ADE-FDTD method for study of two-dimensional electromagnetic wave propagation in lossy lorentz medium

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Abstract. In order to study the reflection of electromagnetic wave in Lorentz medium layer, the finite difference time domain method of auxiliary differential equation (ADE-FDTD) is used to derive the difference formula of two-dimensional TM wave propagating in lossy Lorentz medium, and the reflection coefficient of reflection field is calculated in one-dimensional case. The calculated reflection coefficients coincide very well, which shows that the derived propagation formula of two-dimensional TM wave in lossy Lorentz medium is correct. In addition, the reflection of plane electromagnetic wave by infinite high Lorentz medium layer is also simulated. The results show that the reflection of plane electromagnetic wave by Lorentz dispersive medium layer is correct.

1 Introduction

Many practical engineering problems require broadband analysis, such as the calculation of s parameters, antenna input impedance and target radar cross section. Therefore, it is of great significance to seek efficient broadband algorithm. When broadband electromagnetic waves propagate in complex dispersive media, the dispersive properties of media play an important role. The finite-difference time-domain method (FDTD) was first proposed by Yee [1] in 1966 to solve the direct time-domain numerical solution of Maxwell equation. It treats all kinds of problems as initial value problems, which directly reflects the time-domain nature of electromagnetic field and has wide applicability.

The application of FDTD method to dispersive materials has been studied by many people up to now. The methods used include discrete convolution method based on dispersion relation [2-4], Z transform method [5-6], auxiliary differential equation (ADE) [7-9], and Mobius transform method [10]. ADE method is an effective method for calculating electric field intensity vector ($\vec{E}$) and electric flux density ($\vec{D}$) in dispersive media.

The iteration steps of ADE-FDTD in dispersive media are as follows:

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1) According to the general FDTD formula, the magnetic field at $(n+1/2)$ time $H^{n+1/2}$ is calculated from the known electric field at n time E^n .

2) From the formula $D^{n+1} = D^n + \Delta t(\nabla \times \vec{H})^{n+1/2}$, the electric flux at $(n+1/2)$ time can be calculated iteratively by the magnetic field at $(n+1)$ time.

3) According to the constitutive relation of dispersive medium $D(\omega) = \epsilon_0 \epsilon(\omega) E(\omega)$, the electric field at $(n+1)$ time is calculated by auxiliary differential equation.

4) Go back to step 1) and move on to the next iteration time step.

In the second part of this paper, the formula of two-dimensional TM wave propagating in lossy Lorentz dispersion medium is deduced by ADE-FDTD method. In the third part, the formula is validated in one-dimensional case and the reflection of plane electromagnetic wave by infinite high Lorentz dielectric layer is simulated.

2 Mathematical for mulation

Assuming that the medium is lossy, linear and isotropic Lorentz medium, in order to simplify the calculation, we only consider that the relative dielectric constant varies with frequency (the relative permeability varies with frequency can also be dealt with), and its relative dielectric constant is

$$\epsilon_r(\omega) = \epsilon_\infty + \frac{\sigma}{j\omega\epsilon_0} + \frac{\epsilon_s - \epsilon_\infty}{1 + j2\delta_0(\frac{\omega}{\omega_0}) - (\frac{\omega}{\omega_0})^2} \quad (1)$$

where ϵ_0 is the vacuum dielectric constant, ϵ_∞ is the infinite frequency dielectric constant, ϵ_s is the static dielectric constant, σ is the medium conductivity, the medium resonance frequency, then the relationship between the electric field intensity and the flux density in the medium is as follows.

$$\vec{D}(\omega) = \epsilon_0 \left[\epsilon_\infty + \frac{\sigma}{j\omega\epsilon_0} + \frac{\epsilon_s - \epsilon_\infty}{1 + j2\delta_0(\frac{\omega}{\omega_0}) - (\frac{\omega}{\omega_0})^2} \right] \vec{E}(\omega) \quad (2)$$

$$\text{let } \vec{I}(\omega) = \frac{\sigma}{j\omega\epsilon_0} \vec{E}(\omega), \vec{S}(\omega) = \frac{\epsilon_s - \epsilon_\infty}{1 + j2\delta_0(\frac{\omega}{\omega_0}) - (\frac{\omega}{\omega_0})^2} \vec{E}(\omega), \text{ then equation (2)}$$

can be changed as follow

$$\vec{D}(\omega) = \epsilon_0 [\epsilon_\infty \vec{E}(\omega) + \vec{I}(\omega) + \vec{S}(\omega)] \quad (3)$$

Simultaneous inverse Fourier transform on both sides can be obtained as

$$\vec{D}(t) = \epsilon_0 [\epsilon_\infty \vec{E}(t) + \vec{I}(t) + \vec{S}(t)] \quad (4)$$

According to $\vec{I}(\omega) = \frac{\sigma}{j\omega\epsilon_0} \vec{E}(\omega)$, equation $j\omega\epsilon_0 \vec{I}(\omega) = \sigma \vec{E}(\omega)$ can be obtained

and transformed into the time domain through inverse Fourier transform gives

$$\epsilon_0 \frac{d\vec{I}(t)}{dt} = \sigma \vec{E}(t) \quad (5)$$

Similarly, according to the equation $\vec{S}(\omega) = \frac{\epsilon_s - \epsilon_\infty}{1 + j2\delta_0(\frac{\omega}{\omega_0}) - (\frac{\omega}{\omega_0})^2} \vec{E}(\omega)$, we can get

$$\omega_0^2 \vec{S}(t) + 2\delta_0 \omega_0 \frac{d\vec{S}}{dt} + \frac{d^2 \vec{I}}{dt^2} = \omega_0^2 (\epsilon_s - \epsilon_\infty) \vec{E}(t) \quad (6)$$

where $\vec{I}, \vec{S}$ are the auxiliary differential equation (ADE) we introduced.

In lossy, Lorentz media, for 2-D TM wave propagation in z-direction, the relationship of the components H_x, H_y, E_z, D_z, I and S can be discripted as follow

$$E_z^n = \frac{1}{\epsilon_0 \epsilon_\infty + \sigma \Delta t} [D_z^n - \epsilon_0 I^{n-1} - \epsilon_0 S^n] \quad (7)$$

$$I^n = I^{n-1} + \Delta t \frac{\sigma}{\epsilon_0} E_z^n \quad (8)$$

$$S^n = \frac{2 - \Delta t^2 \omega_0^2}{1 + \Delta t \delta_0 \omega_0} S^{n-1} - \frac{1 - \Delta t \delta_0 \omega_0}{1 + \Delta t \delta_0 \omega_0} S^{n-2} + \frac{\Delta t^2 \omega_0^2 (\epsilon_s - \epsilon_\infty)}{1 + \Delta t \delta_0 \omega_0} E_z^{n-1} \quad (9)$$

$$D_z^{n+\frac{1}{2}} = D_z^{n-\frac{1}{2}} + \Delta t \left[\frac{H_y^n(i+\frac{1}{2}, j) - H_y^n(i-\frac{1}{2}, j)}{\Delta x} - \frac{H_x^n(i, j+\frac{1}{2}) - H_x^n(i, j-\frac{1}{2})}{\Delta y} \right] \quad (10)$$

$$H_x^{n+\frac{1}{2}}(i, j) = H_x^{n-\frac{1}{2}}(i, j) - \frac{1}{\mu_0} \frac{\Delta t}{\Delta y} [E_z^n(i, j+\frac{1}{2}) - E_z^n(i, j-\frac{1}{2})] \quad (11)$$

$$H_y^{n+\frac{1}{2}}(i, j) = H_y^{n-\frac{1}{2}}(i, j) + \frac{1}{\mu_0} \frac{\Delta t}{\Delta x} [E_z^n(i+\frac{1}{2}, j) - E_z^n(i-\frac{1}{2}, j)] \quad (12)$$

3 Simulation result

In order to verify the correctness of the proposed algorithm, we calculate the reflection of a wide-band $E_s(t) = e^{-(t-t_0)^2/T^2}$ ($t_0 = 0.1$ ns, $T_0 = 6$ ps) incident to the interface between the air and the dielectric medium in one-dimensional case.

The thickness of Lorentz media is 12mm, $\varepsilon_s = 3$, $\varepsilon_\infty = 1.5$, $\sigma = 0.0062$ S/m, $\omega_0 = 40\pi \times 10^9$ rad/s, $\delta_0 = 0.1\omega_0$. The reflection coefficient[11] at the air interface between the area 1 and the area 2 theoretically given by

$$\Gamma = \frac{\Gamma_{12} + \Gamma_{23}e^{j2\beta_2d}}{1 + \Gamma_{12}\Gamma_{23}e^{j2\beta_2d}} \quad (13)$$

where d is the thickness of the Lorentz media and β_2 is the phase constant in the Lorentz media. The diagram is shown in Figure 1.

Fig.1. Graph of 1-D description.

Fig. 2. Reflected waveform in the area 1 distance 1mm to the interface.

Figure 2 shows the waveforms of the reflected wave at the interface of area 1 and area 2 with time. The numerical solutions of the incident wave and reflected wave in the time domain are obtained by using the algorithm proposed in this paper. Then the reflection coefficients of the frequency points in the frequency domain are calculated by using the discrete Fourier transform. Figure 3 gives the theoretical results of the reflection coefficients and the utilization formula (13). The comparison of the reflection coefficients of the results

shows that the degree of coincidence between theoretical values and numerical results is very good, which also verifies the correctness of our derivation in the second part.

Fig. 3. Comparison of numerical result and theoretical result of the reflected coefficient between areal and area2

4 Conclusion

The difference formula of two dimensional TM wave propagation in lossy Lorenz medium is derived by using the finite difference time domain method of auxiliary differential equation (ADE). In the one-dimensional case, the reflection field in time domain is calculated for the derived formula. Then the reflection coefficient is calculated by using Fourier's transform to transform the reflection field into frequency domain, and the reflection coefficient coincides with the theoretical derivation of the reflection coefficient. This indicates that the difference formula for the propagation of two-dimensional TM wave in lossy medium is correct. By using the derived formula, the reflection of plane electromagnetic waves in an infinite thickness dielectric layer with a thickness of 12 mm is simulated. The results show that the reflection effect of the dielectric layer is very obvious.

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