

# Distribution of enterprise management resources based on parameters of a surge in demand for product

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**Abstract.** All social and economic processes are characterized by the presence of a large number of low frequencies in the spectrum of the parameters describing them. In this case, it becomes possible to consider individual surges of a random process, which will be characterized by phase, average value and duration. According to these indicators, it is possible to prepare the enterprise for the production of precisely those products for which there is a surge in demand in the near future. It is accepted that the probability of a surge in demand for this type of product after a given time is a Markov nature and is described by the Kolmogorov equation. The overall probability of technology readiness can be defined as the product of the partial readiness probabilities in phase, duration and average amplitude, considering them to be independent events. The unavailability of technology decreases after each dosed management exposure. As a result, a methodology has been developed for the optimal distribution of control resources, which makes it possible to increase the availability of technology by more than a third compared to their equal distribution.

## 1 Introduction

All social and economic processes are characterized by the presence of a large number of low frequencies in the spectrum of the parameters describing them. The usual indicators of a random process will be effective only if there are statistics for a long period of time. However, during this period, changes have occurred that make the process unsteady and, as a result, the classical approach through analysis of variance or correlation analysis may not be suitable in this case. On the other hand, in this case, it becomes possible to consider individual bursts of a random process, which will be characterized by phase, duration and average value. Predicting the onset of a surge in these indicators, it is possible, for example, to prepare an enterprise for the production of precisely those products for which there is a surge in demand in the near future.

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## 2 Theory

Typically, the company is managed according to a pre-compiled schedule [1,2]. For the formation of prices and sales in the market, the company monitors especially carefully [3-5]. On the other hand, a probabilistic description of market and production processes is quite common in the literature [6–10]. Let the likelihood of a surge in demand for this type of product after a given time  $\tau$  have a Markov nature and is described by the Kolmogorov equation [11], presented here in canonical form:

$$\frac{\partial \omega_1(\tau, t)}{\partial t} = b \frac{\partial^2 \omega_1(\tau, t)}{\partial \tau^2} \quad (1)$$

where  $\omega_1$  is the density of the probability described above, depending on the time  $t$ ;  $b$  is the diffusion coefficient.

The readiness of the technology that creates these product at the enterprise should, under control actions, go to an equation similar to (1). Let  $u_1(\tau, t)$  be the number shares of control resources allocated to organize the necessary readiness. We introduce the coefficient of the expense rate of funds  $c_1$ , measured to satisfy the dimension of equation (1) in reverse seconds. Denoting the probability density of technology readiness to meet the emerging demand  $\hat{\omega}_1$ , we have:

$$\frac{\partial \hat{\omega}_1(\tau, t)}{\partial t} = b \frac{\partial^2 \hat{\omega}_1(\tau, t)}{\partial \tau^2} + c_1 u_1(\tau, t) \quad (2)$$

The task of managing the availability of technology is to provide the possibility of production in demand, corresponding to a favorable period of excess (surge) in the price of products on the market over the cost of its manufacture using this technology. This approach dominates the choice of various management strategies [12].

Subtract from (1)-th equation (2) and get:

$$\frac{\partial (\omega_1 - \hat{\omega}_1)}{\partial t} = b \frac{\partial^2 (\omega_1 - \hat{\omega}_1)}{\partial \tau^2} - c_1 u_1 \quad (3)$$

We introduce the notation of probabilistic unavailability of technology

$$S_1 = \omega_1 - \hat{\omega}_1 \quad (4)$$

Then (3) is transformed as follows

$$\frac{\partial S_1}{\partial t} = b \frac{\partial^2 S_1}{\partial \tau^2} - c_1 u_1 \quad (5)$$

Arguing in a similar way, we arrive at an equation linking the probabilistic unavailability of technology  $S_2$  to ensure that the market requires the duration of production of a given product equal to the duration of the surge in demand  $T$  with a control action

$$\frac{\partial S_2}{\partial t} = b \frac{\partial^2 S_2}{\partial T^2} - c_2 u_2 \quad (6)$$

In the same way, we obtain an equation linking the probabilistic unavailability of  $S_3$  technology to provide the average output amplitude of a given product required by the market, equal to the average amplitude of demand surge  $A$ , with a control action.

$$\frac{\partial s_2}{\partial t} = b \frac{\partial^2 s_2}{\partial A^2} - c_3 u_3 \tag{7}$$

To a first approximation, the overall probability of technology readiness can be defined as the product of particular readiness probabilities in phase, duration and average amplitude, considering them to be independent events. The unavailability of the technology decreases after each dosed management impact aimed at correcting the phase, duration of the technology's functioning and the average amplitude of output by  $q_i$ . Considering the managerial impacts to be independent, we will get a general change in technology readiness in the form of a product or an exponential function  $q_i^{u_i}$ . Then the overall probability that describes the readiness of the technology as a whole will be expressed as

$$P = \prod_{i=1}^3 (1 - q_i^{u_i}) \tag{8}$$

To simplify expression (8), we assume that  $P_i = 1 - q_i \rightarrow 1$ . By multiplying and freeing ourselves from the values of the second and higher orders of smallness, we obtain the general unavailability of the technology:

$$Q(\bar{u}) = \sum_{i=1}^3 q_i^{u_i}, \tag{9}$$

The total costs of organizational activities related to increasing the availability of technology in phase, duration and amplitude are expressed as a linear relationship:

$$C = C(\bar{u}) = \sum_{i=1}^3 c_i u_i, \tag{10}$$

The control task is as follows: to find the optimal distribution of the shares of the control actions  $u_i$  to ensure minimum costs at a given level of unavailability of this technology.

To solve the problem of optimal control by the Euler - Lagrange method, we compose the Lagrangian:

$$F(\bar{u}) = \sum_{i=1}^3 c_i u_i + \sum_{i=1}^3 \psi_i \left( \frac{\partial s_i}{\partial t} - b \frac{\partial^2 s_i}{\partial x_i^2} + c_i u_i \right) + \sum_{i=1}^3 \varepsilon_i (Q_i^z - q_i^{u_i}), \tag{11}$$

where for convenience  $x_1 = \tau, x_2 = T, x_3 = A$ , and  $Q_i^z$  - the set value of the unavailability of the technology in phase, duration and amplitude to the satisfaction of market demand. Here, the first term is an integrand, the second requires satisfying the equations of the control object (5,6,7), and the last requires fulfilling the set constraints.

To ensure extreme  $F(\bar{u})$  we compose Euler equations in all variables:

$$\begin{cases} \frac{\partial F(\bar{u})}{\partial u_i} = c_i + \psi_i c_i - \varepsilon_i q_i^{u_i} \ln q_i = 0 \\ \frac{\partial F(\bar{u})}{\partial \psi_i} = \frac{\partial s_i}{\partial t} - b \frac{\partial^2 s_i}{\partial x_i^2} + c_i u_i = 0 \\ \frac{\partial F(\bar{u})}{\partial s_i} - \frac{d}{dt} \left( \frac{\partial F(\bar{u})}{\partial \dot{s}_i} \right) = - \frac{d\psi_i}{dt} = 0 \\ \frac{\partial F(\bar{u})}{\partial \varepsilon_i} = Q_i^z - q_i^{u_i} = 0 \end{cases} \quad i = 1, 2, 3 \tag{12}$$

From the third equation we obtain that  $\psi_i = const, i = 1, 2, 3$

From the first equation of the system we find  $u_i$ :

$$u_i = \frac{\ln \frac{c_i + \psi_i c_i}{\varepsilon_i \ln q_i}}{\ln q_i} = \frac{\ln \frac{a_i}{\varepsilon_i}}{\ln q_i}, \tag{13}$$

Where  $a_i = \frac{c_i + \psi_i c_i}{\ln q_i}$ .

We find the Lagrange multiplier  $\varepsilon_i$ , substituting  $u_i$  from (13) into the fourth equation of system (12):

$$Q_i^z = \sum_{i=1}^3 q_i^{u_i} = \sum_{i=1}^3 \frac{a_i}{\varepsilon_i} = \frac{1}{\varepsilon_i} \sum_{i=1}^3 a_i \tag{14}$$

where from

$$\varepsilon_i = \frac{\sum_{i=1}^n a_i}{Q_i^z} \tag{15}$$

In its final form, the expression for determining the optimal shares of control actions to increase technology readiness is:

$$u_i = \frac{\ln \frac{a_i}{\varepsilon_i}}{\ln q_i} = \frac{\ln a_i - \ln \varepsilon_i}{\ln q_i} = \frac{\ln a_i - \ln \left( \frac{\sum_{i=1}^n a_i}{Q_i^z} \right)}{\ln q_i} = \frac{1}{\ln q_i} \ln \left( \frac{a_i \cdot Q_i^z}{\sum_{i=1}^n a_i} \right) \tag{16}$$

We substitute the resulting expression into the second equation of system (12). This equation can be solved using the Green function [13]. The resulting equation is heterogeneous. Consider its solution under the initial conditions

$$s_i(x_i, 0) = s_{i0}; s_i(0, t) = 0. \tag{17}$$

Introducing in this case  $l_i$  - the boundary of the enterprise's ability to meet the demand for its product, and  $t_f$  - the final control time, we obtain a solution that is written as

$$s_i = \frac{1}{2\sqrt{\pi}} \iint_{00}^{t_f l_i} \frac{1}{\sqrt{b_1(t-\theta)}} \left\{ e^{-\frac{(x_i-\xi)^2}{4b_1(t-\theta)}} - e^{-\frac{(x_i+\xi)^2}{4b_1(t-\theta)}} \right\} c_i u_i(\xi, \theta) d\xi d\theta \tag{18}$$

Substituting expression (16) here for the fraction of the control action, we obtain

$$s_i = \frac{c_i}{\ln q_i} \ln \left( \frac{a_i \cdot Q_i^z}{\sum_{i=1}^n a_i} \right) \frac{l_i}{2\sqrt{\pi}} \int_0^{t_f} \Phi \left( \frac{x_i}{2\sqrt{b_1(t-\tau)}} \right) d\tau, \quad i = 1, 2, 3 \tag{19}$$

Here

$$\Phi(z) = \int_0^z e^{-\alpha^2} d\alpha$$

- error integral

From these equations, having set the final control time  $t_f$ , it is possible to determine the optimal probability of decreasing the unavailability of the technology  $Q_i^z$ , thereby relating it to the nature of the market process, or vice versa, by setting  $Q_i^z$  it is possible to determine the optimal time to reach this value.

### 3 Data and Method

To test the operability of the method of determining the optimal resources to increase technology availability, we set the unavailability level reached at the end of control  $Q_{зад} = 0,01$ , coefficient  $\psi_i = 1$  and  $q_i = 0.001$  for  $i = 1,2,3$ .

Representing the burst in the form of a rectangle, we understand that the largest impact on losses is caused by the erroneous average amplitude of the burst and that it is more difficult to bring the production to a predetermined volume than to stop distributing the funds allocated to individual events to increase readiness by phase, duration and average amplitude, respectively in rubles: 1 - 40000; 2-80000; 3-20000.

We calculate the share of control resources allocated to each of the factors:

$$u_1 = \frac{1}{\ln q_1} \ln \frac{a_1 * Q_{зад}}{\sum_{i=1}^n a_i} = 0,848$$

$$u_2 = 0,7477$$

$$u_3 = 0,9483$$

The funds allocated in accordance with the calculated shares (1-33920 2-59816 3-18966) are plotted on the histogram of Figure 1.



**Fig. 1.** Histograms of funds allocated to increase technology readiness.

### 4 Results and discussion

The total amount of allocated funds is 112702. After dividing them into three factors, you can get the average amount - 37567. Now you can compare the total unavailability of technologies with an optimal solution and an equal distribution of funds according to formula (16). These values are  $2.3332 * 10^{-8}$  and  $3.5934 * 10^{-8}$  in the first and second cases, respectively. Technology availability increases by more than 30%

## 5 Conclusion

Thus, the optimal distribution of enterprise management resources based on the parameters of the surge in demand for product on the market can significantly increase production efficiency. The developed methodology for the optimal distribution of control resources, which makes it possible to increase the availability of the technology by more than a third in comparison with their equal distribution by influencing factors, is of practical importance.

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