Study on Crowning of helical gear shaved by CNC shaving machine with three synchronous axes

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Abstract. Gear Shaving is the most efficient and economical ways to finish gears before heat treatment. The workpiece after shaving operation can reduce the running noise, which often used in the automobile industry. In this paper, we propose a mathematical model of a CNC shaving machine with multi-synchronous axes. The proposed mathematical model can be used to calculate the crowning tooth surface of the gear in parallel-shaving process.

1. Introduction

The gear shaving process is often used in the tooth profile finishing before gear heat treatment process. The workpiece after shaving operation can reduce the running noise, which often used in the automobile industry. In this paper, we propose a mathematical model to simulate the shaving process with crowning motion.

Litvin[1] proposed the basic mesh conditions for the crossed helical gear set. Miao and Dugas [2] develop a mathematical model of the theory plunge shaving. Seol and Litvin [3] proposed a mathematical model of the crowned helical gear after the gear shaving process. Hsu and Fong[4] proposed a mathematical model of parallel crowned shaving process with a rock mechanism. Hsu, Wu, and Tran[5] proposed a shaving method for double crowning that has no natural twist in the tooth flanks on the work gear surfaces is proposed, which uses a variable pressure angle shaving cutter in a parallel gear shaving process.

In this paper, we derive a mathematical models for the shaving cutter and coordinate systems for the CNC shaving machine and shaving gear. We will compare the crowning in the shaving process with two and three synchronous axes CNC shaving machine.

2. Basic geometry of the plunge shaving cutter

Basic meshing conditions for the crossed helical gear set have been derived by Litvin [1] and are applied here to calculate the basic geometry data of the plunge shaving cutter. Assume the following eight data are given: teeth no. N_1 , normal circular tooth thickness s_{pn1} , helix angle β_{p1} of the shaving cutter; teeth no. N_2 , normal circular tooth thickness s_{pn2} , helix angle

β_{p2} of the work gear, normal module m_{pn} and pressure angle α_{pn} of shaving cutter and work gear. Based on Litvin's (1994) work, the following four equations are required to assure the tight meshing between involute crossed helical gears:

$$\gamma_o = (\beta_{o1} \pm \beta_{o2}) \quad (1)$$

$$\tan \alpha_{on} = \tan \alpha_{ot1} \cos \beta_{o1} = \tan \alpha_{ot2} \cos \beta_{o2} \quad (2)$$

$$p_{on} = \frac{m_{pt1} \cos \alpha_{pt1}}{\cos \alpha_{ot1}} \cos \beta_{o1} = \frac{m_{pt2} \cos \alpha_{pt2}}{\cos \alpha_{ot2}} \cos \beta_{o2} \quad (3)$$

$$N_1 \text{inv} \alpha_{ot1} + N_2 \text{inv} \alpha_{ot2} = N_1 \left(\frac{s_{pt1}}{2r_{p1}} + \text{inv} \alpha_{pt1} \right) + N_2 \left(\frac{s_{pt2}}{2r_{p2}} + \text{inv} \alpha_{pt2} \right) - \pi \quad (4)$$

where

$$m_{pt1} = m_n / \cos \beta_{p1},$$

$$m_{pt2} = m_n / \cos \beta_{p2},$$

$$\tan \alpha_{pt1} = \tan \alpha_{pn} / \cos \beta_{p1},$$

$$\tan \alpha_{pt2} = \tan \alpha_{pn} / \cos \beta_{p2},$$

$$s_{pt1} = s_{pn1} / \cos \beta_{p1},$$

$$s_{pt2} = s_{pn2} / \cos \beta_{p2},$$

$$r_{p1} = m_{pt1} N_1 / 2,$$

and $r_{p2} = m_{pt2} N_2 / 2$

Eq. (1) to Eq. (4) correspond to, respectively, the helix angle, pressure angle, circular pitch, and tooth thickness compatibility derived in the normal plane. The “ $\pm$ ” sign

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in Eq. (1) indicates the same/opposite direction of helices between the shaving cutter and the work gear. The four variables, α_{ot1} , α_{ot2} , β_{o1} , and β_{o2} , can be solved by using Eq. (1) to Eq. (4). The operating center distance E_o is solved by using the following equation:

$$E_o = r_{o1} + r_{o2} \quad (5)$$

where

$$r_{o1} = r_{p1} \cos \alpha_{pt1} / \cos \alpha_{ot1}$$

$$r_{o2} = r_{p2} \cos \alpha_{pt2} / \cos \alpha_{ot2}$$

The start of active point (S.A.P.) and meshing point of outside radius(E.A.P.) mean the diameters of the shaving cutter meshing with the form circle and the addendum circle of the work gear respectively. The start of active point (S.A.P.) and meshing point of outside radius are as shown in the following:

$$S.A.P. = \frac{2r_{b1}}{\cos(\tan^{-1}(\frac{\tan \alpha_{ot1} - r_{b2}(\tan \alpha_{f2} - \tan \alpha_{ot2}) \cos \beta_{b1}}{\cos \beta_{b2} r_{b1}}))} \quad (6)$$

$$E.A.P. = \frac{2r_{b1}}{\cos(\tan^{-1}(\frac{\tan \alpha_{ot1} - r_{b2}(\tan \alpha_{a2} - \tan \alpha_{ot2}) \cos \beta_{b1}}{\cos \beta_{b2} r_{b1}}))} \quad (7)$$

where

$$\alpha_{a2} = \cos^{-1}(r_{b2} / r_{a2})$$

$$\alpha_{f2} = \cos^{-1}(r_{f2} / r_{a2})$$

3. Mathematical model of the shaving gears coordinate systems

The coordinate systems for gear shaving machine can be simplified as shown in Fig. 1. The coordinate system S_a is rigidly connected to the frame of the CNC shaving machine. The coordinate systems S_1 and S_2 are rigidly connected to the shaving cutter and the gear, respectively. The parameters ϕ_1 and $\phi_2(\phi)$ are the rotating angle of the shaving cutter and the gear, respectively. The machine movement include the traverse movement along of the axis of the gear z_i , the vertical movement crossed the axis of the gear cz_i , the center distance between the gear and the shaving cutter $E_o - az_i^2$, and the crossed angle between the cutter and the gear γ .

The shaving cutter is made in the form of standard helical gear, and the mathematical model of standard helical gear has been derived by many textbooks [1]. Assume that the position vector $\mathbf{r}_1[x_1(u, v), y_1(u, v), z_1(u, v), 1]^T$ and the unit normal vector $\mathbf{n}_1[n_{x1}(u, v), n_{y1}(u, v), n_{z1}(u, v)]$ of shaving cutter are given and represented in coordinate system S_1 . The locus of the gear $\mathbf{r}_2$ represented in coordinate system S_2 is shown in Eq. (8).

$$\mathbf{r}_2(u, v, \phi, \theta) = \mathbf{M}_{21}(\phi, z_i) \cdot \mathbf{r}_1(u, v) \quad (8)$$

where

$$\mathbf{M}_{21} = \mathbf{M}_{2d} \mathbf{M}_{d1} \quad (9)$$

$$\mathbf{M}_{2d} = \begin{bmatrix} \cos \phi_2 & \sin \phi_2 & 0 & \cos \phi_2 (E_o - az_i^2) - \sin \phi_2 cz_i \\ -\sin \phi_2 & \cos \phi_2 & 0 & -\sin \phi_2 (E_o - az_i^2) - \cos \phi_2 cz_i \\ 0 & 0 & 1 & z_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

Figure 1. Coordinate systems for CNC shaving machine

$$\mathbf{M}_{c1} = \begin{bmatrix} -\cos \phi & -\sin \phi & 0 & 0 \\ \cos \gamma \sin \phi & -\cos \gamma \cos \phi & \sin \gamma & 0 \\ -\sin \gamma \sin \phi & \cos \phi \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

In the shaving process, the cutter is fed into the gear with table traverse in the direction of the gear longitudinal. If the gear needed to crown, the cutter is radial feed-in az_i^2 along the center distance between the gear and the cutter. Therefore, the locus of the gear $\mathbf{r}_2$ is simplified as shown in Eq. (12):

$$\mathbf{r}_2(u, \theta, \phi, z_i) = [x_2(u, \theta, \phi, z_i) \ y_2(u, \theta, \phi, z_i) \ z_2(u, \theta, \phi, z_i) \ 1]^T \quad (12)$$

The equation of meshing between the gear and the shaving cutter is shown in Eq. (13) and (14):

$$f_1(u, \theta, \phi, z_i) = \mathbf{n}_2 \cdot \mathbf{v}_1^{(12)} = \frac{d\phi}{dt} \left(\frac{\partial}{\partial \phi} [x_2(u, \theta, \phi, z_i) \ y_2(u, \theta, \phi, z_i) \ z_2(u, \theta, \phi, z_i)]^T \right) \quad (13)$$

$$f_2(u, \theta, \phi, z_i) = \mathbf{n}_2 \cdot \mathbf{v}_2^{(12)} = \frac{dz_i}{dt} \left(\frac{\partial}{\partial z_i} [x_2(u, \theta, \phi, z_i) \ y_2(u, \theta, \phi, z_i) \ z_2(u, \theta, \phi, z_i)]^T \right) \quad (14)$$

where

$$\mathbf{n}_2 = \begin{bmatrix} n_x(u, \theta) \\ n_y(u, \theta) \\ n_z(u, \theta) \end{bmatrix} \quad (15)$$

L_{12} is the up-submatrix of the order (3×3) of the matrix M_{12} .

Gear shaving is a free cutting process that removes small amounts of metal from the working surface of gear teeth, therefore the operating pitch helical angle will be constant. According to literature the rotating angle between the shaving cutter and the gear is as shown in following equation:

$$\phi = \frac{N_2}{N_1} \phi_2 \pm \frac{\tan \beta_{o2} + c}{r_{o2}} z_2 \quad (+ \text{ for R.H. , } - \text{ for L.H.}) \quad (16)$$

Where N_1 and N_2 are teeth number of the shaving cutter and the gear. The parallel-shaving shaved gear be solved by using Eqs. (12), (13), (14) and (16) simultaneously.

4. Numerical examples

The basic gear data is shown in Table 1. The basic shaving cutter setup on the CNC shaving machine, we adopt the basic meshing condition for a crossed helical gear set derived by Litvin [1]. This example mainly compares gear crowned motion for the shaving machine setting of two axes and the three synchronous axes. The shaving machine setting data is listed in table 2. The simulation crowning amount of the shaved gear of two and three synchronous axes machine as shown in Figure 2 and Figure 3, respectively. The dotted line in the topography indicates the location of the work gear's pitch radius.

Table 1. Basic data for gear, shaving cutter and machine settings.

Work gear data	
Number of teeth	37
Normal module	2.86 mm
Normal circular-tooth thickness	4.197 mm
Normal pressure angle	20°
Helix angle	0° R.H.
Outer diameter	110.727mm
Form diameter	100.67 mm
Face width	16 mm
Shaving cutter data	
Number of teeth	79
Helix angle	12° L.H.
Normal circular-tooth thickness	0.699 mm
Machine settings	
Operating center distance (E_o)	161.584 mm
Operating crossed angle (γ_o)	11.521°

Table 2. Coefficient value of machine setting

	a	c
Case1(two synchronous axes)	0.001	0
Case2(three synchronous axes)	0.001	6.5×10^{-4}

Figure 2. The topography of simulation for Case1 (two synchronous axes)

The results show that whether you use a two-axis or three-axis synchronous shaving machine, the gear tooth surface will be twist. However, using a three-axis synchronous shaving machine will not cause twist in the pitch circle of the gear.

Figure 3. The topography of simulation for Case2 (three synchronous axes)

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