Dynamic behaviors of cylindrical shells with geometric imperfection

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Abstract. For finding out the relationship between vibrational modes and geometric imperfections, the dynamic behavior of cylindrical shells with a local imperfection is analyzed by using numerical simulation of finite element method in this paper. The results show that there are some differences in vibrational modes between cylindrical shells with and without imperfections. They appear that main corrugation of the mode of the higher orders for the shell with a local imperfection can be fastened on the region of the imperfection but the low order ones. The results also show that the vibrational modes of shells depend upon the size, shape and location of the imperfections. The local vibrational modes are discovered and are more obvious for the imperfection of the larger size. These results are helpful to the design of engineering structures.

1 Introduction

The imperfection in structures is inevitable. Structures with imperfection can affect directly the strength, stability and dynamic characteristics of structures. Therefore, it is necessary to study and evaluate the errors and safety caused by the imperfection. In the problem of structures with imperfection, the influence of small geometric imperfection may be considered first. Theoretical approach is proposed for studying the free vibrations of orthotropic cylindrical shells (with small geometric imperfections) [1]. And an analysis method is performed for the nonlinear vibration of a simply supported FGM cylindrical shell with small initial geometric imperfection under complex loads [2]. In slightly larger imperfection, the geometric imperfection is described by specific functions. For beams, a one-dimensional imperfection model is used in the form of the product of trigonometric and hyperbolic functions to describe the various possible geometric imperfections, and the nonlinear vibration is studied for reinforced composite (FG-CNTRC) beams of imperfect shear deformable functionally graded carbon nanotube [3]. A new vibration model is provided which is a rotating cantilever pre-twisted panel with initial exponential function type geometric imperfection and it is pointed that the free vibration of the structures depends upon geometric parameters, imperfection size and location [4]. Based on higher order shear and normal deformation theory, a new non-polynomial is used, then the vibration response of gradient plates with initial geometric imperfections is analyzed according to the theory [5]. The effect of initial geometric imperfections on vibration frequencies is presented for cylindrical shells aid theoretical, numerical and experimental methods [6-8]. By employing the finite element method, nonlinear free

vibration and transient response are studied for plates and shells with geometric imperfections [9-11]. The results show that the behavior of nonlinear vibration can be changed by the imperfections. However, the relationship between vibrational modes and geometric imperfections has not yet been revealed distinctly, and the mechanism needs to be made clear. For this goal, it is attempted to study the problem by using numerical simulation of finite element method in this paper.

2 The fundamental problem and the model

Consider an isotropic homogeneous cylindrical shell with a local imperfection, of which the length is L , the radius is R , the thickness is h . And the imperfection width is l , the depth is d , the circumferential angle is α , the distance to the end is x_1 (see figure 1). The physical parameters of the model are as follows: elastic modulus is E , Poisson's ratio is μ , density is ρ . The dynamic behaviors of the cylindrical shell model with different geometric imperfection are analyzed in the paper by finite element method, including frequency and vibration mode.

3 Comparisons with experimental and analytical results

In order to verify the feasibility of finite element method, the finite element results are compared with the experimental results and the analytical solutions obtained by energy method [12]. The geometric and physical parameters of the model used for comparison are $L=0.2504\text{m}$, $h=0.0381\text{m}$, $R=0.09525\text{m}$, $E=207\text{GPa}$, $\mu=0.28$, $\rho=7860\text{Kg/m}^3$. And the cylindrical shell behaves as a free-body. The circumferential modes $n=1-5$ and the

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longitudinal mode $m=3$ (for the definition of n and m , see Ref. [12]) are chosen for comparisons (see table 1). Compared with the experimental results, the maximum error of finite element method is 3.03%. It can be found

that the results of finite element numerical simulation are very close to those of experiments and analytical solutions, which shows that the finite element method is feasible.

Figure 1. Schematic diagram of a cylindrical shell with a local imperfection

Table 1. Frequencies comparison of $n=1-5$, $m=3$. [Hz]

n	0	1	2	3	4	5
Present	10974	10774	9507	12460	15925	20109
Calculated[12]	10719	10770	9521	12584	16435	20801
Expt.[12]	10874	10803	9588	12717	16391	20737

Table 2. Vibration frequencies of $n=0-11$, $m=1-4$. [Hz]

m	1	2	3	4
$n=0$	159.359	256.114	318.754	478.221
$n=1$	85.279	181.213	290.128	396.510
$n=2$	36.425	87.381	150.785	219.718
$n=3$	26.327	50.862	87.889	132.198
$n=4$	37.984	46.815	65.661	92.691
$n=5$	59.334	62.595	70.700	84.855
$n=6$	86.699	88.346	92.221	99.3125
$n=7$	119.503	120.632	122.995	127.079
$n=8$	157.719	158.643	160.431	163.275
$n=9$	201.398	202.262	203.822	206.109
$n=10$	250.699	251.615	253.009	255.014
$n=11$	305.808	306.820	308.102	309.916

4 Free vibration characteristics of perfect elastic cylindrical shell

The model studied in the paper use steel material, whose $E=206\text{GPa}$, $\mu=0.3$, $\rho=7800\text{ Kg/m}^3$, $L=10\text{m}$, $R=1\text{m}$, $d=0.01\text{m}$. The both ends of the model are fixed. For frequencies and modes, define the number of circumferential corrugation is order n , the number of axial corrugation is branch m . By finite element method, the frequencies of the first four branches whose order is 0 (axisymmetric case) to 11 are shown in table 2. And the vibration modes of different orders and branches are shown in figure 2.

5 Free vibration frequencies for cylindrical shells with geometric imperfection

For studying vibration frequencies of cylindrical shells with geometric imperfection, multiple cases of imperfections are discussed. The results are shown in table 3. In Case 1, the location of imperfection is $x_1=4.5\text{m}$, and the imperfection width is $l=1\text{m}$, angle is $\alpha=2\pi$. Case 2 represents $x_1=2\text{m}$, $l=1\text{m}$, angle is $\alpha=2\pi$. Similarly, in Case 3, $x_1=3.5\text{m}$, $l=3\text{m}$, $\alpha=2\pi$. Imperfection in Case 4 is $x_1=4.5\text{m}$, $l=1\text{m}$, $\alpha=\pi$. In Case 5, $x_1=4.5\text{m}$, $l=1\text{m}$, $\alpha=\pi/4$.

Table 3 presents the lowest ($m=1$) vibration frequencies corresponding to the first 11 modes under imperfections of different depths, widths locations and angles. The results show that imperfection have less impact on lower-order frequencies but larger on higher-orders. And the larger the imperfection size (including depth, width and angle), the more the impact. Furthermore, for higher-order frequencies, the vibration frequencies of the shell with imperfection are generally lower than that without imperfection, and as the order increases, the frequency difference is larger and larger. When $n=11$ and the imperfection size is the largest, the difference reaches a maximum of 74.44%.

Figure 2. Vibration modes

Table 3. Frequencies of $n=0-11$, $m=1$. [Hz]

n		0	1	2	3	4	5
Perfect cylindrical shell		159.359	85.279	36.425	26.327	37.984	59.334
d/h=1/4	Case1	163.397	86.838	36.860	25.878	36.373	56.071
	Case2	158.772	82.371	36.575	26.202	37.405	57.289
	Case3	170.712	90.153	38.004	25.097	32.961	49.129
	Case4	161.371	86.046	36.634	26.054	37.056	57.700
	Case5	160.375	85.805	36.523	26.159	37.468	58.353
d/h=3/4	Case1	172.237	87.547	36.260	25.585	36.338	55.878
	Case2	148.012	81.213	35.958	26.179	37.487	57.191
	Case3	203.059	103.328	42.835	26.451	31.043	38.448
	Case4	165.728	86.135	36.254	25.799	36.672	56.721
	Case5	162.628	85.117	36.256	26.024	37.113	57.416
n		6	7	8	9	10	11
Perfect cylindrical shell		86.699	119.503	157.719	201.398	250.699	305.808
d/h=1/4	Case1	79.983	106.566	135.734	168.039	203.842	243.855
	Case2	80.302	106.579	135.736	167.960	203.847	243.860
	Case3	68.851	92.490	120.350	152.494	188.899	229.574
	Case4	83.215	113.273	146.000	185.187	225.692	271.969
	Case5	84.917	116.698	154.368	193.342	239.842	288.579
d/h=3/4	Case1	78.125	98.146	110.806	115.903	118.091	121.093
	Case2	78.355	98.160	110.806	115.899	118.084	121.097
	Case3	40.268	42.110	46.901	54.867	65.448	78.157
	Case4	80.565	101.761	116.275	123.793	133.626	155.301
	Case5	82.781	110.175	123.163	140.624	168.714	201.609

Since the imperfection has a large influence on the higher-order frequencies, the situations of $n=8$, $m=1-6$ are chosen to study the changes of frequencies of different branches under imperfections of the same angle $\alpha=2\pi$ and width $l=1\text{m}$ (see figure 3). Results show the frequencies increases as the number of branches increases. In addition, the imperfection location, no matter whether on the center or the side of the cylindrical shell, has little effect on frequency.

6 Free vibration modes for cylindrical shells with geometric imperfection

The cylindrical shell modes with the imperfection of the largest size (ratio of imperfection depth to shell thickness $d/h=3/4$, angle $\alpha=2\pi$, location $x_1=3.5\text{m}$, width $l=3\text{ m}$) in all cases are chosen to compare the modes with perfect cylindrical shell as figure 4. According to the comparison,

it can be found that a local geometric imperfection has a more influence on the vibration modes of high orders, but it has little effect on the low-order modes. Thus, only the effect of imperfections of higher-order modes will be shown in the following.

Figure 3. Frequencies of $n=8$, $m=1-6$. [Hz]

Figure 4. Vibration modes of $n=2,7$, $m=1$.

The modes of $n=7,8,9$, $m=1$ are shown in figure 5. In this figure, both in (a) and (b), the angle of imperfection is $\alpha=2\pi$, location $x_1=4.5$, width $l=1$. The difference is that the imperfection depth is in (b) is 3 times that of (a). And the frequencies corresponding to the modes in (a) are $\omega=106.566$ Hz, 135.734 Hz, 168.039 Hz, which in (b) are $\omega=98.146$ Hz, 110.806 Hz, 115.903 Hz. As the figure shows, far from the imperfection, the amplitude is almost zero. And the deeper the imperfection, the more concentrated the corrugations are within region of the imperfection.

Imperfection in figure 6 is all at the condition of $\alpha=2\pi$, $d/h=1/4$, and these vibration modes as the picture shows are order $n=8$, branch $m=1,3,5$. The width of imperfection in (b) is 3 times that of (a). The frequencies corresponding to the modes shown in figure 6 are $\omega=135.734$ Hz, 158.424 Hz, 164.711 Hz, 120.350 Hz, 147.714 Hz and 160.325 Hz respectively. It can be found that corrugations will change significantly at the imperfection.

Figure 7 presents modes of the same order and branches as figure 6. And imperfection in (a) and (b) is $\alpha=2\pi$, $d/h=3/4$, $l=1$, besides, the imperfection in (a) is at the center of the cylindrical shell, but that in (b) is on one side. Their frequencies are $\omega=110.831$ Hz, 158.184 Hz, 163.675 Hz, 110.806 Hz, 157.502 Hz, 163.533 Hz respectively. It is clear that the position of the imperfection has a large influence on the vibration modes, and the vibration corrugations have visible changes at the imperfections at different positions.

Figure 5. Vibration modes of $n=7,8,9$, $m=1$

Figure 8 shows modes of order $n=6,7,8$ of imperfections at the same thickness, location and width

($d/h=3/4$, $x_1=4.5$ m, $l=1$ m) when branch $m=1$, the frequencies of which are $\omega=80.565$ Hz, 101.761 Hz, 116.275 Hz, 82.781 Hz, 110.175 Hz and 123.163 Hz. The imperfection angle in (a) is π , that in (b) is $\pi/4$. By observing these maps, it can be seen the deformation is only concentrated in the imperfection area, and there is almost no deformation outside the imperfection.

Figure 6. Vibration modes of $n=8$, $m=1,3,5$.

Figure 7. Vibration modes of $n=8$, $m=1,3,5$

Figure 8. Vibration modes of $n=6,7,8$, $m=1$.

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Summary

Dynamic behavior of cylindrical shells with a local geometric imperfection is different from the shells

without imperfection. It includes vibrational frequencies and modes. The phenomenon has been found that the imperfections have some influence on the vibration characteristics of cylindrical shells, both for frequencies and modes. Specially, it is more obvious for large-scale imperfections. For the frequency, the results show that imperfection have less impact on the lower-order frequencies but the higher-orders. The frequencies of higher-orders are much lower than the frequencies without imperfection. Besides, the modes display a phenomenon, which the corrugation tends to concentrate on the imperfection location for frequencies of higher-orders. The results show also that the frequency increases with the number of branches and the corrugation is the more complex on the imperfection location for corresponding modes. In addition, frequencies and modes are directly affected by the location, size and depth of the imperfection. These conclusions and results are potential to be used as a design reference for structures with similar geometry.

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