Dynamic Model of Planetary Roller Screw Mechanism with Considering Torsional Degree of Freedom

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Abstract. To predict accurately the dynamics performance of planetary roller screw mechanism, it is necessary to establish its streamline and engineering-compliant dynamic model, which is the basis of mechanical design and precision control of the system. In this paper, the relative displacement between roller and ring gear along the line of action is deduced and the relationship between nature frequencies and the number of rollers is discussed. Considering the torsional stiffness of all components and the thread mesh stiffness based on the Hertzian contact theory, the purely torsional model for planetary roller screw mechanism is presented to reveal the natural frequencies and vibration mode characteristics of the system. The results show that the natural properties of undamped system in planetary roller screw mechanism are mainly reflected by two typical vibration modes: rotational mode and roller mode.

1 Introduction

Planetary roller screw mechanism (PRSM) is a rolling screw transmission device that can convert screw's rotational motion into nut's linear motion or turn the nut's rotational motion into screw's linear motion, which the former one is called the standard PRSM and the latter one is defined as the inverse PRSM. As shown in Fig. 1, the PRSM mainly consists of a screw, a nut, rollers, carriers and ring gears. Owing to its higher speed, larger bearing capacity, higher transmission accuracy, stronger environmental adaptability and longer working life, the PRSM is widely employed in the aerospace and voyage fields, even the medical equipment and precision machine tools. Therefore, it is essential to build a streamlined and engineering-compliant dynamic model of PRSM in this paper, which is helpful to investigate its transmission performance and improve the mechanical design and precision control of the system.

In recent years, there are many researches on PRSM, which are primarily focused on load distribution [1-3], parameter matching [4], friction and wear behaviours [5], axial stiffness [6-7] and kinematic analysis [8-10]. Instead, only few publications on the dynamic characteristic of the PRSM are mentioned.

Jones and Velinsky [9] established the kinematical contact model at the screw/roller and nut/roller interfaces on the basis of the principle of conjugate surfaces. And Jones et al. [11] also proposed the dynamics equations of motion for the screw, rollers, and nut of PRSM by Lagrange's Method and derived their steady-state angular velocities and screw/roller slip velocities. Ma and his colleagues [12] studied the contact characteristics of threaded surfaces under the combined action of normal pressure and tangential friction forces,

and derived not only the velocities of the contact points in the contact ellipse but also the normal pressure and friction force distributions. Fu et al. [13] developed a nonlinear six degrees of freedom dynamic model, in which the load distribution coefficient is introduced to describe the load distribution among threads of PRSM, and studied the transient and steady-state behaviours of the PRSM under a heavy and light external load, respectively. However, the natural frequencies and vibration mode characteristics of the PRSM are not studied in recent researches.

In this paper, a dynamic model of PRSM considering torsional stiffness and mesh stiffness is established, in which the thread mesh stiffness is calculated by using Hertzian contact theory. Then the nature frequencies and the vibration modes of PRSM are investigated, which is used to determine the dynamic characteristics of the system, and the relationship between nature frequencies and the number of rollers is discussed. At last, this paper proposes the direction for further research.

Fig. 1. Mechanical structure of PRSM.

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2 Modelling

2.1 Introduction of PRSM

As shown in Fig. 1, due to the similarities between roller and planetary wheel, the transmission principles of PRSM is similar to that of planetary gear. When the screw rotates, the roller revolves both round the screw and on its own axis, and at the same time, the nut moves in a straight line. The motion and power are transmitted through the threaded contact surfaces.

2.2 Purely torsional model

The purely torsional model of PRSM is established and some assumptions and definitions are described as follows: 1) The torsional line displacement is regarded as the product of the torsional angular displacement of each major component and its gyration radius; 2) The gyration radius of the screw is its nominal radius, which is a half of the pitch diameter of the thread, and the definition of gyration parameters for the nut is the same as that of the screw; 3) The gyration radius of the inner ring gear is a half of its standard pitch circle diameter; 4) The gyration radius of the carrier is considered as the distance between the roller's axis and the geometric center of the carrier, and that of roller is the same. As shown in Fig. 2, the purely torsional model of PRSM mainly considers the effects of the torsional support stiffness, rotational inertia, mass properties of the main components, the thread contact stiffness of the screw/roller and the nut/roller and the gear mesh stiffness between the spur gear located at the outer end of the rollers and the ring gear.

Fig. 2. Pure torsion model.

where K_{su} , K_{nu} , K_{cu} and K_{rn} is the torsional support stiffness of the screw, nut and carrier, respectively; u_s , u_n , u_c and u_r are torsional line displacements of the screw, nut, carrier and ring gear, respectively.

According to the Newton's second law, the undamped free vibration differential equations of system motion are obtained

$$\begin{cases} \frac{J_s}{r_s^2} \ddot{u}_s + \sum_{n=1}^8 K_{sn} d_{sn} + K_{su} u_s = 0 \\ \frac{J_N}{r_N^2} \ddot{u}_N + \sum_{n=1}^8 K_{Nn} d_{Nn} + K_{Nu} u_N = 0 \\ \frac{(J_c + \sum_{n=1}^8 m_{pn} r_c^2)}{r_c^2} \ddot{u}_c + K_{cu} u_c = 0 \\ \frac{J_r}{r_r^2} \ddot{u}_r + \sum_{n=1}^8 K_{rn} d_{rn} + K_{ru} u_r = 0 \\ \frac{J_{R1}}{r_{R1}^2} + K_{s1} d_{s1} - K_{N1} d_{N1} - K_{r1} d_{r1} = 0 \\ \vdots \\ \frac{J_{Rn}}{r_{Rn}^2} + K_{sn} d_{sn} - K_{Nn} d_{Nn} - K_{rn} d_{rn} = 0 \end{cases} \quad (1)$$

where n is the number of the roller; J_s , J_N , J_c and J_r are the rotational inertia of the screw, nut, carrier and ring gear, respectively; r_s , r_N , r_c , r_r are the gyration radii of the screw, nut, carrier and ring gear, respectively; J_{Rn} , m_{Rn} , r_{Rn} are the rotational inertia, mass and gyration radius of the roller; d_{sn} is the relative displacement between the roller n and screw along the z -axis, which can be expressed as

$$d_{sn} = \frac{\theta_s L_s}{2\pi} = \frac{u_s L_s}{r_s \cdot 2\pi} \quad (2)$$

where θ_s is the rotational angular of the screw; L_s is the screw's lead; d_{Nn} is the relative displacement between the roller n and nut along the z -axis, which can be formed as

$$d_{Nn} = \frac{\theta_N L_N}{2\pi} = \frac{u_N L_N}{r_N \cdot 2\pi} \quad (3)$$

where θ_N is the rotational angular of the nut; L_N is the nut's lead; d_{rn} is the relative displacement between the roller n and ring gear along the mesh line, which can be written as

$$d_{rn} = u_r - u_n \quad (4)$$

The Eq. (1) can also be formed as

$$[M] \{\ddot{u}\} + [K_b + K_m] \{u\} = 0 \quad (5)$$

where u is the torsional line displacements; $[M]$ is the mass matrix; $[K_b + K_m]$ is the stiffness matrix, and their formulas can be expressed as

$$\{u\} = \{u_s, u_N, u_c, u_r, u_1, \dots, u_n\}^T \quad (6)$$

$$[M] = \text{diag} \left[\frac{J_s}{r_s^2}, \frac{J_N}{r_N^2}, \frac{J_c + \sum_{n=1}^8 m_{pn} r_c^2}{r_c^2}, \frac{J_r}{r_r^2}, \frac{J_{R1}}{r_{R1}^2}, \dots, \frac{J_{Rn}}{r_{Rn}^2} \right] \quad (7)$$

$$[K_b] = \text{diag}[K_{su}, K_{Nu}, K_{cu}, K_{ru}, 0, \dots, 0] \quad (8)$$

$$[K_m] = \begin{bmatrix} \frac{L_s}{r_s \cdot 2\pi} \sum_{n=1}^n K_{sn} & & & & & & \\ & \frac{L_N}{r_N \cdot 2\pi} \sum_{n=1}^n K_{Nn} & & & & & \\ & & 0 & & & & \\ & & & \sum_{n=1}^n K_{rn} & -\sum_{n=1}^n K_{rm} & & \\ \frac{L_s}{r_s \cdot 2\pi} K_{s1} & -\frac{L_N}{r_N \cdot 2\pi} K_{N1} & 0 & -K_{r1} & K_{r1} & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{L_s}{r_s \cdot 2\pi} K_{sn} & -\frac{L_N}{r_N \cdot 2\pi} K_{Nn} & 0 & -K_{rn} & 0 & \dots & K_{rm} \end{bmatrix} \quad (9)$$

3 Dynamic characteristics analysis

3.1 Calculation of thread contact stiffness

The thread contact stiffness of PRSM, that is the thread meshing stiffness, refers to the ratio of the normal contact load and the contact deformation of the screw/roller or nut/roller. Since the PRSM is subjected to axial loads at run time, it is necessary to project the contact deformation of the thread to the axial direction. The normal deformation of the thread in contact area under the normal contact load is obtained by Hertzian contact theory [14]:

$$\delta = \delta^* \left[\frac{3F_n}{2\sum \rho} \left(\frac{(1-\mu_R^2)}{E_R} + \frac{(1-\mu_x^2)}{E_x} \right) \right]^{2/3} \frac{\sum \rho}{2} \quad (10)$$

where F_n is the normal contact load between two contact threads, and it can be obtained from the axial load F_a ; $\sum \rho$ is the sum function of curvature on two contact surfaces; δ^* relates to the curvature difference function $F(\rho)$, which can be obtained by reference [15]; μ_R , E_R are the Poisson's ratio and Young's modulus of the roller; μ_x , E_x ($x=s$ or N) are the Poisson's ratio and Young's modulus of the screw or nut, respectively.

The deformation of the thread in contact area along the axial direction is shown as

$$\delta_{xR-a} = \delta \cdot \cos \beta_R \cdot \cos \alpha \quad (11)$$

where β_R is the helix angle of the roller; α is the flank angle.

Therefore, the axial contact stiffness of the two components, especially the roller and screw or the roller and nut, can be expressed as:

$$K_{xRn} = \frac{F_a}{\delta_{xp-a}} \quad (12)$$

3.2 Calculation results of parameters

The structure parameters and material properties of the PRSM are presented in Table 1 and Table 2.

The calculation results of related parameters of the dynamic equations are shown in Table 3. The calculation method of the spur gear mesh stiffness can refer to [16].

3.3 Modal analysis

To simplify the calculation, the rollers are assumed equally spaced, the axial load is evenly distributed among threads of PRSM. Each roller and the ring gear mesh stiffness is equal, which is taken as the mean value of the internal mesh stiffness. Moreover, the mass and rotational inertia of the rollers are supposed the same with each other.

Table 1. Structure parameters of PRSM.

Parameters/(Units)	Screw	Roller	Nut
Nominal radius (mm)	12	4	20
Pitch (mm)	2	2	2
Number of starts	5	1	5
Helix angle (°)	7.6	4.5	4.5
Flank angle (°)	45	45	45
Lead(mm)	10	2	10
Roller profile radius(mm)	/	4.596	/
Gear			
Number of teeth	/	16	80
Modulus	/	0.5	0.5
Addendum coefficient	/	1	1
Pressure angle (°)	/	20	20
Helix angle (°)	/	0	0

Table 2. Material properties of PRSM.

Parameters	Material	Young's modulus (Pa)	Poisson's ratio
Screw	GCr15	2.12×10^{11}	0.29
Roller	GCr15	2.12×10^{11}	0.29
Nut	GCr15	2.12×10^{11}	0.29

Solving the problems of the system's natural characteristics can be achieved by obtaining the system's eigenvalues, and the natural frequencies and main modes of the system can be acquired by Eq. (13):

$$\omega_i^2 [M] \varphi_i = [K_b + K_m] \varphi_i \quad (13)$$

where $[M]$ is the mass matrix; $[K_b + K_m]$ is the stiffness matrix; ω_i is the i -th natural frequency; φ_i is the vibration mode corresponding to the i -th natural frequency, which can be expressed as:

$$\varphi_i = [\varphi_s^i, \varphi_N^i, \varphi_c^i, \varphi_r^i, \varphi_1^i, \dots, \varphi_n^i]^T \quad (14)$$

Based on the parameters given in Table 3, the natural properties are studied by the present model. The natural frequencies and their multiplicities associated with each

Table 3. The calculation results of PRSM's parameters.

Parameters (Units)	Screw	Roller	Nut	Carrier	Ring gear
Rotational inertia ($\text{kg} \cdot \text{m}^2$)	1.832×10^{-4}	5.248×10^{-7}	8.68×10^{-4}	5.888×10^{-6}	4.48×10^{-5}
Torsional support stiffness ($\text{N} \cdot \text{m}/\text{rad}$)	2.475×10^4	/	1.522×10^6	6.956×10^6	3.861×10^6
Mass (kg)	1.272	0.0328	2.170	0.023	0.112
Thread contact Stiffness ($\text{N} \cdot \text{m}^{-1}$)	2.078×10^8	/	2.198×10^8	/	/
Spur gear meshing stiffness ($\text{N} \cdot \text{m}^{-1}$)	/	/	/	/	2.464×10^8

mode are computed and listed in Table 4. The results show that the values of the natural frequencies gradually increase as the number of rollers increases except for the first and second-order natural frequencies and the number of multiplicity increases with the raise of roller number.

Table 4. Natural frequencies of PRSM with different number of the rollers (Hz).

Number of rollers	5	7	8	10	12
Multiplicity $l=1$	581	522	499	460	429
	971	835	786	709	650
	1019	1203	1285	1435	1571
	1657	1960	2095	2343	2566
	21666	24103	25233	27353	29320
Multiplicity $l=n-1$	13794	13794	13794	13794	13794

As shown in Table 5, Defining the number of the rollers is seven, the vibration modes of the PRSM can be obtained by Eq. (12), and the results indicate that:

1) The five natural frequencies in condition that the multiplicity is equal to one correspond to five main modes and each mode only represent the torsional vibration characteristic on the parts, and all rollers are subjected to torsional vibration along their own axes in the other four vibration modes except the second-order mode, where the vibration mode of each roller is the same. So these four modes can be defined as rotational mode.

2) The natural frequencies with the occasion of the multiplicity equivalent to $(n-1)$ correspond to different modes, but the motion characteristics of all components are similar, meaning the main parts such as screw, nut, carrier and ring gear do not vibrate while one roller would perform the torsional vibration in each of the modes. Thus, this mode can be defined as the roller mode.

Table 5. Vibration mode of 7-rollers.

	522	835	1203	1960	24103	13794	13794	13794	13794	13794	13794
φ_s^i	0	0	0	0.2481	0	0	0	0	0	0	0
φ_N^i	0	0	0.1291	0	0	0	0	0	0	0	0
φ_c^i	0	1	0	0	0	0	0	0	0	0	0
$\varphi_{r_s}^i$	-0.9986	0	-1	1	-1	0	0	0	0	0	0
φ_1^i	-1	0	-0.9984	0.9923	0.4817	0	0	0	0	0	0
φ_2^i	-1	0	-0.9984	0.9923	0.4817	0	0	0	1	0	0
φ_3^i	-1	0	-0.9984	0.9923	0.4817	0	0	0	0	1	0
φ_4^i	-1	0	-0.9984	0.9923	0.4817	0	0	0	0	0	1
φ_5^i	-1	0	-0.9984	0.9923	0.4817	1	0	0	0	0	0
φ_6^i	-1	0	-0.9984	0.9923	0.4817	0	1	0	0	0	0
φ_7^i	-1	0	-0.9984	0.9923	0.4817	0	0	1	0	0	0

4 Conclusions

In this paper, the natural characteristics of planetary roller screw mechanism considering torsional degree of freedom are investigated: the mesh stiffness of thread and spur gear is calculated respectively, the relative displacement between roller and ring gear along the line of action is deduced; the natural frequencies and vibration modes are computed.

From the numerical results, it can be concluded:

1) The planetary roller screw mechanism has two typical modes of vibration: rotational mode and roller mode.

2) The values of the natural frequencies gradually increase as the number of rollers increases except for the first and second-order natural frequencies and the number of multiplicity increases with the raise of roller number.

3) The research on natural characteristic of planetary roller screw mechanism provides a theoretical basis for designers to predict the dynamics performance of transmission system during the early stages of design, where the resonance point and the parameter sensitive points of the transmission system can be avoided according to the dynamics analysis of the system's natural frequency.

4) Previous studies on the dynamics of planetary gear transmission, such as the establishment of a translation-torsion coupling dynamic model and the steady-state dynamic response can be considered to extend into the dynamics analysis of planetary roller screw mechanism.

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