Automation of assessment of the strain state of metal cutting machine mechanism elements on the basis of response function application

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Abstract. The article dwells upon the assessment of the strain state of metal-cutting machines with the help of the numerical method of finite elements in the form of displacements. A metal-cutting machine is considered as a system consisting of a set of structural elements taking a set position in space in relation to each other, and a part of them may execute relative motions (displacements). The paper suggests a modification of the finite element method on the basis of combining a set of finite elements into a group with the use of the response function. The author provides an analytical calculation method to resolve the problem of assessing the strain state of the machine mechanism elements in linear and non-linear settings. The developed algorithm allows analyzing geometrical, physical and contact non-linearity of metal working. The research provides a practical implementation of the developed method by the examples of assessing the strain state of a rod element modeling the roughness of the contact surface of the machine mechanism elements under temperature and tensile loads.

1 Introduction

At design of metal-cutting machines a relevant task is to make a geometrical image of the structure with a subsequent analysis of its strain state under the action of a system of perturbing factors.

A metal-cutting machine can be considered as a system consisting of a set of structural elements taking a set position in space in relation to each other, and a part of them may execute relative displacements. The relations between components and nodes as well as their interconnections define the type and nature of interaction: stiff (fixed); elastic; thermoelastic; elastoplastic [1-4].

Internal and external perturbing factors changing the state of structural elements induce the violation (deviation) of initial connections and relations thus leading to strains or deviations in the geometrical images of mechanism elements.

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The assessment of the aforementioned deviations is a complex and multi-level task requiring analysis and consideration of various patterns of parts and nodes interaction in machine mechanisms.

At the interaction of components (elements) such phenomena occur which, in some cases at their strain state description, make the researcher reject the assumption of a linear connection between strains and displacements in the material (geometrical non-linearity).

Physical non-linearity is conditioned by the nature of a range of processes, such as occurrence of plastic strains at material cutting and at the contact interaction of bodies.

Geometrical non-linearity occurs at the machining of lowered stiffness elements. One can observe it when using the nodes of machine mechanisms with strained elements (for example, harmonic drives, bearings, etc.). At last, by analogy, it is possible to identify contact non-linearity occurring, for example, at elastic contact of bodies when a functional dependence between the areas of application of contact pressures and external geometrical and power factors is non-linear while the material of the bodies is characterized by physical and geometrical linearity. The preliminary analysis of machining and various patterns of the interaction of elements in the machine nodes leads to the conclusion the models of the error formation mechanism at machining comprise all three types of non-linearity: physical, geometrical and contact [1-6].

The obtained solution may not be a single one for non-linear problems. In such cases the desired solution from a set of solutions is obtained by applying the method of small displacements and basing upon the problem physical nature. At preserving the problem physical content the proposed method allows one to obtain the solution with large increments with a simultaneous use of all the variable intervals of variation. That is why the method can be interpreted as the large increment method.

When solving technical problems, the approximation of volumes or surfaces, elements and nodes with finites elements is widely applied [6-13]. Uniting finite elements in a group is conducted on the basis of the Rayleigh-Ritz method, Galerkin method, the finite difference method, etc. [1]. The relevance of this paper, in contrast to the aforementioned methods, consists in combining finite elements into a group with the use of the response functions applied in the experiment design theory [4-8, 10-12, 14-27].

The purpose of the paper is to develop an algorithm for assessing a stress&strain state of metal-cutting machines on the basis of application of the response function method.

To achieve the set objective, the following research tasks are stated in the paper: develop a concept for solving elastic problems on the basis of the response function method; identify key relations of the finite element method in the form of displacements; develop an algorithm for calculating the matrix for response function coefficients.

2 General provisions of method of assessment of the strain state of elements on the basis of response function application

The modification of the finite element method (FEM) on the basis of the method of the response surface (RSM) is possible in the form of analogy between the expressions setting the connection between the elements of point nodes displacements (stresses) and elements of displacements or stresses of any medium point within the considered FEM continuum on the one hand, and between the input parameters (factors) set by the axes of the MRS design and the responses on the other hand. In both cases a polynomial model is used for such representation [4-6, 28-30].

It is known that the theory of experiment design stipulates for obtaining the response surfaces or functions on the basis of the least square method (LSM). The LSM for making up a group of elements can provide satisfactory results, and the obtained dependencies

describing the stress&strain state of the elastic medium will be correct in case the assumed shape functions satisfy the FEM convergence criteria [4].

The LSM is known [2, 4-6] at making the group, however, its application is complicated by the procedure of a global stiffness matrix formation on differential operators. That is why a useful simplification of the calculation process will be making a resolving system of equations for a whole group of elements on the precise response values in node points. In this case the application of the RSM allows just obtaining approximating dependencies.

The concept of resolving the problems of the elasticity theory on the basis of the method of response functions (RFM) is formed as follows. The researched area fixes a finite number of nodes. The area can be presented as consisting of a finite number of subareas (elements) having common node points. In each element a continuous value is approximated by a polynomial the parameters of which are defined depending on nodal displacements. Displacements in nodal points are called factors and considered to be set in compliance with the experiment designs. The selection of the design is defined by a type of polynoms, solving system and a number of nodes. The region of factor variation is defined on the basis of the a-priori information on the stress&strain state of structures. For the combinations of nodal displacements set in compliance with the design the parameters of mathematical expressions are defined which approximate internal element regions. By the obtained approximating ratios obtain equivalent loads applied in the nodes of separate elements and causing nodal displacements defined by the designs for one element. After defining equivalent forces applied to the nodes of certain elements obtain the equivalent forces called responses by means of summing. These forces operate in the nodes of the group of elements and cause displacements corresponding to the designs. As the factors and the corresponding responses are defined, the least square method is used to define the coefficients of approximating polynoms forming a solving system of equation. This system establishes analytical dependencies between each nodal load and displacements in all nodes. The desired continuum displacements after boundary conditions considering are defined by set external nodal loads and the solution of the solving equation system.

As the internal design of the studied mechanism of the described FEM is simulated with the help of the ECM, the mechanism (metal cutting machine or any other technical device) can be considered as a “glass box”. At the input of such box nodal displacements $\{\delta\}$ are set, at the output of the model equivalent nodal forces are defined $[Y_u]$ (Figure 1). Within the framework of the “glass box” concept the transfer of the output signal after obtaining the roots of the solving system and their assessment through a feedback chain allows correcting the input signal $\{\delta\}$ and building all the cycle again until a satisfactory solution is found $\{\delta\}^0$.

Under this approach main FEM ratios in the form of displacements will be written as follows [4-6]: the finite element in the system of coordinates xyz is defined by nodal points $\{i, j, k, \dots \text{etc.}\}$; displacement inside $\{f\}^{(e)}$ any point inside the element (e) is set by a matrix equation:

$$\{f\}^{(e)} = [N]^{(e)} \{\delta\}^{(e)}, \quad (1)$$

where $[N]^{(e)}$ – shape function matrix; $\{\delta\}^{(e)}$ – nodal displacement vector; $(e) = 1, 2, \dots, n_e$.

Fig. 1. Principal diagram of the algorithm of calculating the strain state of the machine mechanism elements.

At using the method of feasible motions the stiffness matrix of one element takes a form:

$$[K]^{(e)} = \int_{V^{(e)}} [B]^T [D]^{(e)} [B] dv \quad (2)$$

where $V^{(e)}$ – element volume; $[D]^{(e)}$ – matrix containing material elastic constants; $[B]$ – gradient matrix obtained by means of proper differentiating the shape form matrix.

A global stiffness matrix and global column vector are made by combining the elements into a group on the basis of minimizing full system potential energy (Rayleigh - Ritz principle).

$$[K] = \sum_{e=1}^{n_e} [K]^{(e)} \quad (3)$$

$$\{F\} = \sum_{e=1}^{n_e} \{F\}^{(e)} \quad (4)$$

where $\{F\}^{(e)}$ – matrix of external nodal forces applied to the element.

Thus, for FEM, in the form of displacements the resolving equation system will be writtten in a matrix form:

$$[K]\{\delta\} = \{F\}, \quad (5)$$

where $\{\delta\}$ – nodal displacement vector.

At boundary conditions

$$\{\delta\} = 0, k = 1, 2, \dots, n_k \quad (6)$$

where n_k – number of nodes with fixed displacements set by a response function pattern.

RFM matrix calculation algorithm: decompose the machine mechanism structural element into finite elements; form the matrices of design $[X]$ and levels $[U_r]$; identify a finite element and build its stiffness matrix $[K]^{(e)}$; define nodal loads (responses) in a finite element; summing the loads in the finite element nodes define the equivalent systems $[Y]$ of

forces applied to the nodes of all the machine mechanism structure; define the coefficient of the approximating polynomial (response function).

Form a global matrix $[X]$ of experiment design and a matrix of $[U_r]$ equations of factor values. In the matrix $[U_r]$ minimum and maximum values of nodal displacements are provided. For the indexed displacements $\{\delta\}^{(e)}$ of the element a matrix of $[X]^{(e)}$ displacement design for the element, and on the basis of the matrix data form a matrix $[\delta_u]^{(e)}$ of displacement values in the nodes for various experiments u ($u=1, \dots, N_u$, where N_u – number of tests in the experiment).

The forces $[P_u]^{(e)}$ acting in the element nodes for various tests (combinations of displacement values) set by the matrix $[X]^{(e)}$ are obtained by the formula:

$$[K]^{(e)} \{\delta_u\}^{(e)} = [P_u]^{(e)} \quad (7)$$

The equivalent forces $[Y_u]$ applied in the structural nodes and causing displacements defined by the matrices $[X]$ and $[U_r]$ are obtained by the formula:

$$[Y_u] = \sum_{e=1}^{n_e} [P_u]^{(e)} \quad (8)$$

The connection between the displacements at the input of the group of elements (factors) and equivalent nodal forces can be presented in the form of a polynomial model. In a particular case in compliance with the references [6] the ratios of $\{X\}$ and $[Y]$ can be described by a linear model:

$$[b]\{\bar{X}\} = \{Y\}, \quad (9)$$

where $\{\bar{X}\}$ – a column vector x_i of coded values $\{\delta\}$; m – square matrix of coefficients b_{ei} at M_u of unknown parameters, $i \in [0, M_u]$; $\{Y\}$ – response matrix.

$$x_i = 2(\delta_i - \delta_{i \min}) / (\delta_{i \max} - \delta_{i \min}) - 1$$

At orthogonal design for a linear model the coefficients b_{ei} of the system (5) will be obtained after the manipulation:

$$b_{ei} = \frac{l}{N_u} \sum_{u=1}^{N_u} X_{iu} Y_{ul}, \quad u \in [1, N_u], \quad l \in [1, M_u] \quad (10)$$

where Y_{ul} – responses related to l - matrix column $[Y_u]$.

In practice, the displacements in the structural nodes will be obtained by resolving the equation system of the type:

$$[b_l]\{\bar{X}\} = -\{b_0\} + \{F\} \quad (11)$$

where $\{F\}$ – matrix of set external loads; $\{b_0\}$ – coefficient matrix at constant terms; $[b_l]$ – coefficient matrix at unknown $\{X\}\{\bar{X}\}$.

To take into account boundary conditions at making a final system of equations, a two-stage procedure is conducted. This procedure is similar to the one described in the paper [6]. At entering the specified values of displacements the transition to their code values occurs. The solution of the system obtained in such a way is performed by one of the known methods.

Starting the postulation of the mathematical model $y = \varphi(x_1, x_2, \dots, x_k)$, it is necessary to take into account the nature of the a-priori information on the researched process which

can be characterized by three conditions [4-6, 30-33]: the approximating function formula is known, it is required to specify unknown parameters; it is known the response surface coincides with one of the set functions. it is necessary to define this function and specify its parameters; the desired function type is unknown.

It is only known that the function in some region can be approximated by a finite series by some systems of the pre-set functions.

In a general case when the research is conducted at incomplete knowledge of the studied process mechanism the approximating function can be represented by a polynomial with the degree d [9]:

$$y = B_0 + \sum_{i=1}^k B_{ij} x_i x_j + \dots + \xi \quad (12)$$

where B_{ij} – polynomial coefficient; ε – approximation error

In most cases the following models provide sufficient approximation:

First order models:

$$y = b_0 + b_1 x_1 + \dots + b_k x_k + \xi, \quad (13)$$

Second order models:

$$y = b_0 + \sum_{i=1}^k b_i x_i + \sum_{i,j}^k b_{ij} x_i x_j + \xi \quad (14)$$

Models reducible to a linear model by means of manipulation. Multiplying or polynomial model:

$$y = b_0 + \sum_{i=1}^k b_i x_i + \sum_{i,j}^k b_{ij} x_i x_j + \xi, \quad (15)$$

where α – constant; $b_1, b_2, \dots, b_k$ – unknown exponents; ε – multiplying error.

The logarithmation by the equation (15) base reduces the model to a linear form. More complex exponential models are provided in the paper [10]. The coefficients B_{ij} of the approximating polynomial (12) are obtained with the help of of the least square method in compliance with [6] for the matrix equation:

$$\{B_y\} = ([X]^T [X])^{-1} ([X]^T \{Y\}), \quad (16)$$

where $[X]$ – matrix of independent variables or factors $x_1, x_2, \dots, x_k$; $\{Y\}$ – matrix of observation or responses y ; $\{B_y\}$ – coefficient matrix b_y .

In the expanded form the expression (16) will be written as follows:

$$b_y = \sum_{j=0}^k C_{ij} (iy) \quad (17)$$

where C_{ij} – matrix elements $([X]^T [X])^{-1}$; iy – matrices $[X]^T \{Y\}$.

When defining the coefficients with the values x_i are set by their code values by the formula varying each of the factors at various levels:

$$x_i = 2(X - X_{min}) / (X_{max} - X_{min} - 1)$$

The boundaries of the factor variation region and levels are defined in relation to the a-priori data of the process under research. These boundaries can be obtained as a result of

the preliminary calculation, on the basis of experimental research data, by means of setting a range of trials, etc. The combination of levels is defined by the design of a corresponding order. The design order corresponds to the model order, i.e. the degrees of the approximating polynomial.

3 Practical relevance

On the basis of the developed method the authors solved the problems of assessing the strain state of a rod element modeling the roughness of the contact surface of the machine mechanism parts under temperature and tensile loads [6, 7].

The proposed method can be illustrated by the solution of the problem of temperature distribution in a one-dimensional rod with the following parameters: a , b , c . Other rod parameters are given in Figure 2, where P – perimeter, S – cross-section area, L – finite element length, $L=1.5$ cm.

The results of the calculation of a modification system of equations are provided in the Table 1 [10]. It also provides the results of a theoretical solution of the problem and the calculation made by means of a standard FEM procedure.

Fig. 2. Temperature distribution in a one-dimensional rod.

Table 1. Temperature values in the nodes, °C.

Calculation method	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
RFM	150.0	88.8	61.7	49.8	44.8	43.0
FEM	150.0	82.6	59.0	48.6	44.2	42.6
Theoretical solution	150.0	89.9	62.8	50.6	45.2	43.3

Table data comparison proves the RFM gives satisfactory results. Define displacements in the rod under tension. As an example consider a non-linear behavior of roughness presented in the rod form. Consider the rod strain at linear and non-linear dependence of σ and strain ξ . The calculation model is provided in Figure 3. The modulus of elasticity at the linear dependence between σ and ξ . is equal to $E=2\cdot10^5$ MPa, the loading force applied to the rod $P=20000$ H.

A non-linear dependence of σ and ξ . is approximated by the formula [7]:

$$\xi = 0,25\cdot10^{-6}\sigma^2$$

(18)

The range of factors variation is assumed on the a-priori data on the value of rod nodes' motions. Initial motions in the point 1 are assumed to be equal to zero. It is suggested that

the operation of the structure material is described by a stress-strain graph without the yielding area with a steadily decreasing tangential module [10]. The results of the motion calculation in the rod for linear and non-linear models are given in the Table 2. The consistency of results, as it is seen from the Table, is satisfactory.

Fig. 3. The model of calculating displacements in the rod.

Table 2.Temperature values in the nodes, °C .

Model	Motions, mm	
	$u_2 (X_2)$	Subhead $u_3 (X_3)$
Non-linear	0.40457	0.82914
Linear	0.40000	0.80000

The research demonstrated that it is possible to make a new scientific direction in technical calculations on the basis of response functions. This direction have special mathematical, algorithmic and software tools developed.

5 Conclusions

The paper develops an algorithm for assessing the strain state of the parts of metal cutting machine mechanisms on the bases of the response function application.

A comparative analysis of the FEM and RFM showed that in some cases at the calculation of production machines one can apply the finite element method based upon the least square method.

The author obtained key analytical dependencies for a modified method of finite elements based upon the approaches used by the response function method.

A sufficient method precision was proved by the comparison of the obtained numerical results on the basis of response functions for non-linear rod tension and the heat transfer problems with known analytical results.

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