains unresolved.

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