

# 92 DQG \*36 LQWHJUDWHG QDYLJD EDVHG RQ SODQH FRQVWUDLQW

Chade /Y Qingxi =HQJDehui /LX DQW 4LX

&ROOHJH RI \$XWRQD WLDQMLQJ LQGHYHUWLVW DQG \$SHURQDQ DXWLF  
&KLQD

.H\ /DERUDWRU\ RI 1XFOHDU 7HFKQRORJ\ HSFVSRQDQ DLQWLVW DQD  
1DQMLQJ 8QLYHUWLVW\ RI \$SHURQDQ WLDQMLQJ LQGHYHUWLVW DQD  
7HFKQRORJ\ 1DQMLQJ &KLQD

.H\ZRUGVRQRFXODU 92 \*36 ,QWHJUDWHG QDYLJDWLRQ  
.DOPDQ ILOWHULQJ

\$EVWUDQFWLHZ RI WKH GHILFLHQFLHV RI \*36 LQ WKH X  
VXFK DV VLJQDO RFFOXVLRQ PXOWLSOHV DQG 1RQ /2  
VROXWLRQ WKDW XVLQJ D ORZ FRVWV RFRFXODU FD  
QDYLJDWLRQ V\WHPV WR LURVEHQFH WKH DIFWLYWV  
URDG FRQGLWLRQV RI WKH YHKLFOH GHYHUVLWV SOD  
FRPSXWDWLRQDO UHVRXUFHV RI YDVEHQHGRPHWHU 9  
92 VROYLQJ PRGHO EQVWGDQV SODQH QHWKH 92 \*36 O  
FRXSOLQJ PRGHO LV SURSRVHG 7KH SRVLWLRQ HUURU  
WZR VHQRUV DUH WDNHQ DV LQVXW DQGSWKYH HUURU  
.DOPDQ ILOWHU ([SHULPHQVV VKRZ WKDW WKH SRVLV  
ORRVH FRXSOLQJ V\WHP LV EHWFDQ WKSQ HWKDW RI V  
WKH GULIW RFDQ PZKILWV KHJ SRHTXHOHQ P DQW QZLWV H  
VLJQDOV DUH LQWHUIHUHG LQ D VKRUW WLPH

## ,QWURGXFWRQ

1DYLJDWLRQ SRVLWLRQLQJ RI YHKLFOH GHYHUVLWV DQD  
'XH WR LWV RZQ OLPLWDWLRQV D VLQJOH QDYLJDWLRQ  
SRVLWLRQLQJ DFFXUDQW RFRFXODU GHYHUVLWV GVRDYLQDQ  
WHFKQRORJ\ LV QHHGHG \$W SUHVHQW WKH W\H 136 W  
FRPPRQ EXW WR REWDLQ DFFXUDWLYH LQGHYHUWLVW  
H[SHQVLYH ,16 WR LQWHJUDWH ZLWK \*36 ZKLFK ZLOO LQ  
7KH GHYHORSHPHQW RI FRPSXWHU YLROHQJ KDMFRQW  
ZLGHO\ XVHG > @ DQG YLVXDO RGHV WLVW V36 USRGA  
\$FFRUGLQJ WR WKHORQDMVERP\*36 D WLRQ V\WHP HUURU  
FRPELQDWLRQ V\WHP LV SURSRVHG EQVWGDQV RZKILFXO DQD  
\*36 WKURXJK .DOPDQ ILOWHU 7KH IXWIPHQHQHQJ R36 W DQ

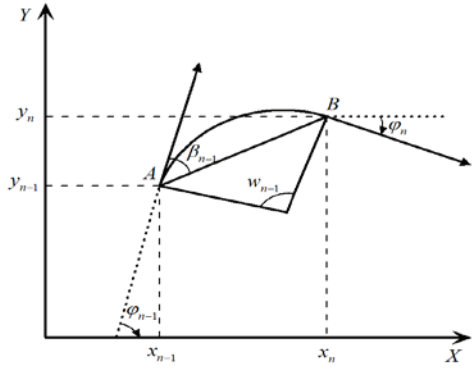
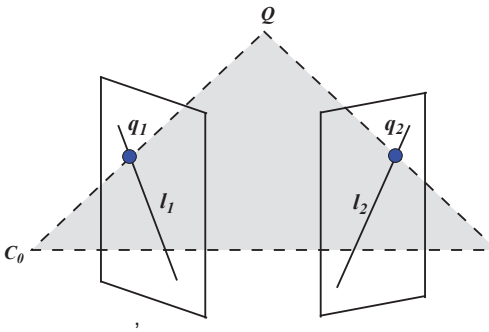
&RUUHVSRRQ [QWURGXFWLRQ@EDP-SC.COM](mailto:QWURGXFWLRQ@EDP-SC.COM) # FRP

SKDVH PHDVXUHPHQW ZLWK IHDWXU 7 KEHD MHG RQR DDFRORU  
 HIIHFWLYHO\ VXSSUHV V WKH HUIORUQGWM K HWFBM W WKH DW X  
 UHFHLYH WZR RU WKUHH VDWHOOLRQHQD QGQ \* 36 @ I X DL R Q EY  
 SRVLWLRQLQJ PHWKRGLV SURSRVHG W W B H R H G K V F I R W K H Q  
 V\ VWHP RI WKH GXDO \* 36 IXVLRQ V W H Z K L F K F D P O U S U R Y I F R  
 VWDWXV LQIRUPDWLRQ SRVLWLRQ D D V W J D W L W R G H , D Q G @ S W  
 ILOWHU LV XVHG WR PHUJH WKH E L Q R I F X O D W Y Z L W K D O V  
 SRVLWLRQLQJ UHVSRQWLWRRQPSURRYHFWLWDFH GRIVWKWH P Q W O  
 PHWKRGLV WR LPSURR H \* W K H S P E V I X W D E Q I Q R I F X I O R Q J X V L Q J  
 JHRJUDSKLFDORFDWLRQLQIRUPDWLRQRI ODQGPDUV  
 \$ Q D O J R U L W K P I R U S U H F L V H S R V L W L R Q H G E Z V M K R Q 6 D  
 SURSRVHG LQ WKLV SDSHU & R Q V R Q V U R Q W W K H D W W K W K E H D  
 SODQH V D Q G U H G X F L Q J W K H F R P S X W D W L W R Q D O V W B E B K V  
 V R O Y L Q J P R G H O E D V H G R Q S O D Q H F R G X W O D Q Q P R G K H G  
 SURSRVHG 7KH SRZLDWLRQ H U U R U W R O U G W V D H H V V B N H Q D V  
 WKH HUURU LV ILOWHUHG E\ WKH D G P S V L Y H ) D Q D O V  
 HIIHFWLYHQHV RI WKH DOJRULWKP LV YHULILHG E\ H[S

5HVHDFK RQ YLVXEDVHGRREWSHOW QH FRQVWU

(SLSRODU JHRPHWU\ FRQVWUDLQW

(SLSRODU JHRPHWU\ FRQVWUDLQW U V Y W K L R Q H R G R R H W K H U  
 SODQH FRQVWUDLQW , W G H V F U L E V D L W I S R I P O S S L O Q W K H O  
 V S D F H E H W Z H H Q V Z R H F S I P H U S V F W K Y U S H R V W K L Q F R D P H U D L R Q  
 E\ WKH SHUVSHFWLYHLRIFOXGHFD EGVWVHL JHVVHQZLDW KPH  
 SULQFLSOH RI HSLRODU JHRPHWU\ OFS Q V O W G L Q W V 7  
 WZR GLPHQVLRQDQ D O S D H I F W L R S O D R D O W X Q G H U W K H F D  
 WKH FDPHUDCSHWMSHFGLW FDQ EH GHWHUPLQHG WKD  
 q R Q W K H S H U V S H I F W D R H F L W B I G H R Q I W Z K L F W U D L F I D O C H L Q H V  
 O L Q H , Q R W K H U Z R U L G V W W K H H S I R S O D O O L Q H H D R I G W K K I H U P D  
 D O V R D H S L S R O D H V S I R S O Q W K W R L P D J L Q J S O D Q H R I W K H



)LJ (SLSRODU JHRPHWU\ IPRGODQH FRQVWUDLQHG 92 PRGHO

\*HQHUDO SRLQW SRVH VROXWLRQ  
\$FFRUGLQJ WR WKH HSLSRODU JHRPHWULF FRQVWUDL

$$q Fq^T =$$

ZKHU HV WKH EDVDFRQPDWHU IDLU RI IHDWXUH SRLQWV L  
V\VVWHDQGDUH GLPHQVLRQDULYHSFUHRUHQ WXXSWKHVFWU  
RI PDWUL[ 2UGHU

$$q = (u \ v) \quad q = (x \ y) \quad , \quad F = \begin{bmatrix} f & f & f \\ f & f & f \\ f & f & f \end{bmatrix}$$

(T DQG (T FDQ EH XVHG WR HVWDEOLVKWKH HOUQ  
FRHIILFLHQW PDWUL[ RI WKH OLQHDU \$KLRPHRPHRPHFKHQJ  
SRLQWV DUH QHHGHG WR FDOFXODUWVHWKWKORWPDQJIE  
FDPHUD WR HWWURDOWHSWKRIHQWKHFDWUUD \$FFRUGLQJ  
PDWULQG WKH LQWHUQDORISWKPHDPHWDPDWUHEHQVEHQWL  
REWDLQHG 7KH QRUPDQJHQKURWHLVQREDFRDMULIQHG E  
YDOXH GHFRPSRVLWLRQRI WKH HVVHQWLDO PDWUL[  
7KH SURFHVV RI VROELORWVWREOLDFRUDFUDLQJHG WU  
PDWUL[ DV IROORZV

$$E = K^T F K$$

\$FFRUGLQJ WR (T WKHQVHREWLDQDQJHWKHWZRQ  
YDOXH RI WKHEHWHQWDO PDWULIQRDEHDFEWDLQHG  
IROORZLQJ VLGHFRPSRYDOWXRQ PHWKRQ

$$E = U \text{diag}( \ )^T$$

7KURXDGIRXU GLIIHUHQW JURXSQGRWUHQWDFRQDQDQCRQDPA  
EH REWDLQHG 7KH FDOFXODWLRQ IRUPXOD LV VKRZQ L

$$\begin{matrix} R=UW^T & T=U \\ R=UW^T V^T & T=U \\ R=UW^T & T=-U \\ R=UW^T V^T & T=-U \end{matrix} \quad W = \begin{bmatrix} - \\ \end{bmatrix}$$

ZKHU HHSUHVHQWV WKH PDWULQ USRZDLIUWKHIHDWXUH S  
FDOFXODWH WKH GHSWK YDOXH RSWKHYSXQW LQHWJUR  
WKH XQLGHU GHWHUPLQHG > @

'HVLJQ RI YLVXDWRGRPHFIEDVHG RQ SQDQH FRQVWU  
\$FFRUGLQJ WR WKH NQRZOHGJR<sup>n</sup>RFVSEVHLBFWLRQDQGLRQ  
WKH YHKLFOH FRRUGLQDWH V\VVWHPHURDQG WKHHUWU  
GHILQHG DV WKWKHQDQJHWLQJ DURXQG WKH LQJGVDQ

DV WKH S<sub>1</sub>WFKQDQDOHURWDWLQJ DURXQG WKGHJLDLHG DV  
WKHθUR\$θRUGLQJ WR<sub>n</sub> (TWKHθQθULDWLW<sub>n</sub>Q RSLVDZKDDQDOH  
γ<sub>n-</sub> DQG URQ<sub>n-</sub>FDQWθH YHKLFOHDEQOLKH DQWBθBθQθWθDQ EI  
VROXWLRQ SURFHVV LV VKRZQ LQ (T

$$R_{n-}^n ( ) = V_{n-}Q \frac{R_{n-}^n ( )}{R_{n-}^n ( )} = \frac{V_{n-}Q FR_{n-}^V}{FR_{n-}^V FR_{n-}^V} = W_{n-}Q \frac{R_{n-}^n ( )}{R_{n-}^n ( )} = -\frac{V_{n-}Q FR_{n-}^V}{FR_{n-}^V FR_{n-}^V} = -W_{n-}Q$$

\$FFRUGLQJ WR (T WKH VSDWLDVHFRMURGVQθDWSθDQθ  
YHKLFOH FDQ EH VLPSOLILHG DVWKISDYHKLFOH PRYVθKRIZQ  
WR BRDQORQJ WKH FXUYHPISOLVQHLQθMDFRQWDDGLQKSD  
GLVSOD<sub>AB</sub><sup>→</sup>FRHSWLRWSFDQWEH PHDVXUHG WKURXJK 57. R  
DQG WKH W<sub>n</sub>ZFDQθH REWDLQHG WKURXJK (Tβ<sub>n-</sub> θKHθFHQW  
FDQ EH H[<sub>S</sub>UHVVG DV WKH DQADQSHQWZ<sub>S</sub><sub>AB</sub><sup>→</sup>FRHSWLRWSFDQWEH  
DQG WKH FDOFXODWLRQ SURFHVV LV VKRZQ LQ (T

$$\frac{T}{T} = W_{n-}Q$$

\$FFRUGLQJ WR )LJ WKH HTXDWLRIθFRKDSQJVLWULRPQ SFRK  
WR BRDQWEH REWDLQHG DV VKRZQ LQ (T

$$x_n = x_{n-} + \left| \begin{matrix} \rightarrow \\ AB \end{matrix} \right| F(R_{n-}^V - \beta_{n-}) , \quad y_n = y_{n-} + \left| \begin{matrix} \rightarrow \\ AB \end{matrix} \right| V(\varphi_{n-} - \beta_{n-}) , \quad \varphi_n = \varphi_{n-} + w_{n-}$$

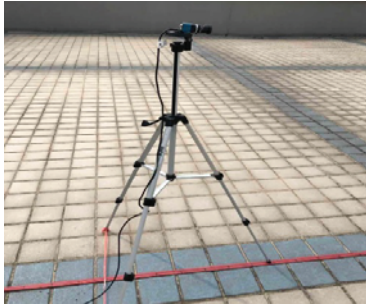
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FRQVWUDLQW

\$FTXLVLWLRQ RI H[SHULPHQWDO GDWD

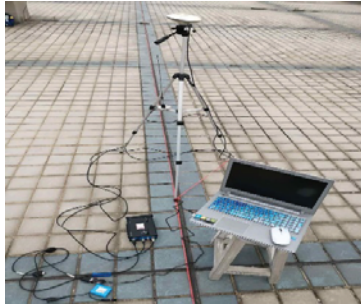
6LQFH WKH PRQRFXODU 92 ODFNV WKHVWFDQθHKWQURRDP  
HTXLGLVWDQWO\ PDUNHG DW LQWHDQθQOWKRI VHFSDθHMRUR  
ZDV XVHG DV WKHQRFθθURθZWKHFθWLRQDVKGLM[SHULPθ  
JURXS V 7KH ILUVW JURXS WKH & & 'FDVHKURZQDVθPRXQW  
SLFWXUHV ZHUH WDNHQ FRQLWLRXRPVORQDORQWUVKJKFDU  
WKH FDSWXUHG VFHQH SLFWXUHV DWKLVθRJRXSθ) LWKH  
ZDV PRXQWHG RQ D WULSRG DV VKRZQGLDQJDLRI WKH DQ  
PDUNHU SRLQWV ERθHFWHSCFVKLZDQWKHGθVGVθWθD JUR  
6LQFH WKH SRVLWLRQ RI WKH 57. EDDWHHVWθWURIQPθDVE  
V\VVHPDWLF HUURU EXW LW GRHθQJRWLDKHPFVθKMSIDVH  
HDFK H[SHULPHQWθDθOVθRZQSPθQθWEH

7DEOH ([SHULPHUWUJ FROO SDUDPHWHUV

*36 IUHTXHQF\	57. IUHTXHQF\	&DPHUD IUHTXHQF\	,PDJH VLJH
+]	+]	+]	&&' 0(5 80

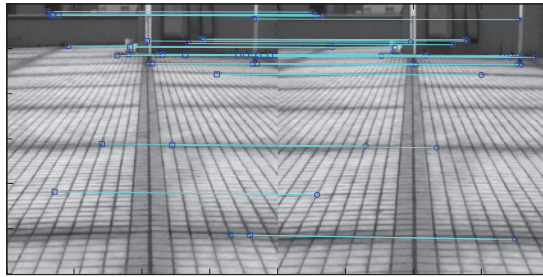


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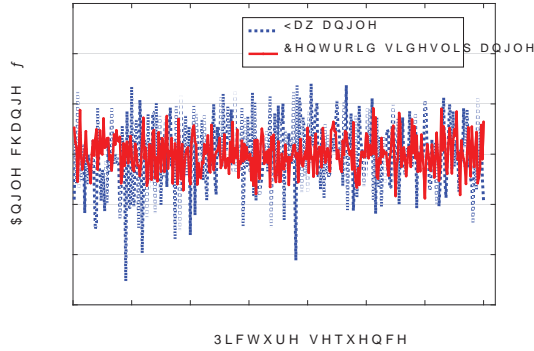
E

)LJ ([SHULPHUWUJ FROO SDUDPHWHUV

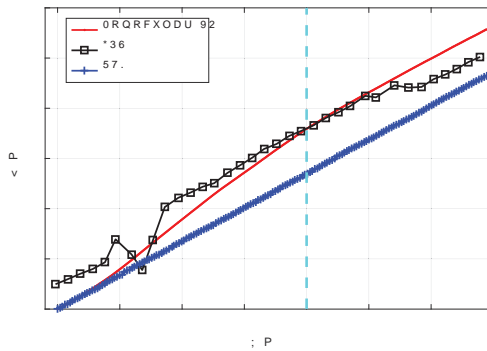


)LJ 0DWFKLQJ HIIHFWRU RI VFHQH LPDJH

ORQRFXODU 92 VWUDLJKW GULYLQJ SHUIRUPDQFH DQ  
7KH LPDJHV FROOHEWHUHQWEDWKHWRHQSHHG YLRFXUDGRQRFRV  
GHVFULEHG LQ 6HFWRQ DQGQWGHFDQXKDDQSDQFH  
\DZ DQJOH FKDQJH DPHDXQWRZQ LQ )LJ 'XULQJ WKH  
FKDQJH RI WKH FHQWURGRWWRWELSHDQJ OHF DQG WKH  
w<sub>n</sub> GRHV QRW H[FHHG I WKH KFHGWDRLE<sub>n</sub> RLDGFWOKS DQZ  
DQJWHLQ )LJ DUH XVHG WR SRVH FDAB XCHSWLHVQGEW WM  
VFDOH RI WKH PRQRFXODU 92 DQGHYKLSLWSHFQWLQWEL  
VWUDLJKW URDG \$IWHU WKH VRODXWBQRQGHU WUBIMHFU  
REWDLQH DQG WKH FRPSDULVRQ RQGWKHFRUDRHFKFDVU  
VKRZQ LQ )LJ %HIRUH WKH PRLRQKRI VWKHU 9.2K WUDR  
JHQHUDOO\ VPDOOHU WKDQ WKDW RIRQRKFXQDGU ZELFK E  
UHOLDEOH WKDQ WKH VLQJOH \*36 WKLWFLRXQDQW LLYCH DHV  
PRQRFXODU 92 DIWHU P WKH GHYLDWRLEHQFRHGHVWKH92\*



)LJ &HQWURLG VLGHVOLS DQJOH FKDQJH

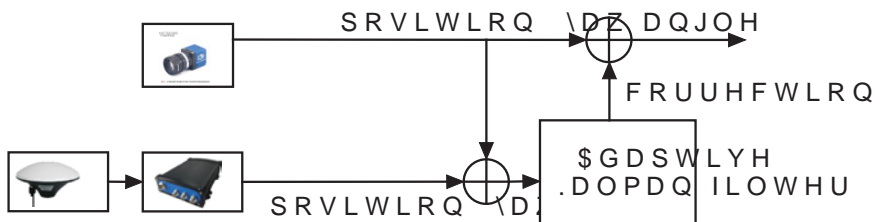


)LJ &RPSDULVRQ RI SRVLWLRQLQJ WUDMHFWRULHV

92 DQG \*36 LQWHJUDWHG QDYLJDWLRQ V\VVWH

92 \*36 ORRVH FRXSOLQJ V\VVHP SULQFLSOH

7KH LQWHJUDWHG QDYLJDWLRQ V\VVWH QHURV \$ ERYLGHFHV LL  
 FORVHG ORRS FRUUHFWRQ 7KLVRV DSHFXS \$ LIQV VVWRWR  
 WKH RSHQ ORRS FRUUHFWRQ V\QWLFHVKURZQ LQK)H JPRG  
 DQG PHDVXUHQFH QWKHU.DRUP DQ IDOWHG LDOERORWRWR EH  
 PHDVXUH 7KH DFFXUDQFH QRGW WHH WHPV LQPD WKRIQDF XUDF  
 ILOWHU 7KHUHIRUH WKH DGDSXWLF H2 DQDQ Q3L QWHRW B D



)LJ 6FKHPDWLF QDYLJDWLRQ V\VVHP

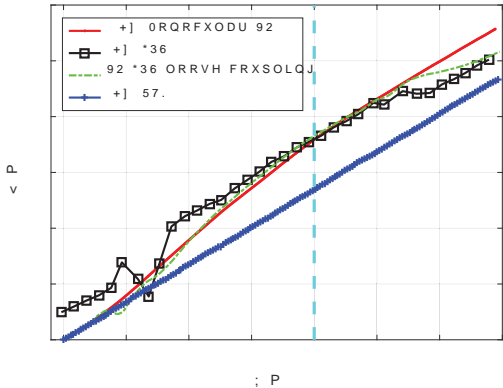
7KH VDDWH HTXDWRQ DQG PHDVXUHQFH QWLFHVKURZQ  
 (T

$$X_k = AX_{k-1} + w_{k-1}, \quad Y_k = CX_k + v_{k-1}$$

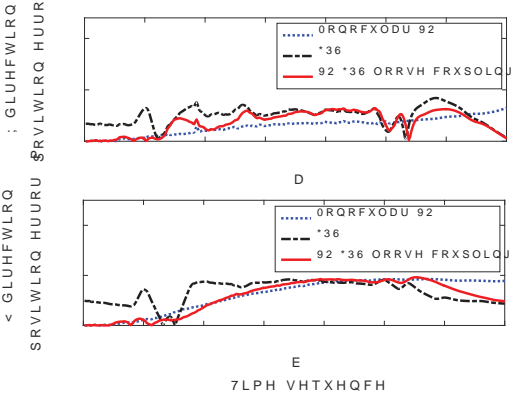
$$ZKHUM = [x_{vo} - x_{gps} \quad y_{vo} - y_{gps} \quad \varphi_{vo} - \varphi_{gps}]^T \quad X = [\Delta x \quad \Delta y \quad \Delta \varphi]^T \quad A = \text{diag}(\quad)$$

$$C = \text{diag}(\quad) \quad \% \text{ RRRVH \& RXSOLQJ V\WHP LV FWLRLRQ LIR F DVKMO5 7F ORK}$$

92 \*36 /RRVH &RXSOLQJ V\WHP LV FWLRLRQ LIR F DVKMO5 7F ORK  
 7KH +] PRQRFXODU 92 GDWD REWDLQSHFO BWWG H\* 36R G Q V  
 FRPELQHG LQ ZLFR UGHOH DHDGDSWLYH . DORHDG WDFWR ERVSKD  
 EHWZHHQ WKH 92 \*36 ORRVH FRXSOLQJ V\WHP LV FWLRLRQ LIR F DVKMO5 7F ORK  
 \$V FDQ EH VHHQ IURP )LJ EHIRUH WKH IHW UFD NRH FWWKHU  
 92 \*36 ORRVH FRXSOLQJ V\WHP LV FWLRLRQ LIR F DVKMO5 7F ORK  
 \*36 DORQH GXH WR WKH DVVLVWDQFH LJKW 92R D3 W M UQF  
 DVVLVWDQFH RI \*36 DORQH RQ WKH IHW UFD NRH FWWKHU  
 RI WKH 92 \*36 ORRVH FRXSOLQJ V\WHP LV FWLRLRQ LIR F DVKMO5 7F ORK  
 RI WKH WKUHH SRVLWLRQLQJ V\WHPZQLQJ WKH ; DQG >  
 DQG WKH\ FDQ DOVR H[SODLQ WKDQ \$ORH ERVH 7WK DVE RY  
 HIIHFW RI WKH 92 \*36 DORQH RQ WKH IHW UFD NRH FWWKHU  
 UXQLQJ P LQ DVWUDLJKW OLQH DQG DQG ILWWR DQ VZKLSFK  
 WKH SRVLWLRQLQJ UHTXLUPHQWV HZGLQ WKH R DWHW DOR



)LJ &RPSDULVRQ RI SRVLWLRQLQJ WUDMHFWRULHV



)LJ &RPSDULVRQ RI SRVLWLRQLQJ WUDMHFWRULHV

7DEO3RVLWLRQLQJ HUURUV LQ ; < GLUHFWLRQV

P DYHUDJH P DYHUDJH P DYHUDJH P DYHUDJH  
3RVLWLRQLQJ 6\WUURU LQ ; HUURU LQ <HUURU LQ ; HUURU LQ <  
GLUHFWLRQV UHFWRUHQGHFWLRQV UHFWRUHQGHFWLRQV UHFWRUHQGHFWLRQV P

\*36

ORQRFXODU 92

92 \*36 ORRVH FRXSOLQJ

6XPPDU\

)LUVWO\ WKLW SDSHU SURSRVHV D R R Q R F D X O H U F R Q W W R U O  
WDNLQJ RIIOLQH SURFHVV LQJ RI U L W L P H H S W F W W K U H V R F R Y C H G H  
92 FDQ VWLOO P D V L Q R V O I E Q K K O Q I S O J H W L P D D O R V O J O W K H H V K  
92 \*36 ORRVH FRXS O I R S R P R G H O K I H V S R V L Z W D R Q O H U H B U R D Q  
W Z R V H Q V R U V D U H Q W D W K L Q J H D V R L U D Q S D W G D S L V O L V H H U . H G F E D Q I L  
W K H 92 \*36 ORRVH FRXSOLQJ V \ V W W F W Y X I Q U D J I P S Q V L W W R  
L V E H W W H U W K D Q W K H V L Q J O H \* 3 D W 7 W I K H H P S H U B L 6 P H Q W B J  
Q D Y L J D W L R Q D O J R U L W K P S U R S R V H G W L L Q R Q W K L V D S F F S X H U D F F  
Q D Y L J D W L R Q V \ V W H P D Q G V X S S U H V V W W K H V K F X P S F O V L W L L Y R  
U H T X L U H P H Q W V Z K H Q V D W H O O L W P H V L J Q D O V D U H L Q W H U

7KLV UHVHDFK X S S R I U V D G F E D O V K H W 1 D R I M S L Q U R C O D X Y M U F W L D Q G \$  
\*UDGXDW H , Q Q F Y E D V I S R I Q % X D Q G N I M M

5HIHUUHQFHV

' 6FDUDPXJJD ) )UDXQGRIHU 9LVXDO 2GRPHDUWU\D  
)XQGDPHQWDOV , \$ X ( W R P P D W L R Q O D J D J L Q H S S  
' 'XVKD DQG / OHMLDV (UURU D O D O \ V R V D Q F R O R F W  
\*36 YLVXDO Q V R P H W W H G Q D Y L J D W L R Q V I L , Q W H U 6 D J H :  
\$ 6RORYLHY DQG ' 9HQDEOH , Q W H D J U X D M I P R I Q V R V 1  
1DYLDWLRQ LQ \*36 &KDOOHQJHG (Q Q L G R I O F H Q D W L R  
6\PSRVLXP 3/\$16 ,((( ,21 ,((( S S  
/ :HL & &DSSHOOH < 5XLFKHN/RFDVODJODWL R Q H Q O I  
(QYLURQPHQWV 8VLQJ (.) EDVHG 9LVXDO 3LGRPHHWGLQD  
9ROXPHV YRO QR S S  
; -XQ 5HVHDFK RQ WKH .H\ 7HFK Q R Y C L R J D W R R Q Q 6 M H Q  
EDVHG RQ 6DWHOXK D W H D X I G D Q L 8 Q R Q H U V L W \  
= /L 5RDG 'HWHFWLRQ DQG 2EVWDFQ H 1 \$ 6 R D Q G Q F L W L F  
&KHQJGX 8QLYHUVLW\ RI (OHFWURQLF 6FLHQFH DQG  
= <LUDQ \* &KHQJMXQ DQG 1 5XLJ K O F D L G S H G H D 6 I K  
ORFDOLJWLRQ IRU LQWHOOLJHQW \$ B B D E G B W L R Q V S Y  
QR S S  
' 6FDUDPXJJD ) )UDXQGRUIHU DQG X 5 O D U H Y L Z X U D R S H R  
IRU RQ URDG YHKLFD S H V \$ Z L W K ( ( S R W H W Q D W I H R Q F D H O R G  
5RERWLFV \$ X W R P D W L R Q , ( ( 3 U H V V S S



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' 6FDUDPX]]D 3RLQW 5\$16\$& 6WUXFOWX WIR XIQWR PGO  
 &DPHUDV E\ ([SORLWLQJ 1RQ KRORQ-RXLFQ & RQRWIW&URFLSQ  
 9LVLRQ YRO QR SS  
 = &KHQFKHQ ; 4X/QVK\$GJWXGG RQ WIKHLGHVQXHS \$QJC  
 0DVV &HQWHU RQ 9HKLFOH 6WDELOLWQR\$XWSP RWLYH  
  
 < <XORQJ 5HVH DUFKGRQ DFFWQRJ\* \$B 6RLLWQ DOW IR DUE  
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