

Study on the Vibration Suppression Method of Urban Railway Vehicles Based on a Composite Dynamic Vibration Absorber

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Abstract. To reduce the bounce and the pitch vibration of carbody, a vertical dynamic model for urban rail vehicles is established to analyze the vibration response of the carbody in the low frequency range. In this paper, different methods of single-degree-of-freedom dynamic vibration absorber to suppress the vibration for carbody are investigated. The limits of single-degree-of-freedom dynamic vibration absorber to the vibration reduction effect of carbody are pointed out. After that, the design of a composite dynamic vibration absorber including a double oscillator structure is introduced. A vibration discreteness index is used to evaluate dynamic vibration absorbers with various designs for the vibration damping performance. Finally, the vibration reduction performance of the composite dynamic vibration absorber is verified by Sperling's riding index. The results demonstrate that the performance of the single degree of freedom dynamic vibration absorber attached to a carbody may increase the vibration within a partial scope, when the peak frequency of vibration is far away from the design frequency. The installation of the composite dynamic vibration absorber vibration provides gentler running experience for passengers.

1 Introduction

During the operation of rail transit vehicles, the vibration caused by rail irregularities is transmitted to the carbody through the suspension system, which has a certain impact on the ride stability and comfort of passengers. On the one hand, the vibration energy of the carbody is weakened by the transmission and diffusion of elastic waves; on the other, the damping effect depending on the vibration medium is weakened. The mass-spring-damping structure of the dynamic vibration absorber (DVA) has become a representative device for effectively suppressing vibration^[1-3].

To effectively improve the stability of vehicle operation, domestic and foreign scholars have conducted a lot of research on the DVA to suppress the vertical vibration reduction of the carbody in several years^[4-16]. Foo E. et al. adopted the principle of "skyhook damping" to suppress the elastic vibration of the carbody due to the lightweight design by installing a DVA on the carbody^[4]. Wu P B. et al. proposed to install the under-vehicle equipment on the undercarriage as a DVA to suppress the elastic vibration of the EMU carbody, and analyzed the influence of the suspension parameters of the equipment on the vibration of the carbody. Theoretical research found that light equipment and heavy equipment are installed away from the centre of the carbody, the vibration damping performance of the DVA is limited^[5-8]. Based on the theory of DVA, Wen Y P. et al proposed a design method of a single-degree-of-freedom (SDOF) DVA, which can effectively reduce the bounce vibration

of the carbody, and comprehensively consider the vibration characteristics at different working conditions^[9-11]. Based on the fixed-point theory, the frequency ratio and damping ratio of the DVA with negative stiffness solved by Shen Y et al. and the simulation results that the DVA can greatly reduce the resonance amplitude, widen the vibration frequency range, and even keep the amplitude of the main system small in the whole frequency range^[12-13]. Combing with multibody dynamics software Simpack, Tomioka T et al. put forward a continuous model in which the carbody seen as elastic which may realize the three-dimensional elastic vibration control of the carbody by the DVA^[14-16]. To sum up, most of the previous DVAs are used to control the elastic vibration of the carbody, and few of them are used to control the rigid vibration, especially the dynamic DVAs which can absorb both the bounce and pitch vibration of the carbody. Thus, there is lack of a damping method for simultaneously suppressing the nodding and floating vibration in the rigid vibration of rail vehicles.

In this paper, combined with vibration characteristics of urban rail vehicles to establish the vertical dynamic model of the elastic carbody of urban rail including the composite vibration absorber (CDVA), and the double oscillator structure of the CDVA with two design frequencies to suppress the bounce and pitch vibration of the carbody is confirmed, and puts forward numerical indexes for quantitatively evaluating the vibration damping performance of the carbody. Finally, the paper compares and analyses the design of the SDOF DVA.

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The study is conducted from the perspective of the comparison between SDOF DVA and CDVA.

2 Establishment of system model

Fig. 1 is a vertically coupled vibration model of a vehicle system including a composite vibration absorber.

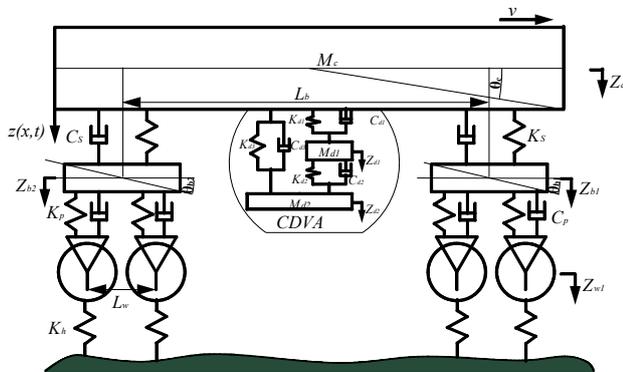


Figure 1. The vertical dynamics model of urban rail vehicles with CDVA

The eight components in the vehicle system include 12 degrees of freedom, respectively corresponding to the two degrees of freedom of the bounce motion Z_c and the pitch motion θ_c of the carbody, the four degrees of freedom of the bounce motion Z_{bi} ($i=1,2$) of the bogie and the pitch motion θ_{bi} ($i=1,2$) at the I and II position end, the four degrees of freedom of the bounce motion Z_{wi} of the two front and rear wheelsets and the two degrees of freedom of the bounce motion Z_{di} ($i=1,2$) of the CDVA (including two oscillators and vibration coupling between oscillator 1 and 2), and the carbody, the bogies and the wheelsets are regarded as rigid bodies. The velocity of the vehicle is v , t is a time variable. The parameters of the model are shown in Table 1.

2.1 Solution of model

Table 1. Urban rail vehicles dynamic parameters and meanings.

Meaning	Parameter	Value
carbody mass	M_c	39t
wheelset mass	M_w	1.1185t
bogie mass	M_b	2.6t
length of carbody	L	21.88m
half-length between truck centers	L_b	7.85m
half wheelbase	L_w	1.25m
vertical equivalent stiffness of primary suspension	k_p	2400kN·m ⁻¹
vertical equivalent stiffness of secondary suspension	k_s	700kN·m ⁻¹
vertical damping of primary suspension	c_p	50kN·s·m ⁻¹
vertical damping of secondary suspension	c_s	130kN·s·m ⁻¹
moment of inertia for carbody nodding	I_c	2300t·m ²
moment of inertia for bogie nodding	I_b	1.424t·m ²

According to Lagrange equation, the expression of vertical dynamics of multi-free vehicle system including CDVA can be formulated as below:

$$\begin{bmatrix} \mathbf{M}_{cc} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{dd} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{Z}}_c \\ \ddot{\mathbf{Z}}_d \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{cc} & \mathbf{C}_{cd} \\ \mathbf{C}_{dc} & \mathbf{C}_{dd} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{Z}}_c \\ \dot{\mathbf{Z}}_d \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{cc} & \mathbf{K}_{cd} \\ \mathbf{K}_{dc} & \mathbf{K}_{dd} \end{bmatrix} \begin{Bmatrix} \mathbf{Z}_c \\ \mathbf{Z}_d \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_c \\ \mathbf{F}_d \end{Bmatrix} \quad (1)$$

Where, the footnotes c and d respectively represents the vehicle system without the composite vibration absorber and with the CDVA; \mathbf{M} , \mathbf{C} , \mathbf{K} and \mathbf{F} respectively represents the mass matrix, damping matrix, and \mathbf{Z} are the generalized acceleration vector, the generalized velocity vector and the generalized displacement vector of components:

$\dot{\mathbf{Z}}$ and \mathbf{Z} are the generalized acceleration vector, the generalized velocity vector and the generalized displacement vector of components.

Considering the wheelset-time lag characteristics between the four wheelsets in vehicle operation, the system is transformed from multi-excitation input into single, thus obtain the relation matrix between the irregular excitation of the other wheelsets and the first wheelset:

$$\mathbf{q}(\omega) = \mathbf{Q} \mathbf{q}_1(\omega) \quad (2)$$

Where, \mathbf{Q} is the time delay relation matrix between the first wheel and other wheelsets, and the specific expression is:

$$\mathbf{Q} = \begin{bmatrix} 1 & e^{-j\omega T_1} & e^{-j\omega T_2} & e^{-j\omega T_3} \end{bmatrix}^T \quad (3)$$

Where u is the velocity of the vehicle, $T_1 = 2L_w/u$, $T_2 = 2L_b/u$, $T_3 = 2(L_w + L_b)/u$.

Taking the track irregularity excitation q_1 as the input, the equation (1) can be presented as:

$$(-\omega^2 \mathbf{M} + j\omega \mathbf{C} + \mathbf{K}) \mathbf{Z}(\omega) = \mathbf{K}_f \mathbf{q}_1(\omega) \quad (4)$$

Taking the influence of composite vibration absorber into consideration, \mathbf{M} is mass matrix of components in the vehicle system, \mathbf{C} and \mathbf{K} are damping and stiffness matrix of system considering influence of composite vibration absorber respectively, \mathbf{K}_f transfer matrix between vehicle and wheel excitation.

To analyze the vibration characteristics of rail vehicle system, the frequency response function $H(\omega)_{\dot{z}_i-q_1}$ of acceleration response of each component of the vehicle system to random irregularities q_1 is solved:

$$H(\omega)_{\dot{z}_i-q_1} = -\omega^2 q_1(\omega)^{-1} Z_i(\omega) = -\omega^2 h_i(\omega) \quad (5)$$

Where, $q_1(\omega)$ is acceleration power spectral density (PSD), $h_i(\omega) (i=1,2,\dots,12)$ the displacement vector is the system frequency response characteristic. $h_1(\omega)$ is the displacement frequency response characteristics of bounce motion of carbody, $h_8(\omega)$ is displacement frequency response characteristics of pitch motion of carbody; $h_{11}(\omega)$ and $h_{12}(\omega)$ are displacement frequency response characteristics of two oscillator of CDVA, thus the vibration response at any position a of the carbody is:

$$H(a, \omega) = H_1(\omega) + (0.5l - a)H_8(\omega) \quad (6)$$

Where, $H_1(\omega)$ is the frequency response function corresponding to the displacement frequency response $h_1(\omega)$ of the bounce motion of the carbody; $H_8(\omega)$ is the frequency response function corresponding to the displacement frequency response $h_8(\omega)$ of the pitch motion of the carbody.

$$G_{\dot{z}_i}(\omega) = |H(a, \omega)_{\dot{z}_i-q_1}|^2 G_{q_1}(\omega) \quad (7)$$

Where, $G_{q_1}(\omega)$ is track irregularity excitation.

2.2 The natural frequency of CDVA

The vibration equations of the CDVA with upper and lower oscillators designed is as follows:

$$\begin{cases} M_{d1} \ddot{z}_{d1} = -k_{d1} z_{d1} - c_{d1} \dot{z}_{d1} - \\ k_{d2} (z_{d1} - z_{d2}) - c_{d2} (\dot{z}_{d1} - \dot{z}_{d2}) \\ M_{d2} \ddot{z}_{d2} = -k_{d3} z_{d2} - c_{d3} \dot{z}_{d2} + \\ k_{d2} (z_{d1} - z_{d2}) + c_{d2} (\dot{z}_{d1} - \dot{z}_{d2}) \end{cases} \quad (8)$$

Where, M_{d1} and M_{d2} are the masses of the upper and lower oscillators respectively; K_{d1} and C_{d1} (K_{d3} and C_{d3}) are stiffness and damping between the upper oscillator 1 (lower oscillator 1) and the carbody respectively; Z_{d1} and Z_{d2} are the vertical displacements

of oscillator 1 and oscillator 2 in the two-degree-of-freedom model.

When the damping effect is not considered, the natural frequency of the CDVA can be written as:

$$\begin{vmatrix} K_{d1} + K_{d2} - M_{d1}\omega^2 & -K_{d2} \\ -K_{d2} & K_{d3} + K_{d2} - M_{d2}\omega^2 \end{vmatrix} = 0 \quad (9)$$

Where, $\omega = 2\pi f_d$, f_d is the natural frequency of vibration absorber.

According to equation (9), obviously, natural frequency of the CDVA is determined by the mass and stiffness of the two oscillators, and the magnitude of the natural frequency affects the damping effect of the absorber. By determining the mass of the upper and lower oscillators, also, K_{d2} equals K_{d3} , the relationship between natural frequency and the mass or stiffness of the absorber can be obtained by simplifying:

$$f_{d1,2}^2 = (AK_{d1} + BK_{d2}) \mp \sqrt{(AK_{d1} - CK_{d2})^2 + DK_{d1}K_{d2}} \quad (10)$$

Where, A , B , C and D all can be expressed by M_{d1} and M_{d2} .

3 Vibration response of the carbody

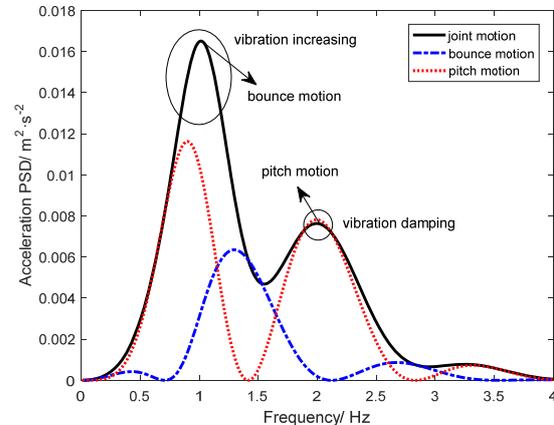


Figure 2. Vibration response of carbody end.

Taking the American six-level spectrum as input, the vibration response at each position of the carbody can be obtained. Where the total displacement of the vertical vibration of the carbody defined as a joint motion, and the vibration response at a position is $H(a, \omega)$, it is formed by superimposing bounce and pitch motion of the carbody, the vibration responses are $H_1(\omega)$ and $H_8(\omega)$. The acceleration PSD at the end of the carbody when the velocity is 80 km/h and the vehicle is unloaded is selected as an example for analysis, as shown in Fig. 2:

As can be seen from Fig. 2, the vibration response at the end of the carbody is observed with the acceleration PSD as the ordinate. in the low frequency band, the carbody closing motion produces two obvious peaks, the

first vibration peak is dominated by bounce vibration, and the second vibration peak is dominated by carbody pitch vibration. The vibration response of the carbody closing motion is not a simple linear superposition of the bounce motion and the pitch motion. There is also a coupling vibration between the two motions. Thus, the vibration shows different degrees of enhancement or attenuation on the acceleration PSD.

4 Disadvantages of the SDOF DVA

In the rail vehicle system, the carbody is regarded as the main vibration system. Based on the resonance principle of the DVA, the SDOF DVA can absorb the vibration energy of the carbody. As additional equipment, the quality of the vibration absorber ensures its vibration reduction effect, and the restrictions of factors such as economy and difficulty of arrangement should be considered. Following research, the mass ratio of the vibration absorber is selected as: $\mu=0.1$.

On the premise of determining the quality of the absorber, the key to vibration reduction of a SDOF DVA is to determine the design frequency f_d . When the vehicle is operating at 80 km/h with no load, the carbody has two different vibration peak frequencies. The peak frequency of body heave motion is 1.01 Hz and the peak frequency of pitch motion is 2.00 Hz respectively. For the above frequencies, when designing a SDOF DVA, there are two designs I and II are designs of SDOF DVA with different design frequencies, and the vibration reduction effect is shown in Fig. 3.

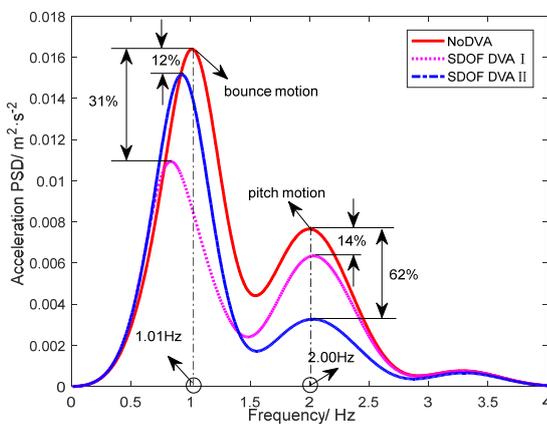


Figure 3. Vibration reduction performance of single-degree freedom vibration absorber

As can be seen from Fig. 3, the SDOF DVA designed for different vibration peaks can effectively reduce the bounce vibration of the carbody, especially when the SDOF DVA is designed for 1.01 Hz, its vibration reduction effect on the bounce motion of the carbody at the peak frequency is about 31%; When the design of SDOF DVA is designed for 2.0 Hz, the damping effect of the SDOF DVA on the pitch motion of the carbody at the peak frequency is 62%; However, the effect of SDOF DVA design on the peak frequency of pitch motion and that of SDOF DVA design on the peak frequency of bounce vibration are only 14% and 12%, respectively.

Because the SDOF DVA can only be designed for a certain target vibration reduction frequency, also the target vibration reduction frequency is single.

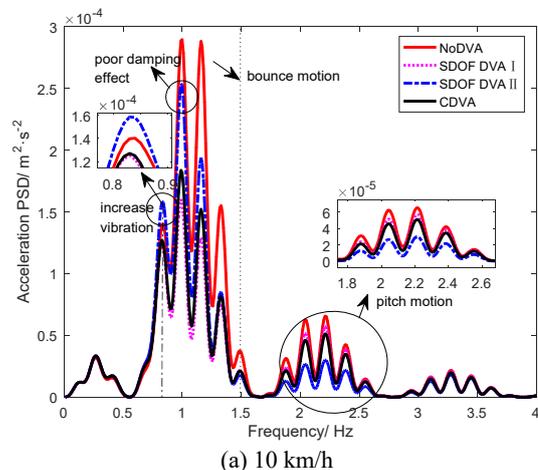
To sum up, the SDOF DVA can achieve the effect of wide-band vibration reduction to a certain extent. However, when there are two target vibration reduction frequencies, and the design frequency of the DVA is far away from a certain peak frequency of vibration, the design of the SDOF DVA will have limitations and the vibration reduction effect is limited. Therefore, the paper proposes a method of vibration reduction of the carbody by using the CDVA.

5. Superiority of Compound DVA

According to the design of the SDOF DVA, when the vehicle at a velocity of 80 km/h under no-load conditions, the bounce and pitch vibration of the carbody are dominant in the frequency range of 0~4 Hz, and there are two different peak frequencies on 1.01 Hz and 2.00 Hz, the target vibration damping frequency f_{j1} and f_{j2} , they are correspond to the natural frequencies of the CDVA. With equation (9), thus the natural the frequency of the CDVA is determined by the mass and stiffness of the two oscillators. On the premise that the total mass of the absorber is constant, the mass and stiffness of the oscillators need to be determined, so, the stiffness can be expressed by the mass and natural frequency of the absorber:

$$\begin{cases} K_{d1} = \frac{4\lambda - (2M_{d1}M_{d2} + 4M_{d2}^2) \cdot (\lambda - \tau)}{4M_{d1}M_{d2}^2 + M_{d2}^3} \\ K_{d2} = \frac{4(\lambda - \tau)}{4M_{d1} + M_{d2}} \\ K_{d3} = \frac{4(\lambda - \tau)}{4M_{d1} + M_{d2}} \end{cases} \quad (11)$$

Where, λ and τ can be represented by M_{d1} and M_{d2} f_{d1} and f_{d2} as follows:



(a) 10 km/h

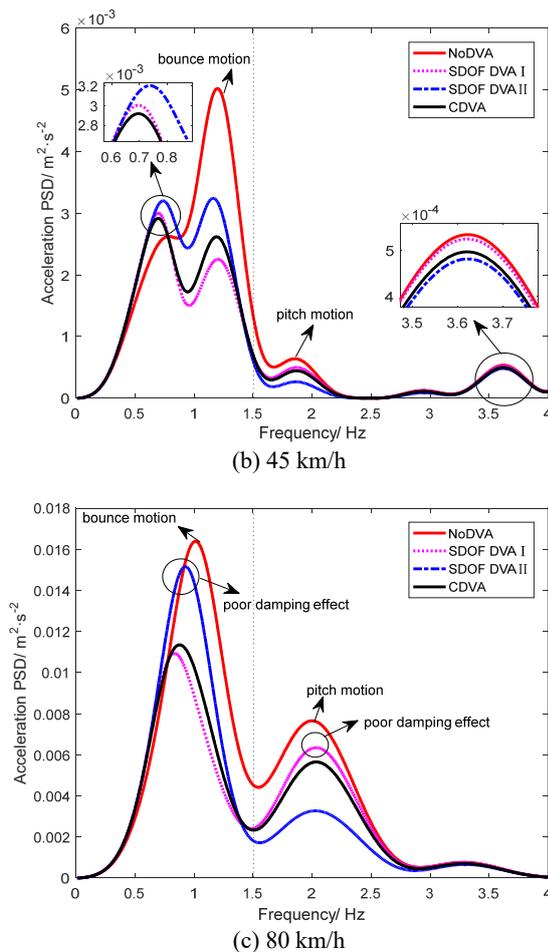


Figure 4. Influence of SDOF DVA and CDVA on vertical vibration of rail carbody

$$\lambda = M_{d1}M_{d2}\pi^2(f_{d1}^2 + f_{d2}^2) \quad (12)$$

$$\tau = M_{d2}\pi^2 \sqrt{M_{d1}^2(f_{d2}^2 - f_{d1}^2) - M_{d1}M_{d2}f_{d1}^2f_{d2}^2} \quad (13)$$

When the natural frequencies of the absorber are determined, the structural parameters of the CDVA can be obtained.

In order to verify the damping effect of CDVA, considering the frequent changes in vehicle velocity caused by the short distance between urban vehicle traffic stations and vehicle starting and braking frequently, the acceleration PSD at the end position of the carbody under three vehicle velocity conditions, at low velocity 10 km/h, at medium velocity 45 km/h and at high velocity 80 km/h, is investigated, the damping effect of CDVA is compared with the SDOF DVA, as shown in Fig. 4.

As can be seen from Fig. 4, with the change of the vehicle velocity of the urban rail vehicle, the difference reflected on the vibration response at the end of the carbody lies in difference for the vibration characteristics. The CDVA is used to reduce the vibration of the carbody at different operating velocities of the carbody. Although the vibration reduction effect of the CDVA is insufficient compared with the design of the SDOF DVA for the single peak frequency, the vibration reduction effect of the CDVA is obvious at the peak frequencies of floating, sinking and nodding movements, and there is no obvious

vibration increase phenomenon in the design of the SDOF DVA in Fig. 4(b). Therefore, the CDVA is not confined to vibration reduction aiming at a certain peak value. By dispersing the total mass of the DVA onto the two oscillators and utilizing the targeted vibration reduction effect of the respective design frequencies of the two oscillators, the problem of the design limitation of the SDOF DVA is treated in a compromise way to realize the effect of wider frequency vibration reduction of the carbody. Compared with the Fig.5, acceleration PSD with the increase of vehicle velocity increases in order of magnitude, and the peak values in each frequency band tend to be concentrated from scattered. This is because when urban rail vehicles are running at high velocity, the unfavourable wavelength that has the greatest impact on the carbody vibration gradually increases, and the irregularity of the track in this wavelength range will excite the body with lower natural vibration frequency. The shorter the wavelength, the more times the internal excitation occurs between units.

To describe the vibration reduction effect of the CDVA and the SDOF DVA in the bounce vibration and pitch vibration frequency bands more intuitively, the vibration is divided into two frequency bands: the first band is 0~1.5 Hz, mainly bounce vibration; The second segment is 1.5~4 Hz, mainly pitch vibration. The vibration discreteness in each frequency band is calculated separately. Considering the vibration characteristics of the vehicle during operation, the bounce vibration of the carbody is significantly larger than the pitch vibration in numerical value. Therefore, when analyse the vibration damping effect of the CDVA and the SDOF DVA quantitatively, the vibration discreteness of each frequency segment obtained needs to be multiplied by the weighting coefficient of the corresponding frequency segment, namely, the comprehensive index of vibration discreteness. The value is small, which means the vibration energy under this velocity condition is small. The specific expression is:

$$CVD = \sum_{k=1}^2 \left(\sum \left(G_{z_c}(f_{jk}) - \overline{G_{z_c}(f_{jk})} \right)^2 \cdot a_k \right) \quad (14)$$

Where, a_k ($\sum_{k=1}^2 a_k = 1$) is weighted coefficient of different vibration frequency segments, $G_{z_c}(f_{jk})$ is acceleration PSD of carbody vibration acceleration at f_j Hz in k th vibration frequency band. $\overline{G_{z_c}(f_{jk})}$ is the average of the acceleration PSD of carbody vibration in the range of k th frequency.

As can be seen from Fig. 5, the CDVA has the lower index of vibration dispersion under the conditions of low velocity of 10 km/h, medium velocity of 45 km/h or high velocity of 80 km/h, that is to say, the lower the vibration dispersion degree of the CDVA is compared with the SDOF DVA, and the obvious the vibration reduction effect is CDVA, SDOF DVA I is after, the SDOF DVA II is last. Because of the double-oscillator structure of the CDVA, the distribution of mass and the targeted

damping of the two design frequencies contribute significantly to the damping at the peak frequency of the carbody. As can be seen from Figs. 5(a), (b) and (c) that with the increase of velocity, the dispersion index of the vehicle increases gradually in numerical, that is, the greater the dispersion degree of the acceleration PSD value of the carbody in the modified frequency band, the greater the vibration response of the vehicle. Own to the increase of velocity, the intensification of the function between the wheel and rail of the vehicle, thus the vibration energy of the carbody increases, and the greater the acceleration PSD value, the greater the dispersion degree.

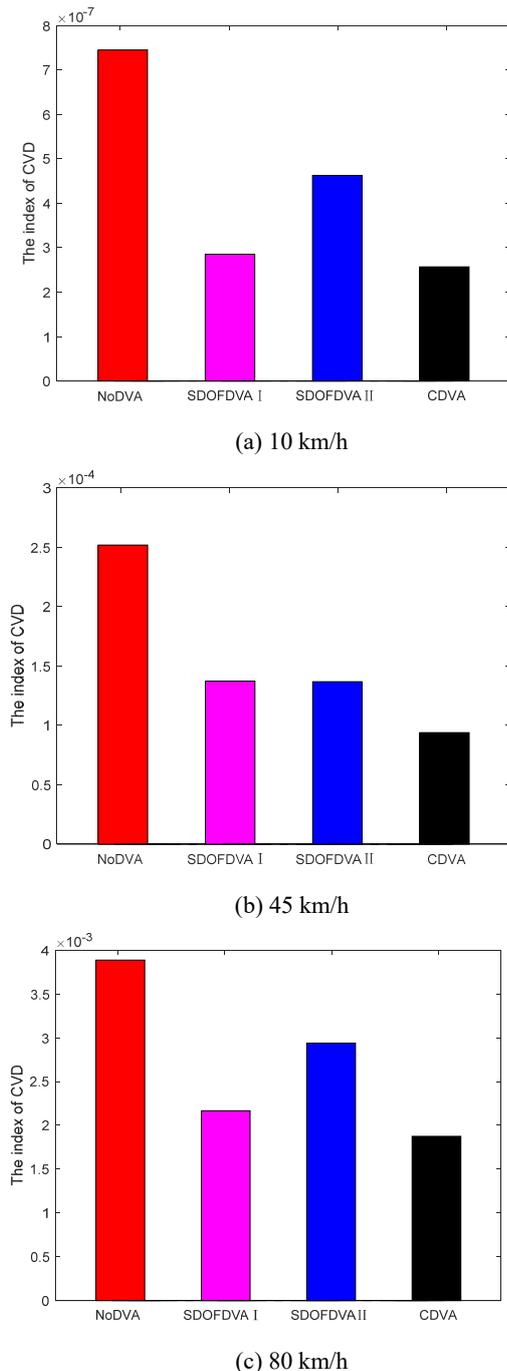


Figure 5. Influence of SDOF DVA and CDVA on vertical vibration of rail carbody.

6. Sperling's riding index verification

Ride comfort depends on different dynamic performance standards and subjective feelings of passengers. Vibration of different factors such as vehicle conditions, track areas and operating conditions will reduce ride comfort of passengers. According to the specific needs of the railway management department for traffic details, there are also a wide range of methods for evaluating ride comfort [17-18]. Sperling's riding index is adopted in this paper. The result is intuitive and more suitable for comparison of two or more operating conditions.

In this paper, the product of impulse and vibration kinetic energy is used as a measurement standard to evaluate the running stability of vehicles. The formula is:

$$W_z = 2.7^{10} (z_0 f^5 F(f))^{0.5} = 0.896^{10} (F(f) / f)^{0.5} \quad (15)$$

Where, z_0 is amplitude, f is frequency, $a = z_0(2\pi f)^2$ is acceleration, $F(f)$ is weighted coefficient related to vibration frequency

For vertical vibration, there are four different expressions of the weighting coefficient for the difference of the value range. Through spectrum analysis, the stationarity index of each frequency band is solved. Therefore, the total stationarity index W_{total} in the whole frequency band is:

$$W_{total} = (W_{z1}^{10} + W_{z2}^{10} + \dots + W_{zm}^{10})^{0.1} \quad (m \leq 4) \quad (16)$$

The stability performance decreases with the increase of the W_{total} , and the vehicle running stability is always evaluated to be in an excellent level when the value increases.

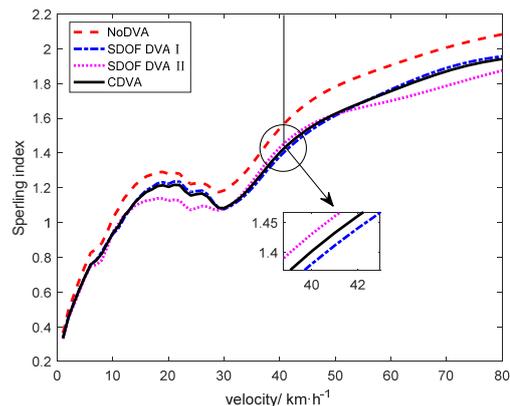


Figure 6. Sperling index of the comfort of the railway vehicle.

Fig. 6 is Sperling's riding index under no-load condition when the rail vehicle is equipped with CDVA. As can be seen from Fig. 6, with the increase of velocity, the vertical stability index of rail vehicles generally shows an upward trend and tends to be flat. When the vehicle is running under the conditions of unpowered DVA, installation of SDOF DVA and CDVA, the stability of the vehicle is kept in an excellent state. However, with the increase of the velocity of rail vehicles, the absolute value of the difference between the vertical

stability indexes of the vehicles with or without the DVA shows an increasing trend, which indicates that the dynamic DVA has more room to improve the stability of the vehicles at medium and high velocity and the vibration reduction effect is more obvious. Compared with the SDOF DVA, the stability index of the CDVA is always in a relatively stable state, and generally better than that of the SDOF design, and there is no phenomenon that the stability index of the SDOF design fluctuates greatly up and down. Therefore, under the condition of full velocity, the stability index value of the vehicle is lower and more stable, which can effectively improve the running stability of the rail vehicle and the riding comfort of the vehicle. Especially in medium and high velocity operation, its vibration reduction effect is more significant.

7 Conclusions

(1) Through the vertical dynamic model of the carbody-CDVA, the vibration response of the carbody shows the coupling of bounce vibration and pitch vibration in low frequency band, and the acceleration PSD shows the enhancement and attenuation of vibration in different degrees. The SDOF DVA can suppress the above two kinds of vibrations of the carbody respectively, but once the design frequency is far from the peak frequency, the vibration reduction effect of the carbody is suppressed and there exists vibration increase within a partial scope.

(2) In order to ensure that the dynamic DVA can suppress bounce vibration and pitch vibration in low frequency band, the parameter design of the CDVA with double oscillator structure is proposed. By using the principle that the mass and design frequency of the two oscillators jointly act on the rigid vibration of the carbody, the CDVA can effectively control the multi-peak vibration in low frequency band of the vehicle.

(3) The vibration discreteness comprehensive index established by using the acceleration PSD at low, medium and high velocity to quantitatively evaluate the vibration damping effect of the dynamic DVA, the smaller the value, the smaller the dispersion degree of vehicle vibration, the smoother the vibration response of the vehicle, and the better the vibration damping effect. Under the same working condition, the damping effect of the CDVA is better than that of the SDOF DVA.

Acknowledgements

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