

An inverse method for design and characterisation of acoustic materials

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Abstract. This paper presents applications of an inverse method for the design and characterisation of anisotropic elastic material properties of acoustic porous materials. Full field 3D displacements under static surface loads are used as targets in the inverse estimation to fit a material model of an equivalent solid to the measurement data. Test cases of artificial open-cell foams are used, and the accuracy of the results are verified. The method is shown to be able to successfully characterise both isotropic and anisotropic elastic material properties. The paper demonstrates a way to reduce costs by characterising material properties based on the design model without a need for manufacturing and additional experimental tests.

1 Introduction

Acoustic materials are popular in the industry for sound absorption, soundproof, sound wave manipulation, etc. Among those, porous materials are one of the most used for sound absorption [1]. The majority of the studies to date have concentrated on conventional porous materials of equivalent isotropic or transverse isotropic [2-4]. However, the properties of lightweight porous materials are highly dependent on the microstructure and geometry, possibly holding anisotropy in the macroscopic properties of the material in production process or design, as the melamine foam with non-zero shear-compression coupling moduli and negative Poisson's ratio [5]. In addition, some applications require porous acoustic materials exhibiting anisotropic behaviour in certain direction. e.g., anisotropic acoustic metamaterials designed to manipulate sound waves [6]. The progress observed in manufacturing technologies, such as 3-D Printer, micro- and nano-manufacturing techniques, open a door to design artificial materials for specific mechanical applications, and motivate the characterisation of anisotropic materials by giving designed model. However, the few studies of artificial porous materials have not provided a way to guide the design of acoustic foams based on the application functions. Thus, developing or applying a method that can characterise the material properties during early design phases becomes critical.

Several contributions have attempted to characterise acoustical porous material properties over the last decades from experimental, analytical and numerical methods, see e.g. [4, 5, 7-13], but limited to isotropic or transversely isotropic materials and omitting the

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three-dimensional local effects. Inverse estimation is a widely used method for the characterisation of elastic material properties of porous materials [7, 9, 11-13]. In recent works by the authors in [14], related to the characterisation of porous elastic foams, an inverse method was proposed, and the accuracy was numerically verified.

The aim of the present paper is to address the inverse method in context of design and characterisation of anisotropic porous foams. The paper is organised as follows. First, the method in paper [14] is adapted for its application to acoustic porous materials and the microstructures are simplified by open-cell Kelvin cells. The method is then applied to the characterisation of isotropic and anisotropic artificial porous foams and the results are presented and discussed with respect to previous works. The paper is ends with conclusions and perspectives.

2 Method and structure introduction

An inverse estimation methodology for the different moduli in the static Hooke's matrix \mathbf{H} of open-cell materials was developed by the authors in [14]. The data used for the approach consists of displacements of target Kelvin model and displacements of predicted solid homogeneous model, see Fig. 1. The Hooke's matrix of the predicted model is varied within an optimisation routine until the difference between target and predicted displacements is minimised. The displacements are obtained as the result of a static compression and a shear load by a displacement of 0.01 mm between two flat plates of a cubic sample of material in each orthogonal direction, x , y and z . A schematic representation of the measurement setup is given in Fig. 2 for a compression of the material in the y -direction. In addition to the measurement of the displacements, the static force needed to compress the material sample must be measured for a complete characterisation of the Hooke's matrix. More details about the method and modelling set-up may be find in [11].

The frame of the porous material is assumed to have anisotropic material properties and the influence of pressure variations in the air are neglected under a quasi-static loading [15]. The frame behaves as an equivalent homogeneous solid yielding the constitutive equation for the frame in the form of a Hooke's law such that

$$\boldsymbol{\sigma} = \mathbf{H}\boldsymbol{\varepsilon} \quad (1)$$

$$\boldsymbol{\sigma} = \{ \sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23} \} \quad (2)$$

$$\boldsymbol{\varepsilon} = \{ \varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}, \varepsilon_{13}, \varepsilon_{23} \} \quad (3)$$

where \mathbf{H} is the static, elastic Hooke's matrix in stiffness form, $\boldsymbol{\sigma}$ is the stress, and $\boldsymbol{\varepsilon}$ is the strain vector. The fully anisotropic elasticity requires 21 independent parameters in the elastic Hooke's tensor.

The anisotropic elastic properties of porous materials may arise from diverse reasons, e.g. manufacturing, pre-strain, distributions, geometry shape, etc. Since the purpose of the paper is to demonstrate the method in design applications, a straightforward approach by modification of Kelvin cell microstructures is used to generate anisotropic foam materials. The Kelvin cell has a relatively simple geometry but can generate complex enough anisotropic material properties [2-4, 8, 13, 16-18].

The general Kelvin unit cell has 14 polyhedron sides including eight hexagonal faces and six square faces. Its cell size is denoted as S_x , S_y and S_z in the x , y and z -direction respectively, see Fig. 3. Isotropic Kelvin cells and anisotropic Kelvin cells of mixed struts are given as examples in Fig. 3. Geometric and material properties of the Kelvin cells are

shown in Table 1, where the circular strut shape was used for the sake of simplicity. The Young's modulus of the matrix material is E and the Poisson's modulus is ν .

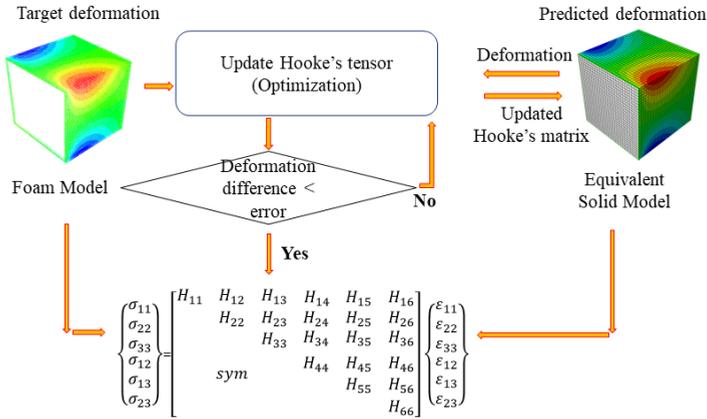


Fig. 1. Inverse method flowchart: minimisation of the displacement difference on free faces (colour contour plot) between the target and predicted models.

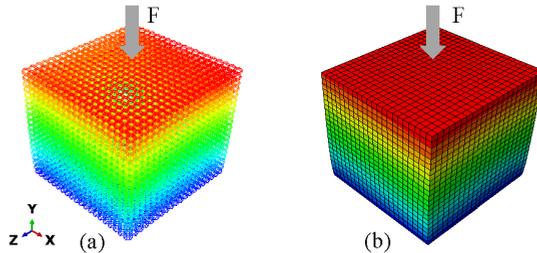


Fig. 2. Representation of the Kelvin cell and solid model in the case of a compression along y -axis.

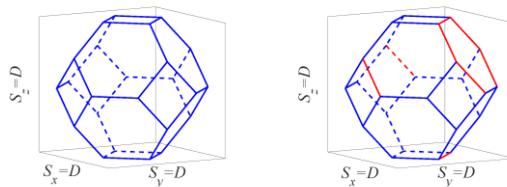


Fig. 3. Sketch of a unit Kelvin cell: a) isotropic cell, c) anisotropic cell with mixed struts of weak (red lines) and normal struts.

Table 1. Geometric and material properties of the Kelvin cells.

Quantity	Symbol	Isotropic	Anisotropic	Unit
Cell width in x, y, z -direction	D	3	3	mm
Cross section radius of normal struts	r_n	0.25	0.25	mm
Cross section radius of weak struts	r_w	-	0.13	mm
Solid density of strut material	ρ	1600	1600	kg/m^3
Young's modulus of strut material	E	563	563	MPa
Poisson's ratio of strut material	ν	0.3	0.3	-

3 Result and discussion

Figs. 4 and 5 show that the deformation components of the equivalent model are nearly the same as the target model loads with a very small deviation. All deformation under compression and shearing are recovered in the equivalent model. The displacement differences between the predicted and the target displacements for each applied load are less than 0.07%. The negligible difference may be due to various reasons, e.g., boundary condition, different element type between the target and predicted model, etc.

Table 1 shows the predicted Hooke's tensors of the isotropic and anisotropic cases, and the difference between them. The results show that the mixed cross-section struts introduced anisotropic properties to the KC model, i.e., all Hooke's tensor components $H_{ij} < 0$ for the mixed struts case. The location of reduced cross-sections struts dominates the shear-compression coupling effects. The terms of compression and shearing moduli, H_{ii} , decreased by around 30% due to the weak struts. The results also indicate that the modification of specific struts in the open-cell geometry, e.g., changing cross-section areas, could provide a way to tune the moduli of the anisotropic Hooke's matrix.

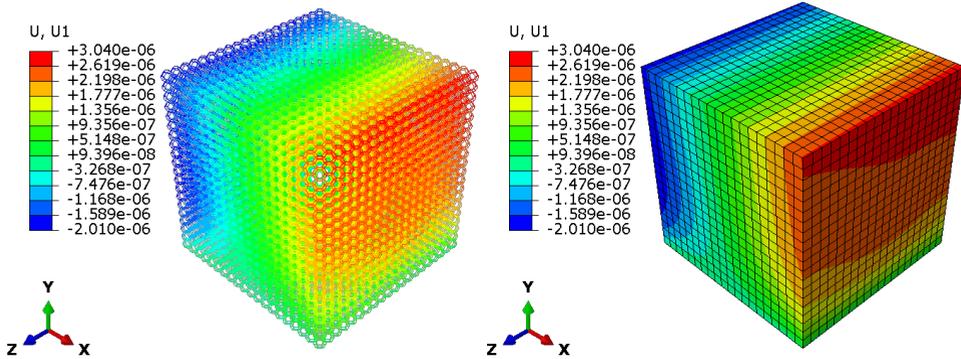


Fig. 4. Compression load case of the anisotropic design with zero displacements at the $y=y_{min}$ face, imposed displacement of 0.01mm in the y -direction at the $y=y_{max}$ face, results shown are the displacement component in the x -direction for the a) anisotropic Kelvin model and b) the predicted equivalent solid model.

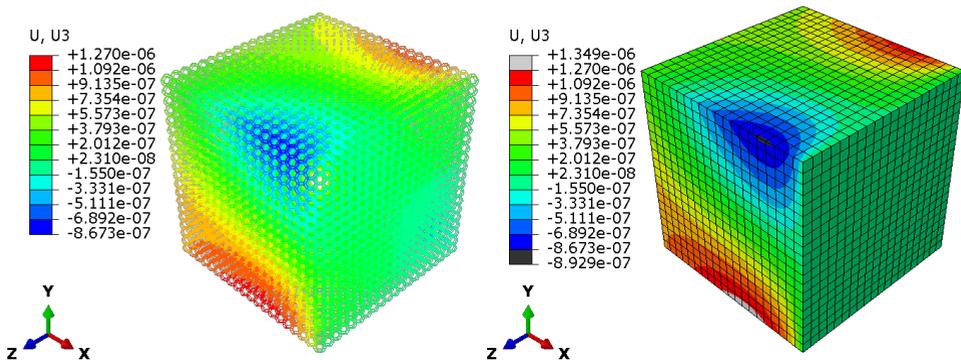


Fig. 5. Shear load case of the anisotropic design with zero displacements at the $x=x_{max}$ face, shear displacement 0.01 mm in the y -direction at the $x=x_{min}$ face, results shown are the displacement component in the z -direction for the a) anisotropic Kelvin model and b) the predicted equivalent solid models.

Table 2. Predicted Hooke’s tensor components: isotropic and mixed struts anisotropic Kelvin cells.

H_{ij}	Isotropic	Anisotropic	$(H_{\text{aniso}}-H_{\text{iso}})/H_{\text{iso}}$ (%)
H_{11}	14.626	10.2648	-29.8189
H_{12}	9.6332	7.1356	-25.9269
H_{22}	14.699	10.3190	-29.7972
H_{13}	9.576	7.0583	-26.2919
H_{23}	9.6108	7.1030	-26.0938
H_{33}	14.610	10.1921	-30.2367
H_{14}	0	0.2952	--
H_{24}	0	0.2922	--
H_{34}	0	0.2196	--
H_{44}	2.5993	1.7084	-34.2748
H_{15}	0	0.2847	--
H_{25}	0	0.2140	--
H_{35}	0	0.2844	--
H_{45}	0	0.3010	--
H_{55}	2.6141	1.6983	-35.0318
H_{16}	0	0.2187	--
H_{26}	0	0.2926	--
H_{36}	0	0.2963	--
H_{46}	0	0.2921	--
H_{56}	0	0.3043	--
H_{66}	2.6167	1.7151	-34.4539

4 Conclusion

The paper shows that the elastic Hooke's tensor including 21 parameters is successfully characterised for anisotropic porous foams using an proposed inverse estimation methodology by the authors. Deformations of the target foam under compression and shear loads are recovered in the equivalent model. Both the Young's modulus and the shear modulus are identified in the isotropic model, and the full Hooke’s tensor is properly characterised in the anisotropic model. This highlights, therefore, that the inverse method could apply both to isotropic and anisotropic foams. Additionally, it is shown that changing

cross-section areas of struts provides a possible way to tune the moduli in the Hooke's tensor. Extending this preliminary study to several modification including the analysis of several forms of the influence of geometry, or the influence of solid matrix parameters, are required in order to further apply the inverse method to acoustic material design.

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