

Coupling matrix synthesis of general chebyshev filters

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Abstract. A single optimization algorithm based on SolvOpt that synthesizes coupling matrices for cross-coupled microwave filters is presented. The rules for setting initial values of SolvOpt are proposed to find global minimum of the cost function. SolvOpt method provides faster convergence and higher accuracy to find the final solution compared with hybrid optimization algorithms. Application examples illustrate the excellent performance and the validity of this method.

1 Introduction

Filtering structures with increasingly stringent requirements can often be met only by using cross-coupled resonators to generate finite transmission zeros. Both analytical and numerical methods for the synthesis of coupling matrices corresponding to cross-coupled filters have been extensively studied. A fundamental analytical theory of cross-coupled resonator bandpass filters was developed in the 1970s by Atia and Williams [1]. A slightly different, widely used, analytical technique based on generating the Chebyshev filtering functions with prescribed transmission zeros was advanced by Cameron [2]. Cameron further proposed “ $N + 2$ ” CM synthesis techniques for microwave filters with source/load-multiresonator coupling [3]. These analytical techniques produce a full coupling matrix (CM) which must be transformed to a form suitable for realizations by repeated matrix similarity transformations. The main difficulty with these methods is that the sequence of matrix transformations is not known in advance and may be difficult to derive, for example box sections configurations. Numerical methods can strictly enforce the desired topology compared with analytical methods. Amari proposed CM synthesis of microwave filters based on the local optimization method [4,5], which relies upon on the provision of a good initial guess. However, how to set initial guess values is not discussed. Recently, a class of hybrid optimization methods combining local search methods with global methods has been reported [6,7]. For example, the paper in [6] presented a method consists of a Levenberg-

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Marquardt algorithm for a local optimizer and genetic algorithm for a global optimizer, respectively. In [7] a genetic algorithm is combined with a sequential quadratic programming local search method to form a hybrid method. These hybrid optimization methods can find a global minimum, however, they need more iteration, and the process of synthesizing CM becomes very complex.

A single optimization method based on SolvOpt that synthesizes CM for cross-coupled microwave filters is presented in this paper. SolvOpt is a solver for local optimization problems. Local optimization methods relies upon on the provision of a good initial guess at the solution, however, synthesizing CM by optimization is not a purely mathematical problem, and considering that the filters can be realized on the physical structure, the limited range of values of CM elements can be known in advance, so, we can easily guess a good initial values for SolvOpt algorithm to synthesize CM. The rules for setting initial values of SolvOpt optimization method are proposed. One can judge whether a final solution is a global optimum from the cost function value of the solution, because the value of cost function is zero in theory. So, local search method based on SolvOpt can also be guaranteed to find a global solution.

2 Coupling matrix synthesis using solvopt method

For any two-port lossless filter network, the transmission function S_{21} and reflection function S_{11} may be expressed as

$$|S_{21}(\Omega)|^2 = \frac{1}{1 + \varepsilon^2 C_N^2(\Omega)} \quad , \quad |S_{11}(\Omega)|^2 = 1 - |S_{21}(\Omega)|^2 \quad (1)$$

Where Ω is the normalized frequency variable and ε is a ripple constant related to the passband return loss RL by $\varepsilon = [10^{RL/10} - 1]^{-1/2}$. C_N is the general Chebyshev filtering function given in [2, eq.(3)], which is determined by the filter order N and transmission zeros Ω_n ($n=1, 2 \dots N$). Here, $S_n = j\Omega_n$ is the location of the n th transmission zero in the complex s -plane, Cameron has proved that the number of transmission zeros with finite locations m must satisfy $m \leq N$, those zeros without finite locations must be placed at infinity. However, the two-port networks without source/load-multiresonator coupling will realize a maximum of $N-2$ finite-location transmission zeros [2, 3]. Amari has given a rigorous proof for the maximum number of finite transmission zeros of cross-coupled filters with a given topology [8, 9].

The S -parameters describe the response of a two-port filter network. The relation between S -parameters and the CM can be expressed as: for the case of “ N ” CM [4, eq.(8)]

$$S_{21} = -2j\sqrt{R_1 R_N} [A^{-1}]_{N,1} \quad , \quad S_{11} = 1 + 2jR_1 [A^{-1}]_{1,1} \quad (2)$$

for the case of “ $N+2$ ” CM with source/load-multiresonator coupling case [5, eq.(4)],

$$S_{21} = -2j[A^{-1}]_{N+2,1} \quad , \quad S_{11} = 1 + 2j[A^{-1}]_{1,1} \quad (3)$$

The normalized load and source resistors R_1 and R_N can be accurately calculated in this paper using Cameron's analytical method [2], and this is different with other optimization methods.

The elements of CM $M_{i,j}$ are known as the coupling coefficients and varying their values causes the response to change. The aim of the CM synthesis process is to select CM which causes (2) or (3) to produce a filter response coincide with the response obtained from (1).

The selection of an appropriate cost function is important for the success of any optimization method. The cost function given by Amari [4] is used for the current work as

$$K(x) = \sum_{k=1}^Q |S_{11}(\Omega_{pk})|^2 + \sum_{k=1}^P |S_{21}(\Omega_{zk})|^2 + (|S_{11}(\Omega=1)| - c)^2 + (|S_{11}(\Omega=-1)| - c)^2. \quad (4)$$

Here, $c = \varepsilon / \sqrt{1 + \varepsilon^2}$, P and Q are the number of finite transmission and reflection zeros, respectively, Ω_{zk} and Ω_{pk} are the location of the k th transmission and reflection zero at the normalized frequency, respectively. The variable x represents the set of control variables at the current iteration, that is, the elements of CM. The nonzero CM element $M_{i,j}$ will be used as independent variables in the optimization process. The gradient of the cost function needs to be used in SolvOpt algorithm. The gradient of the $|S_{11}|$ and $|S_{21}|$ with respect to $M_{i,j}$ was given in [4] for “ N ” CM case and [5] for “ $N+2$ ” CM case. The gradient of the cost function with respect to an independent variable $M_{i,j}$ can be derived from (4) as

$$\begin{aligned} \frac{\partial K}{\partial M_{i,j}} = & \sum_{k=1}^Q 2|S_{11}(\Omega_{pk})| \frac{\partial |S_{11}(\Omega_{pk})|}{\partial M_{i,j}} + \sum_{k=1}^P 2|S_{21}(\Omega_{zk})| \frac{\partial |S_{21}(\Omega_{zk})|}{\partial M_{i,j}} + 2(|S_{11}(\Omega=1)| - c) \frac{\partial |S_{11}(\Omega=1)|}{\partial M_{i,j}} \\ & + 2(|S_{11}(\Omega=-1)| - c) \frac{\partial |S_{11}(\Omega=-1)|}{\partial M_{i,j}}. \end{aligned} \quad (5)$$

The SolvOpt (Solver for local optimization problems) is concerned with minimization or maximization of nonlinear, possibly non-smooth objective functions or solve a nonlinear optimization problem [10]. This algorithm is selected to take advantage of the fast local search and simple usage properties. Its usage has a form: $[x,f]=\text{solvopt}(x,'fun','gradfun')$, where, the left-hand side is output variables: x is the solution point and f is the value of the cost function at the solution point, the right-hand side is input variables: x is a vector of the starting point, 'fun' provides the name of the M-file (M-function) of the cost function and 'gradfun' provides the name of the M-file that returns the gradient vector of the cost function at a point x . More details are introduced in [10].

Although SolvOpt optimization methods relies on the provision of a good initial guess at the solution, considering that the filters can be realized on the physical structure, Generally, magnitudes of the direct coupling coefficients are bounded by 0.1 and 1, and the cross couplings by 0 and 0.8. Rules of setting initial values for SolvOpt are proposed as follow:

For “ N ” CM, all cross and self couplings set to specific value ranged from zero to 0.2 and all direct couplings to specific value ranged from 0.4 to 0.6.

For “ $N+2$ ” CM, direct couplings $M_{S,1}$ (source to resonator 1) and $M_{L,N}$ (load to resonator N) set to 1, the rules of setting all remaining CM elements are the same as those of “ N ” CM.

We can synthesize “ N ” or “ $N+2$ ” CM easily and efficiently by minimizing a cost function based on the rules above of setting initial values.

The SolvOpt optimization algorithm begins with an initial set of control variables x , which consists of elements of CM, according to the rules given in this paper. It can be repeatedly performed for more accurate solution; the solution will be used as the initial values of the next iteration. SolvOpt algorithm will terminate, when the value of the cost function reaches a target value. However, when maximum iterations have been performed and a target value of the cost function has not been satisfied, set of control variables x will be re-initialized according to setting rules. Usually, the value of the cost function with a good initial guess

will reaches a value below 1.0×10^{-10} when two or three iterations have been performed. Generally, desired accuracy of the cost function will be obtained, when the guess number of initial set of control variables is one or two according to rules proposed in this paper.

3 Examples

In this section, for the verification of the presented method, it is applied to two examples of filter synthesis. Coupling schemes of three filters are shown in Fig. 1. In Fig.1, solid circle represents source or load; hollow circle represents the resonators; dashed line represents the cross coupling and solid line represents the direct coupling.

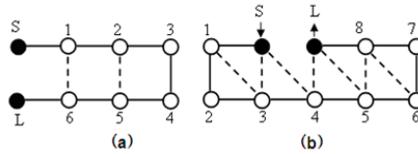


Fig. 1. Coupling schemes of three filters (a) filter 1. (b) filter 2.

3.1 Symmetric 6th-order filter (Filter 1)

This is an example of synthesise “ N ” CM. We consider a symmetric 6th-order filter with four transmission zeros at ± 1.592692 and ± 2.132335 and a passband maximum return loss of 20 dB (filter 1). The six reflection zeros locates at ± 0.9734 , ± 0.7498 , ± 0.2893 and $R_1=R_N = 0.9904$, which are calculated using Cameron’s method [2]. Coupling scheme of this filter is shown in Fig. 1(a). The initial guess of control variables, x , for this example consists of the following 7 variables, corresponds to setting all direct couplings, $M_{i,i+1}$, for $i=1,2, \dots, 5$, to 0.5; the cross couplings, $M_{2,5}$, and $M_{1,6}$ to zero. The value of the cost function in (4) reaches 1.636×10^{-10} when one iteration has been performed. Nonzero elements of “ N ” CM of filter 1 can be obtained as

$$M_{56} = M_{12} = 0.8300; M_{45} = M_{23} = 0.5788; M_{34} = 0.7061; M_{25} = -0.1660; M_{16} = 0.0194.$$

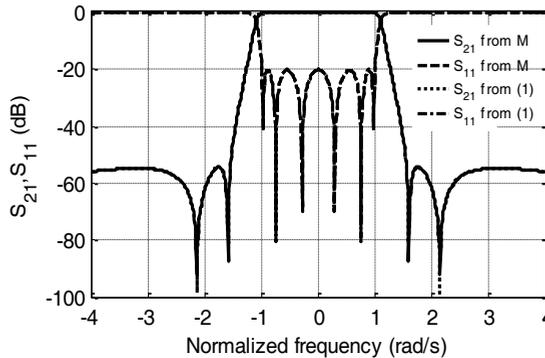


Fig. 2. The frequency responses of filter 1 (synthesized), as obtained from CM and the filtering function in (1).

3.2 Asymmetric 8th-order filter (Filter 2)

This is an example of synthesise “ $N+2$ ” CM. We consider an asymmetric 8th-order filter with seven transmission zeros (in this case, three real-axis and two complex pairs) at -1.196 ,

-1.45 , 1.62 , $-0.148 \pm j0.9040$, and $0.49 \pm j0.955$ and a passband maximum return loss of 20 dB (filter 2), these specifications are given in [6]. The eight reflection zeros are located at 0.97428 , 0.78139 , -0.98691 , -0.87098 , -0.61416 , 0.46504 , -0.26391 , and 0.10520 , which are calculated using Cameron's method [2]. Coupling scheme of this filter is shown in Fig. 1(b). The initial guess of control variables, x , for this example consists of the following 24 variables, corresponds to setting all direct couplings, $M_{i,i+1}$, for $i=1,2, \dots, 7$, to 0.6; the cross couplings, $M_{1,3}$, $M_{3,3}$, $M_{3,4}$, $M_{L,4}$, $M_{L,5}$, $M_{5,8}$, and $M_{6,8}$ to 0.2; the self-couplings, $M_{i,i}$, for $i=1,2, \dots, 8$, to 0.2 and source/load to resonator direct couplings $M_{S,1}$ and $M_{L,8}$ set to 1. The value of the cost function in (4) reaches 5.738×10^{-13} when two iterations have been performed. Nonzero elements of " $N+2$ " CM of filter 2 can be obtained as follows:

$$M_{1,2} = 0.7232, M_{2,3} = 0.4009, M_{3,4} = 0.5777, M_{4,5} = 0.5488, M_{5,6} = 0.5273, M_{6,7} = -0.5801,$$

$$M_{7,8} = -0.8101; M_{S,1} = 0.9710, M_{L,8} = 0.9884;$$

$$M_{1,3} = -0.3356, M_{S,3} = -0.0451, M_{S,4} = -0.1903, M_{L,4} = -0.0639, M_{L,5} = 0.0024, M_{S,8} = 0.0301,$$

$$M_{6,8} = -0.1449;$$

$$M_{1,1} = -0.0511, M_{2,2} = 0.5995, M_{3,3} = -0.1954, M_{4,4} = -0.0485, M_{5,5} = -0.0243, M_{6,6} = -0.0519, M_{7,7} = 0.1930, M_{8,8} = -0.0111.$$

Both the frequency response of the prototype as computed from (1) and that computed directly from the CM are shown in Fig. 3. The excellent agreement between the two, the difference is not visible in the figure, shows the accuracy of the SolvOpt method.

For the comparison, the value of cost function is equal to 1.203×10^{-6} , calculated by substituting CM in [6, 17(a), p.2164] into (4). More than 50 iterations are needed to converge for this example using the hybrid method in [6], however, SolvOpt method only needs two iterations to converge to 5.738×10^{-13} .

As can be seen, the proposed method provides faster convergence and higher accuracy to find the final solution than hybrid method in [6, 7].

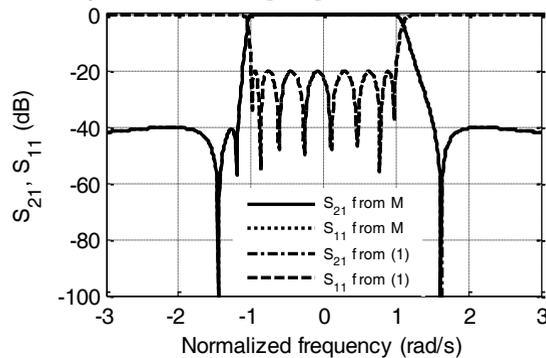


Fig. 3. The frequency responses of filter 2 (synthesized), as obtained from CM and the filtering function in (1).

4 Summary

A single SolvOpt algorithm that synthesizes coupling matrix for cross-coupled microwave filters with or without source/load-multiresonator coupling has been presented, and its initial set has been proposed for fast convergence and good accuracy. The method has been applied to synthesis of filters with varied orders and symmetries and has yielded excellent results, which show simplicity, efficiency and accuracy of SolvOpt method, even for filter responses with large numbers of control variables to be optimized. The proposed SolvOpt algorithm

simplifies the process of extracting CM and provides faster convergence and higher accuracy to find the final solution, compared hybrid optimization methods.

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