

# Mixed mode I/II/III fracture assessment of PMMA using a new test fixture

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**Abstract.** Brittle and quasi-brittle fracture has been recognized as the main failure mechanism in various structural components. Due to complexity of the components' geometry and loading condition, a combination of tensile, in-plane shear and out of plane shear loading results in final fracture of cracked components. Hence, it is important to evaluate the fracture behaviour of different materials under a mixed mode loading condition which is close to the real-life loading of the structural parts. In this paper, a new loading device for general mixed mode I/II/III fracture tests is designed and recommended. Finite element analyses are performed to study the fracture parameters of the test specimens under different mixed mode loading conditions. According to the numerical results, the designed loading fixture can generate a wide varieties of mode mixities from pure tensile mode to pure in-plane and out of plane shear modes. Fracture tests were performed on pre-cracked specimens made of polymethyl methacrylate (PMMA) to check the accuracy of the proposed fixture. The proposed fixture was proved to be applicable for general mixed mode fracture evaluation of brittle materials.

## 1 Introduction

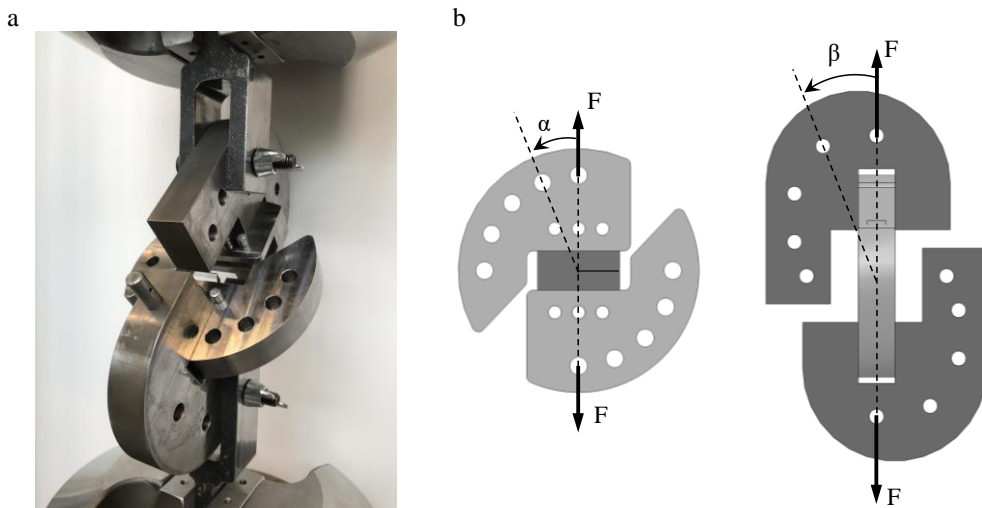
Various fracture testing procedures have been suggested in the past to obtain the mixed mode fracture behaviour of different materials under a combination of tensile mode and in-plane shear mode (i.e. mixed mode I/II) [1-3]. Application of different loading angles on a symmetrically pre-cracked specimen and inducing inclined pre-crack in a test specimen under axial loading are the main concepts of the available mixed mode fracture testing methods. In addition to the classic case of mixed mode I/II fracture, some experimental studies were conducted by researchers to evaluate the mixed mode I/III and II/III fracture behaviour of pre-cracked components [4-7]. Compared to the case of mixed mode I/II, relatively fewer test procedures are available for mixed mode I/III and II/III loading conditions.

Even though the results of mixed mode I/II, I/III and II/III fracture tests give a good understanding of the overall behaviour of material in presence of cracks, however, considering the real-life components, a combination of all three modes of fracture can result in final failure of the parts. As a matter of fact, performing mixed mode I/II/III fracture tests requires complex specimen geometries and loading conditions. Hence, there are very limited test configurations for experimental investigation of mixed mode I/II/III fracture

[8,9]. In this research a testing fixture has been evaluated, which allows to use a relatively simple test specimen for complex mixed mode fracture tests. The proposed apparatus has simple configuration, inexpensive fabrication procedure, convenience of testing set up and also the ability of introducing general mixed mode loading conditions. The capabilities and accuracy of the new fixture have been evaluated using numerical and experimental analyses on PMMA specimens.

## 1 Fracture testing fixture

The mixed mode fracture test configuration is presented in Fig. 1. The fixture includes two separate parts namely C-fixture (also known as Arcan fixture) and J-fixture. While C- fixture is used for applying in-plane mixed mode loading, J- fixture is used to apply out of plane mixed mode loads. Depending on the angle of rotation in each fixture different 3D mode mixities can be obtained. Five loading holes with equal angular position are drilled on each half of both C- and J- fixtures, which are used for applying various mixed mode loading conditions in the test specimen. The positions of loading holes were chosen in a way to have a wide range of mode mixities among pure mode I ( $\alpha = 0^\circ, \beta = 0^\circ$ ), pure mode II ( $\alpha = 90^\circ, \beta = 0^\circ$ ) and pure mode III ( $\alpha = 0^\circ, \beta = 90^\circ$ ), where  $\alpha$  and  $\beta$  are the angular positions of the C- and J- fixtures. The test specimen has a thickness of 10 mm with an induced pre-crack of  $a/w = 0.5$ , where  $a$  is the crack length and  $W$  is the width of specimen. The test specimen is fixed in the fixture using pins on both sides of the specimen.



**Fig. 1.** The fixture for mixed mode I/II/III fracture tests; (a) test configuration and (b) schematic view of C- and J- fixture.

## 2 Numerical analysis

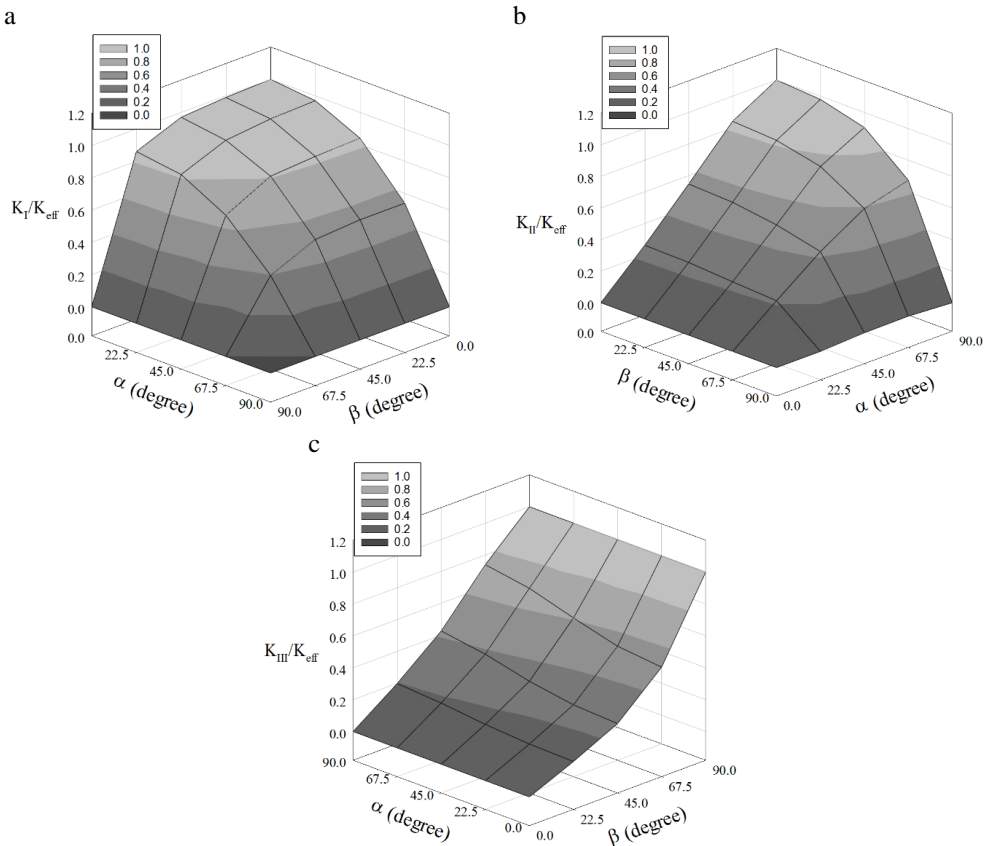
Finite element analyses were performed using ABAQUS 6.14 standard finite element code to evaluate the efficiency of the test fixture in applying 25 different mixed mode loading conditions. For this aim, stress intensity factors (SIF) of the cracked specimen was calculated under different loading conditions. Linear elastic finite element analyses were conducted on the test configuration by considering an elastic modulus of 2.9 GPa and Poisson's ratio 0.35 for PMMA. The variations of SIFs with respect to the loading angles of

C-fixture and J-fixture are shown in Fig. 2. The represented results in Fig. 2 were obtained by deviding the SIF values at the mid-thickness of the model to the effective SIF value obtained from the following equation

$$K_{eff} = \sqrt{K_I^2 + K_{II}^2 + K_{III}^2} \tag{1}$$

where  $K_I$ ,  $K_{II}$  and  $K_{III}$  are the SIF values related to mode I, mode II and mode III portions of the stress at crack tip.

According to Fig. 2, pure mode I and pure mode II loading conditions can be obtained under  $(\alpha=0^\circ, \beta=0^\circ)$  and  $(\alpha=90^\circ, \beta=0^\circ)$  loading angles, while pure mode III can be obtained when  $\beta=90^\circ$  independent of the C-fixture angle,  $\alpha$ . Hence, in order to perform mode III fracture experiments using the new fixture, it would be enough to test the samples  $\beta=90^\circ$  and only one angle of the C-fixture.



**Fig. 2.** The normalized SIFs under various loading conditions; (a) mode I SIF, (b) mode II SIF, and (c) mode III SIF.

### 3 Mixed mode fracture tests

The test specimens were machined from a PMMA sheet of 10 mm thick using computer numerical control (CNC) machine and 0.4 mm thick circular saw blade was used to create initial notch in the test specimens followed by fine cutting of the notch tip to make the final crack length of each specimen equal to  $a/W = 0.5$ . A series of fracture tests were performed

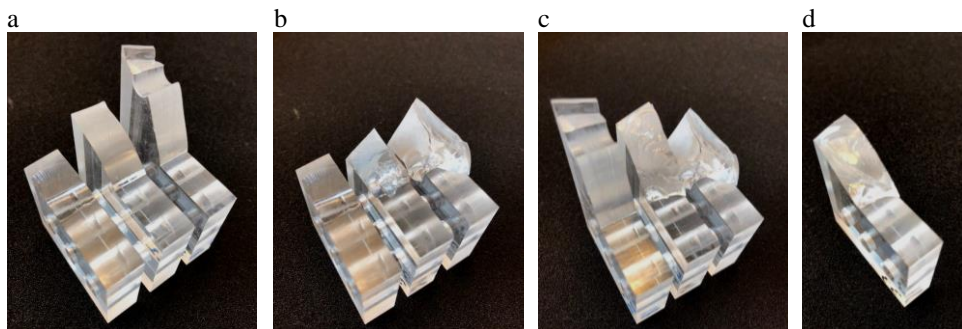
on the specimens with a displacement rate of 1 mm/min to examine the applicability and efficiency of the loading fixture for mixed mode I/II/III fracture tests. The external load was applied to the fixture using two forks connected to the J-fixture (see Fig. 1). This load was then transferred to the test specimen through the pins on each side of the specimen. All the test specimens were fractured suddenly from the crack tip and the load displacement curves represented linear behaviour confirming the brittle fracture behaviour of PMMA under various loading conditions.

The peak values of the load-displacement curves were extracted and considered as fracture loads of the tested specimens. These values were then used for calculating the critical SIF values from the FE analysis. The details of the fracture tests conducted under different loading conditions are presented in Table 1, where  $F_c$  represents the average fracture load,  $K_{I_f}$ ,  $K_{II_f}$ ,  $K_{III_f}$ , and  $K_{eff \ f}$  represent critical mode I, mode II, mode III and effective SIFs corresponding to the fracture loads and obtained at the mid-thickness of the FE models. The average fracture toughness of PMMA under pure mode I,  $K_{IC}$ , pure mode II,  $K_{IIC}$ , and pure mode III,  $K_{IIIC}$  were obtained as 18.2 MPa√mm, 36.8 MPa√mm, and 102.5 MPa√mm, respectively.

**Table 1.** Mixed mode fracture strength of tested PMMA specimens.

Loading mode	$\alpha$ (deg)	$\beta$ (deg)	$F_c$ (N)	$K_{I_f}$ (MPa√mm)	$K_{II_f}$ (MPa√mm)	$K_{III_f}$ (MPa√mm)	$K_{eff \ f}$ (MPa√mm)
I	0	0	796.8 ± 86.8	18.2	0	0	18.2
II	90	0	1295.4 ± 13.7	0	36.8	0	36.8
III	0	90	2180.1 ± 219.2	0	0	102.5	102.5
I/II	45	0	779.7 ± 79.3	16.5	10.8	0	19.8
I/III	0	45	1285.9 ± 76.9	36.7	0	9.7	38.0
II/III	90	45	1277.9 ± 57.7	0	39.9	18.8	44.2
I/II/III	45	45	1025.0 ± 35.1	26.7	16.2	10.4	32.9

The representative fracture surfaces of the tested specimens are illustrated in Fig. 3. The measured in-plane and out-of-plane fracture angles,  $\theta_f$ ,  $\phi_f$  obtained by direct measurement from the fractured samples are presented in Table 2. According to the experimental observations, except the pure mode I loading case where crack extension occurred along the initial crack plane, in mixed mode I/II loading condition the crack kinks in the loading plane and in mixed mode I/III loading the crack twists with respect to the initial crack plane. A combination of both in-plane and out of plane crack extension was observed for the case of mixed mode II/III and the general case of mixed mode I/II/III.



**Fig. 3.** Representative fractured CTS specimens under different loading conditions; from left to right (a) mode I, mixed mode I/II, mode II, (b) mode I, mixed mode I/III, mode III, (c) mode II, mixed mode II/III, mode III, and (d) mixed mode I/II/III.

**Table 2.** Fracture angles of the specimens tested using the mixed mode loading fixture.

Loading mode	$\alpha$ (deg)	$\beta$ (deg)	$\theta_f$ (deg)	$\phi_f$ (deg)
I	0	0	0	0
II	90	0	68	0
III	0	90	0	42
I/II	45	0	35	0
I/III	0	45	0	37
II/III	90	45	40	16
I/II/III	45	45	25	18

## 4 Conclusions

Applicability of a new loading fixture for general mixed mode I/II/III fracture studies was evaluated using numerical and experimental analyses. Finite element analyses were performed on the designed set-up and the results revealed the capability of the loading fixture to apply various loading conditions including pure modes and mixed modes of fracture. A series of fracture tests were conducted on pre-cracked PMMA specimens to practically confirm the performance of the fixture. Fracture loads and fracture angles were obtained from the experiments. The proposed fracture test fixture and its associated test specimen is recommended as a practical and suitable test set-up for any type of mixed mode fracture studies.

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