Minimization torque ripple for SRM based on flux linkage partition in DB-DTFC

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Abstract. This paper proposes a novel deadbeat torque and flux control (DB-DTFC) to reduce torque ripple for switched reluctance motor (SRM). DB-DTFC combines the advantages of direct torque control (DTC) and space-vector modulation (SVM). DB-DTFC leads current vector control into DTC in order to find the equation between torque and current through deadbeat prediction theory i.e. a beat reaches a given point. In addition, the deadbeat calculation module here is similar to that of permanent magnet synchronous motor. Based on *dq*0 reference frame of SRM, the most suitable *dq*0 axis current of next moment corresponding to different torque errors is calculated and predicted. According to the calculated *dq*0 axis current, the optimal space voltage vectors can be selected to reduce torque ripple. In order to verify the effectiveness and correctness of the proposed scheme, DB-DTFC is verified and compared with the DTC-SVM by simulation.

1 Introduction

As a new type of reluctance motor, switched reluctance motor (SRM) has attracted much attention because of its outstanding advantages, such as simple and robust structure, wide speed range, good reliability and high efficiency [1]. However, double salient structure and the principle of minimum reluctance lead to SRM's nonlinear strong coupling characteristic. This characteristic is an important reason for large noise and torque ripple of SRM [2].

Traditional current chopper control and angle control can not make speed regulation and torque ripple reduction compatible perfectly, then space-vector modulation (SVM) and direct torque control (DTC) are proposed. With the development of DTC, many DTC-based control method has been explored. [3] proposes direct instantaneous torque control (DITC) based on double hysteresis control. [4] proposed the direct torque and predictive flux control method based on model prediction of SRM.

Current vector control (CVC) is an important part of vector control. The traditional CVC is only closed loop control of stator current in the *d-q* coordinate system without torque closed loop. When the idea of CVC is introduced into DTC, the relationship between torque and current can be found. Deadbeat direct toque flux control (DB-DTFC) of induction machine (IM) [5] is proposed. DB-DTFC not only has the advantages of DTC and SVM, but also can calculate the *d-q* axis current through torque error, so as to the selection of optimal voltage vectors. Because of these advantages, DB-DTFC has been applied interior permanent magnet synchronous machine (IPMSM) [6] and synchronous reluctance motor [7].

Since the promotion of DB-DTFC, many scholars have made contributions on improving DB-DTFC for IPMSM. In [8] deadbeat direct current control is proposed to reduce computation load of DB-DTFC . Although there is no torque closed loop in this method, the author use the idea of maximum torque per ampere to combine the torque and current. [9] proposes the loss minimization DB-DTFC from the point of loss and proved the validity of the method. Based on DTC-SVM , reference [10] proposes an optimal deadbeat control method which combined adjacent voltage vectors with optimal voltage vectors. Although DB-DTFC is widely used and developed in other motors, the application of DB-DTFC in SRM has been greatly limited. The reasons of the restriction are uncertain torque expression and the absence of rotor flux linkage.

According to [11], the mathematical model of *dq*0 coordinate system makes it possible for DB-DTFC to be applied on SRM. [12] proposes a new current vector control method based on the *dq*0 coordinate system and emphasizes the effect of zero-phase current on the torque control. Another vector control about speed-sensorless SRM drive is also proposed based on the *dq*0 coordinate system [13].

Since DB-DTFC have both advantages of DTC and SVM, this paper proposes a novel DB-DTFC of SRM based on *dq*0 coordinate system and deadbeat prediction theory. The *dq*0 coordinate system is established to find the equation between *dq*0 axis current and torque. As for the deadbeat prediction, it is used to find the optimal current and voltage under different torque errors. Due to

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the difference between SRM and IPMSM, DB-DTFC of SRM is different from that of IPMSM, especially in the case of zero phase current optimization. The proposed method uses torque flux decoupling control and finds the optimal *dq*0 axis current. DB-DTFC is verified the validity and correctness by simulation. It is also compared with DTC-SVM for the performance of torque ripple under different conditions.

2 Principle of DB-DTFC scheme

2.1. Mathematical model of static coordinate

According to Kirchhoff's Law of Voltage, the k-phase voltage equation of SRM is:

$$u_k = R_k i_k + \frac{d\psi_k}{dt} \tag{1}$$

Expression (1) can also be written in matrix form as

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = R \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + p(\begin{bmatrix} L_a & 0 & 0 \\ 0 & L_b & 0 \\ 0 & 0 & L_c \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}) \tag{2}$$

where R is the stator resistance; i_a , i_b , i_c are the phase currents of A, B, C phase, respectively; L_a , L_b , L_c are self-inductance of the corresponding phase; p is differential operator.

Among the matrix neglecting the mutual inductance, the self-inductance part is expressed as:

$$L_k(\theta_r) = L_{dc} + L_{ac}\cos(2\theta_r - (\mathrm{k}-1)\frac{2\pi}{N}) \tag{3}$$

where L_{dc} and L_{ac} are DC self-inductance and DC self-induction amplitude, respectively; θ_r is rotor position angle and *N* is phase number, $k = 1,2,...,N,$.

The instantaneous torque expression of SRM can be defined as:

$$T_e = \frac{P}{2}(i_a^2 \frac{\partial L_a}{\partial \theta_r} + i_b^2 \frac{\partial L_b}{\partial \theta_r} + i_c^2 \frac{\partial L_c}{\partial \theta_r}) \tag{4}$$

where T_e is the instantaneous torque; P is magnetic pole number.

2.2 Mathematical model of rotating coordinate

According to literature [11], “rotor flux” generated by the DC current, when DC current is applied to each circuit. The rotating coordinate system is established by rotating the “rotor flux”. In the synchronous rotating coordinate, current can be expressed as:

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos 2\theta & \cos(2\theta - \frac{2\pi}{3}) & \cos(2\theta + \frac{2\pi}{3}) \\ -\sin 2\theta & -\sin(2\theta - \frac{2\pi}{3}) & -\sin(2\theta + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{5}$$

where θ is the electrical angle; i_q and i_d are the *q*-axis and *d*-axis components of i_a , i_b and i_c , i_0 is zero-phase current.

Both substituting (3), (5) into (2) and ignoring asynchronous components can get the new matrix:

$$\begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix} = R \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} + \begin{bmatrix} L_{dc} & 0 & \frac{L_{ac}}{\sqrt{2}} \\ 0 & L_{dc} & 0 \\ \frac{L_{ac}}{\sqrt{2}} & 0 & L_{dc} \end{bmatrix} p(\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}) + 2\omega_r \begin{bmatrix} 0 & -L_{dc} & 0 \\ L_{dc} & 0 & \frac{L_{ac}}{\sqrt{2}} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} \tag{6}$$

2.3 Derivation of deadbeat control algorithms

Considering (4) as the torque expression in deadbeat control, when the speed in constant, $\partial L_a/\partial \theta_r$, $\partial L_b/\partial \theta_r$ and $\partial L_c/\partial \theta_r$ are determined by motor parameters. They can be regarded as constants. Therefore when the sampling time is small enough, only the three-phase current needs to be adjusted to control the torque increase and decrease .

Substituting (5) into (4), i_a , i_b and i_c can be replaced by i_q , i_d and i_0 to achieve the same control effect of torque as three-phase stator current. According to the decoupling control of DB-DTFC based on IPMSM [6], flux and torque can be decoupled into two independent systems. Torque is affected both i_q and i_0 [12], flux is only controlled by i_d .

In the torque control, considering i_q , i_0 as two different variables and considering torque as a constant, one quadratic equation about i_q and torque can be obtained, the other quadratic equation about i_0 and torque can also be obtained.

For the first quadratic equation about i_q and torque, the smaller value $i_q^*(k)$ can be acquired by adopting the quadratic formula. $i_q^*(k)$ can be expressed as:

$$i_q^*(k) = \frac{E_1}{D_1} \tag{7}$$

$$\begin{cases} D_1 = 4a\sin^2 2\theta + 4c\cos^2(\frac{\pi}{6} + 2\theta) + 4b\sin^2(\frac{\pi}{3} + 2\theta) \\ E_1 = 2i_d(a\sin 4\theta + b\sin(\frac{2\pi}{3} + 4\theta) - c\sin(\frac{\pi}{3} + 4\theta)) \\ \quad - \left\{ \begin{array}{l} (8a\sin^2 2\theta + 8c\cos^2(\frac{\pi}{6} + 2\theta) + 8b\sin^2(\frac{\pi}{3} + 2\theta))* \\ [\frac{6\Delta T_e}{P} - 2i_d^2(a\cos^2 2\theta + b\cos^2(\frac{\pi}{3} + 2\theta) + c\sin^2(\frac{\pi}{6} + 2\theta)) - \\ i_0^2(a+b+c) + 2\sqrt{2}i_0 i_d(b\cos(\frac{\pi}{3} + 2\theta) + c\sin(\frac{\pi}{6} + 2\theta) - a\cos 2\theta)] \\ +[2i_d(a\sin 4\theta + b\sin(\frac{2\pi}{3} + 4\theta) - c\sin(\frac{\pi}{3} + 4\theta)) \\ +2\sqrt{2}i_0(a\sin 2\theta + c\cos(\frac{\pi}{6} + 2\theta) - b\sin(\frac{\pi}{3} + 2\theta))]^2 \end{array} \right\}^{\frac{1}{2}} \\ \quad + 2\sqrt{2}i_0(a\sin 2\theta + c\cos(\frac{\pi}{6} + 2\theta) - b\sin(\frac{\pi}{3} + 2\theta)) \end{cases}$$

where $a = \partial L_a/\partial \theta_r$, $b = \partial L_b/\partial \theta_r$ and $c = \partial L_c/\partial \theta_r$; T_e is replaced by ΔT_e for reflecting the change of torque and $\Delta T_e = T_e^* - T_e(k)$. $a = \partial L_a/\partial \theta_r$, $b = \partial L_b/\partial \theta_r$ and $c = \partial L_c/\partial \theta_r$.

As for the other quadratic equation about i_0 and torque, the method of solving the minimum value $i_0^*(k)$ is similar to that of $i_q^*(k)$, $i_0^*(k)$ can be expressed as:

$$i_0^*(k)=\frac{E_2}{D_2} \tag{8}$$

$$\begin{cases} D_2=2(a+b+c) \\ E_2=2i_d(a\sin4\theta+b\sin(\frac{2\pi}{3}+4\theta)-c\sin(\frac{\pi}{3}+4\theta)) \\ \quad -\left\{\begin{aligned} &[2\sqrt{2}i_d(b\cos(\frac{\pi}{3}+2\theta)+c\sin(\frac{\pi}{6}+2\theta)-a\cos2\theta) \\ &2\sqrt{2}i_q(a\sin2\theta+c\cos(\frac{\pi}{6}+2\theta)-b\sin(\frac{\pi}{3}+2\theta)]^2 \\ &4(a+b+c)*[\frac{6\Delta T_e}{P}-2i_d{}^2(a\cos^2 2\theta+b\cos^2(\frac{\pi}{3}+2\theta)+c\sin^2(\frac{\pi}{6}+2\theta)) \\ &-2i_q{}^2(a\sin^2 2\theta+c\cos^2(\frac{\pi}{6}+2\theta)+b\sin^2(\frac{\pi}{3}+2\theta)) \\ &+2i_di_q(a\sin4\theta+b\sin(\frac{2\pi}{3}+4\theta)-c\sin(\frac{\pi}{3}+4\theta))] \end{aligned}\right\}^{\frac{1}{2}} \\ \quad +2\sqrt{2}i_q(a\sin2\theta+c\cos(\frac{\pi}{6}+2\theta)-b\sin(\frac{\pi}{3}+2\theta)) \end{cases}$$

The two factors i_q and i_0 affecting the torque need to be set to $i_q(k+1)=i_q^*(k)$, $i_0(k+1)=i_0^*(k)$ and make sure that the values of the next moment are the smallest current values.

Due to the short sampling time, current can be discretized. In the discrete time, substituting $i_q^*(k)$ and $i_0^*(k)$ into (6), *q*-axis voltage $u_{sq}^*(k)$ for torque control can be obtained as:

$$u_{sq}^*(k)=R_si_q^*(k)+L_{dc}\frac{i_q^*(k)-i_q(k)}{T_s}+2\omega_r(L_{dc}i_d(k)+\frac{L_{ac}}{\sqrt{2}}i_0{}^*(k)) \tag{9}$$

As for the flux control, i_d controls flux directly and *d*-axis voltage $u_{sd}^*(k)$ for flux can also be obtained from:

$$\psi_s(k+1)=\psi_s^*=u_{qds}(k)T_s+\psi_s(k) \tag{10}$$

In order to simplify the calculation, the motor model is oriented in the direction of stator and the flux linkage coincides with the d-axis, so as to make ψ_{sq} equal to zero. Then discretizing the flux linkage can get the flux control expression about d-axis voltage $u_{sd}^*(k)$ to make the next moment flux reach the given value.

$$u_{sd}^*(k)=\frac{\left|\psi_s^*-\psi_{sd}(k)\right|}{T_s}+R_si_{sd}(k) \tag{11}$$

$u_{sq}^*(k)$ and $u_{sd}^*(k)$ are the set of optimal voltage vectors for the *k*th-control cycle. They satisfy the given conditions for flux and torque. In this way, the DTC torque flux hysteresis loop can be replaced and the deadbeat regulation of the switched reluctance motor can be achieved successfully.

2.4 Space-vector modulation

$u_{sq}^*(k)$ and $u_{sd}^*(k)$ can be transformed into $u_\alpha(k)$ and $u_\beta(k)$ by inverse rotation coordinate system which can be directly used in space-vector control module. Based on voltage vector principle of SRM, the basic voltage vectors of different sectors are redefined.

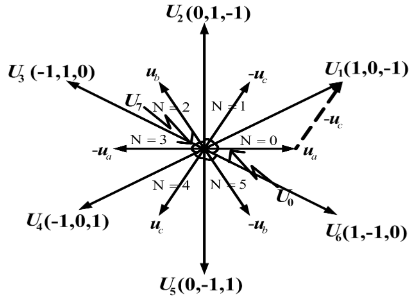


Fig. 1 Basic space voltage vectors of SVM

According to volt-second balance principle, if the voltage vector is calculated and needed in the first sector, $u_\alpha(k)$ and $u_\beta(k)$ can be expressed as:

$$\begin{cases} U_\alpha=\dfrac{T_1}{T_s}\left|U_1\right|\cos(\dfrac{\pi}{6})+\dfrac{T_2}{T_s}\left|U_2\right|\cos(\dfrac{\pi}{2}) \\ U_\beta=\dfrac{T_1}{T_s}\left|U_1\right|\sin(\dfrac{\pi}{6})-\dfrac{T_2}{T_s}\left|U_2\right|\sin(\dfrac{\pi}{2}) \end{cases} \tag{12}$$

Since the unique three-phase asymmetric bridge power converter of SRM, the maximum amplitude of SRM space voltage vector is $\sqrt{3}U_{dc}/2$ and $\left|U_1\right|=\left|U_2\right|=\sqrt{3}U_{dc}/2$. Among them, U_{dc} is the bus voltage. The time of different sector vectors can be calculated as table.1.

Tab. 1 Voltage working time under different sectors

Secto	0	1	2	3	4	5
T_1	$T_6=-T_z$	$T_1=T_x$	$T_2=T_y$	$T_3=T_z$	$T_4=-T_x$	$T_5=-T_y$
T_2	$T_1=T_y$	$T_2=T_z$	$T_3=-T_x$	$T_4=-T_y$	$T_5=-T_z$	$T_6=T_x$
T_0	$T_0(T_7)=T_\mathrm{s}-T_1-T_6$					

where T_x, T_y and T_z can be expressed as

$$\begin{cases} T_x=\dfrac{4T_sU_\alpha}{3U_{dc}} \\ T_y=\dfrac{2T_s(U_\alpha+\sqrt{3}U_\beta)}{3U_{dc}} \\ T_z=\dfrac{2T_s(\sqrt{3}U_\beta-U_\alpha)}{3U_{dc}} \end{cases} \tag{13}$$

If $T_1+T_2\geq T_s$, there will need time saturation processing, T_1 and T_2 need to be redefined as:

$$\begin{cases} T_1=\dfrac{T_1}{T_1+T_2}T_s \\ T_2=\dfrac{T_2}{T_1+T_2}T_s \end{cases} \tag{14}$$

3 Principle of DB-DTFC scheme

3.1 Structure of DB-DTFC scheme

On the basis of control rate of DB-DTFC, $u_{sq}^*(k)$ and $u_{sd}^*(k)$ are selected. And then $u_\alpha(k)$ and $u_\beta(k)$ which are transformed from $u_{sq}^*(k)$ and $u_{sd}^*(k)$ can be input into the SVM module. SVM can synthetize the new voltage vectors by judging the basic voltage vectors selected by $u_\alpha(k)$, $u_\beta(k)$ and calculating the action time of the basic voltage vectors. In addition, the sector judgement module in SVM is similar to that of DTC. The control signal corresponding to the new voltage vectors synthesized by SVM drives the motor

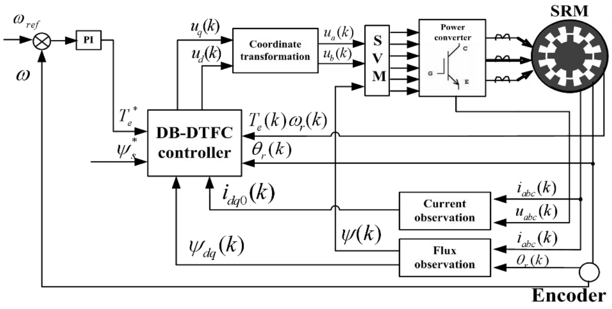


Fig. 3. Control block diagram of DB-DTFC

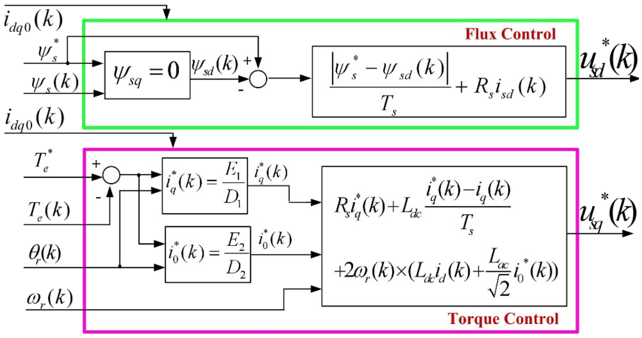


Fig. 4. The control rate of DB-DTFC

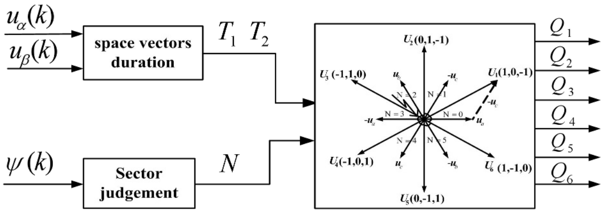


Fig. 5. Control block of SVM

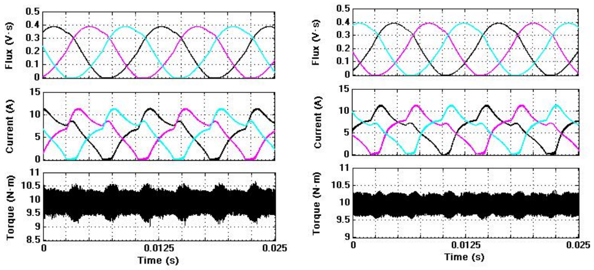
3.1 Analysis of simulation results

The simulation results of proposed DB-DTFC and DTC-SVM will be analyzed and compared in this part. Relying on Matlab/simulink the simluation can be established based on the three-phase 12/8-pole SRM, and the rated voltage is 520V.

In order to assess the effect of different algorithms. The torque error T_{RC} can be defined as follow:

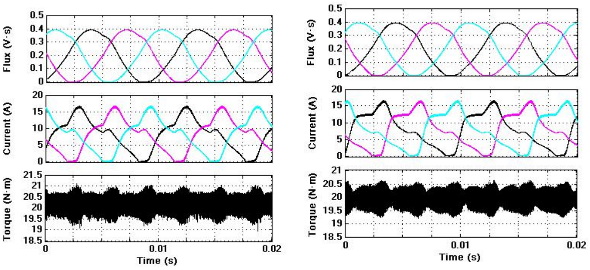
$$T_{RC}=T_{e-\max}-T_{e-\min} \tag{15}$$

$T_{e-\max}$ and $T_{e-\min}$ are the maximum and minimum values of torque respectively. Fig.6 is the waveform of 600rpm with 10N·m load torque, the optimization of T_{RC} is not obvious as that of Fig.7 and Fig.8 with high speed loading.



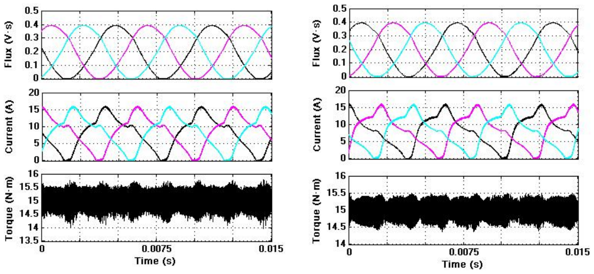
(*a*) (*b*)

Fig. 6. *ω*=650*rpm*, *T_L*=10*N·m* (*a*)DTC-SVM (*b*) proposed DB-DTFC



(*a*) (*b*)

Fig. 7. *ω*=*800rpm*, *T_L*=*20N·m* (*a*)DTC-SVM (*b*) proposed DB-DTFC



(*a*) (*b*)

Fig. 8. *ω*=1200*rpm*, *T_L*=15*N·m* (*a*)DTC-SVM (*b*) proposed DB-DTFC

Tab. 2 Torque ripple error comparison

Index	**DTC-SVM** T_{RC}	**DB-DTFC** T_{RC}
650rpm，10*N·m*	1.65	0.81
800rpm，20*N·m*	2.15	1.36
1200rpm，15*N·m*	1.85	1.05

From the torque waveform, the two method both have good control to the torque. But the more excellent control performance of DB-DTFC can be seen than that of DTC. The torque ripple becomes smaller than that of DTC-SVM. Table 2 is a numerical comparison of torque ripple error about DTC and DB-DTFC. From table 2, the value of torque ripple error can be acquired in detail. Under different conditions, the torque ripple of

DB-DTFC decreases by 0.8*N·m* compared with DTC-SVM.

4 Conclusion

To solve the problem of switched SRM torque ripple, DB-DTFC is proposed based on *dq*0 coordinate system. DB-DTFC is based on DTC-SVM and introduces current vector into the control system. But the flux and torque control of DB-DTFC is different from DTC-SVM. Especially, decoupling flux and torque and adopting *dq*0 axis currents control respectively. Using i_q , i_0 control the torque and using i_d controls the flux. The flux control is similar to DB-DTFC based on IPMSM. But the zero phase current is taken into account in the torque control. DB-DTFC is compared with DTC-SVM by simulation on the stable state. The better performance of DB-DTFC is reflected in the torque ripple values. The simulation results can also verify the validity and feasibility of DB-DTFC.

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