Development of a Robust UFIR Filter with Consensus on Estimates for Missing Data and unknown noise statistics over WSNs

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Abstract. Wireless sensor networks (WSN) are often deployed in harsh environments, where electromagnetic interference, damaged sensors, or the landscape itself cause the network to suffer from faulty links and missing data. In this paper, we develop an unbiased finite impulse response (UFIR) filtering algorithm for optimal consensus on estimates in distributed WSN. Simulations are provided assuming two possible scenarios with missing data. The results show that the distributed UFIR filter is more robust than the distributed Kalman filter against missing data.

1 Introduction

The wireless sensor networks (WSN) have found applications in diverse areas in recent years due to a noticeable progress in the miniaturization and cost reduction of smart sensors, which allow providing a more ubiquitous and large scale deployment. Accordingly, the use of the WSN technology can be found in healthcare, object tracking, environmental monitoring, etc. [1, 2].

A distinctive characteristic of WSN is the ability of a massive nodes deployment, which allows a desired quantity Q to be measured by a big number of sensors. In the real world environments, measurements are often subject to noise and missing data due to unstable links. Therefore, optimal estimation is required along with adequate sensor fusion techniques [3–6], which must demonstrate a sufficient robustness against missing data, model errors (mis-modeling), and incomplete information about noise statistics. On the other hand, the restrictions of WSN caused by limited battery life and processing power put a stress on the development of the algorithms to ensure that these limited resources are exploited efficiently.

In order to improve battery life and minimize the computational burden, distributed filtering has been introduced to estimate Q in real time [7, 8]. Under the distributed filtering, each node is tasked with the estimation of Q and a consensus protocol is implemented by averaging the estimates, measurements, or information matrices [9] so that all the nodes agree in a common value called the group decision value [10].

The low computational burden and optimal estimation, make the Kalman filter (KF) a very popular sensor fusion technique [11]. Based on the KF approach, many authors have addressed the consensus problem in WSNs. In [12], the author has proposed a KF structure that requires each

node to locally aggregate its measurement and covariance matrix with those of its neighbors and, in a posterior step, compute the estimate using a KF with a consensus term. In [13], the KF approach has been developed for local estimation and a consensus matrix as fusion technique. In [14], the authors have presented an algorithm based on the KF to address an issue with missing data. Let us notice that the KF optimality has important requirements such as a complete knowledge of the noise statistics, the noise distribution must be strictly Gaussian, an adequate model must be used, and a knowledge of the initial conditions is mandatory [15–18]. If these requirements are not met, the performance of the KF may drastically degrade and become unacceptable for real world WSN applications [19].

It has been proven in [20–23] that better robustness can be achieved by using filters operating on finite data horizons. Under such an assumption, a moving average estimator has been designed in [24] for weak observability. A consensus finite-horizon H_∞ approach was developed in [25] under missing measurements. In [26], an unbiased finite impulse response (UFIR) filter was developed for consensus on measurements. Although this filter demonstrates an ability to be more robust and adequate than the KF for WSN, it was designed under the condition that all sensors measure the same state at the same time. In [19], the UFIR structure has been developed for consensus on estimates, but the consensus factor was obtained through a previous analysis and without a mathematical background. Also, the effect of missing measurements on the filter performance was not considered. The issue of missing measurements has been addressed by the KF and UFIR approaches. In [27] an extended KF was modified to address the issue of missing measurement while in [28] a UFIR filter was developed as a robust estimator that neglects noise statistic. Regarding WSNs, in [29] a KF was modeled for intermittent observations and in [30] a UFIR alternative for missing and delayed data was developed.

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In this work, we further develop the results obtained in [19] by providing a mathematical expression for the consensus factor and also include a prediction step to overcome the issue of missing data. The rest of the paper is organized as follows. In Section 2, the problem is formulated and the model presented. Section 3 presents a predictive KF algorithm. The design of a tracking predictive dUFIR filter is given both in the batch and fast iterative forms in Section 4. Simulations are provided in Section 5 and conclusions are given in Section 6.

2 Problem Formulation and Preliminaries

Consider the dynamics of a quantity Q given for a distributed WSN with the following discrete K -states space equations,

$$x_k = F_k x_{k-1} + B_k w_k, \quad (1)$$

$$\bar{y}_k^{(i)} = H_k^{(i)} (F_k x_{k-1}), \quad (2)$$

$$y_k^{(i)} = \gamma_k (H_k^{(i)} x_k + v_k^{(i)}) + (1 - \gamma_k) \bar{y}_k^{(i)}, \quad (3)$$

$$y_k = H_k x_k + v_k, \quad (4)$$

where $x_k \in \mathbb{R}^K$, $u_k \in \mathbb{R}^M$, $F_k \in \mathbb{R}^{K \times K}$, $E_k \in \mathbb{R}^{K \times M}$, and $B_k \in \mathbb{R}^{K \times L}$. The i th, $i \in [1, n]$, is a part of the WSN regarded as an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where each vertex $v^{(i)} \in \mathcal{V}$ is a node and each link is an edge of a set $\mathcal{E}$, for $i \in \mathcal{I} = \{1, \dots, n\}$ and $n = |\mathcal{V}|$ with J inclusive neighbors.

Each node measures x_k by $y_k^{(i)} \in \mathbb{R}^p$, $p \leq K$, with $H_k^{(i)} \in \mathbb{R}^{p \times K}$. Local data $y_k^{(i)}$ are united in the observation vector $y_k = [y_k^{(1)T} \dots y_k^{(J)T}]^T \in \mathbb{R}^{Jp}$ with $H_k = [H_k^{(1)T} \dots H_k^{(J)T}]^T \in \mathbb{R}^{Jp \times K}$. Noise vectors $w_k \in \mathbb{R}^L$ and $v_k = [v_k^{(1)T} \dots v_k^{(n)T}]^T \in \mathbb{R}^{Jp}$ are zero mean, not obligatorily white Gaussian, uncorrelated, and with the covariances $Q_k = E\{w_k w_k^T\} \in \mathbb{R}^{L \times L}$, $R_k = \text{diag}[R_k^{(1)T} \dots R_k^{(n)T}]^T \in \mathbb{R}^{Jp \times Jp}$, and $R_k^{(i)} = E\{v_k^{(i)} v_k^{(i)T}\}$. A binary variable γ_k serves as an indicator of whether a measurement exist ($\gamma_k = 1$) or not ($\gamma_k = 0$), in which case the measurement prediction $\bar{y}_k^{(i)}$ (2) is used by substituting x_{k-1} with the estimate.

3 Predictive Distributed Kalman Filter

In order to assert a better robustness of the distributed UFIR (dUFIR) filter against missing data, a comparison with the distributed KF (dKF) is mandatory. For this reason, we first develop a modification of the KF algorithm introduced in [12] by including a predictive feature (lines 3–5).

The logic behind the dKF, makes it mandatory for the first step to ensure that the prediction from every first order neighbor is available such that in a posterior step the implementation of a local KF is provided with a consensus on estimates from neighbors.

Algorithm 1: Predictive dKF Algorithm

Data: $P_0^{(i)}, Q_k, R_k^{(j)}, \bar{x}_k^{(j)}, y_k^{(j)}, \bar{x}_0^{(i)} = x_0$

Result: $\hat{x}_k^{(i)}$

```

1 begin
2   for  $k = 0 : \infty$  do
3     if  $\gamma_k = 0$  then
4        $y_k^{(j)} = H_k^{(j)} F_k \hat{x}_{k-1}^{(j)}$ ;
5     end if
6      $z_k^{(j)} = H_k^{(j)T} R_k^{(j)-1} y_k^{(j)}$ ,  $\forall j \in J$ ;
7      $s_k^{(i)} = \sum_{j \in J} z_k^{(j)}$ ;
8      $Z_k^{(j)} = H_k^{(j)T} R_k^{(j)-1} H_k^{(j)}$ ,  $\forall j \in J$ ;
9      $S_k^{(i)} = \sum_{j \in J} Z_k^{(j)}$ ;
10     $M_k^{(i)} = (P_k^{(i)-1} + S_k^{(i)})^{-1}$ ;
11     $\hat{x}_k^{(i)} =$ 
12       $\bar{x}_k^{(i)} + M_k^{(i)} (s_k^{(i)} - S_k^{(i)} \bar{x}_k^{(i)}) + \epsilon M_k^{(i)} \sum_{j \in J} (\bar{x}_k^{(j)} - \bar{x}_k^{(i)})$ ;
13     $P_k^{(i)} \leftarrow F_k M_k^{(i)} F_k^T + B_k Q_k B_k^T$ ;
14     $\bar{x}_k^{(i)} \leftarrow F_k \hat{x}_k^{(i)}$ ;
15  end for

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4 Predictive Distributed UFIR Filter

To obtain optimum estimates and achieve a consensus on estimates, we formulate the distributed estimate as

$$\hat{x}_k^c = \hat{x}_k + \lambda_k^{\text{opt}} \Sigma_k, \quad (5)$$

where the centralized and individual estimates, $\hat{x}_k$ and $\hat{x}_k^{(i)}$ respectively, are obtained through

$$\hat{x}_k = K_{m,k} Y_{m,k}, \quad (6)$$

$$\hat{x}_k^{(i)} = C_{m,k}^{(i)} Y_{m,k}^{(i)} \quad (7)$$

and $\Sigma_k = \sum_j (\hat{x}_k^{(j)} - \hat{x}_k^{(i)})$ is a consensus protocol that minimizes the disagreement between the first-order neighbors [12].

A consensus factor λ_k^{opt} is chosen such that the root mean squared error (RMSE) is minimized by

$$\lambda_k^{\text{opt}} = \arg \min_{\lambda_k} \{\text{tr } P(\lambda_k)\} \quad (8)$$

with $P(\lambda_k) = E\{(x - \hat{x}^{ic})(x - \hat{x}^{ic})^T\}$ as the relevant error covariance.

4.1 Batch Distributed UFIR Filter Design

To determine gains $K_{m,k}$ and $C_{m,k}^{(i)}$, we express the model equations (1)–(4) in the extended state space form over horizon N as described in [19, 31],

$$X_{m,k} = A_{m,k} x_m + D_{m,k} W_{m,k}, \quad (9)$$

$$Y_{m,k} = C_{m,k} x_m + M_{m,k} W_{m,k} + V_{m,k}, \quad (10)$$

$$Y_{m,k}^{(i)} = C_{m,k}^{(i)} x_m + M_{m,k}^{(i)} W_{m,k} + V_{m,k}^{(i)}, \quad (11)$$

where $X_{m,k} = [x_m^T x_{m+1}^T \dots x_k^T]^T$, $Y_{m,k} = [y_m^T y_{m+1}^T \dots y_k^T]^T$, $W_{m,k} = [w_m^T w_{m+1}^T \dots w_k^T]^T$, $V_{m,k} = [v_m^T v_{m+1}^T \dots v_k^T]^T$, $Y_{m,k}^{(i)} = [y_m^{(i)T} y_{m+1}^{(i)T} \dots y_k^{(i)T}]^T$, $V_{m,k}^{(i)} = [v_m^{(i)T} v_{m+1}^{(i)T} \dots v_k^{(i)T}]^T$, and the extended matrices are

$$A_{m,k} = [I \ F_{m+1}^T \dots (\mathcal{F}_k^{m+1})^T]^T, \quad (12)$$

$$D_{m,k} = \begin{bmatrix} B_m & 0 & \dots & 0 & 0 \\ F_{m+1}B_m & B_{m+1} & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathcal{F}_{k-1}^{m+1}B_m & \mathcal{F}_{k-1}^{m+2}B_{m+1} & \dots & B_{k-1} & 0 \\ \mathcal{F}_k^{m+1}B_m & \mathcal{F}_k^{m+2}B_{m+1} & \dots & F_k B_{k-1} & B_k \end{bmatrix}, \quad (13)$$

$C_{m,k} = \bar{C}_{m,k}A_{m,k}$, $M_{m,k} = \bar{C}_{m,k}D_{m,k}$, $C_{m,k}^{(i)} = \bar{C}_{m,k}^{(i)}A_{m,k}$, $M_{m,k}^{(i)} = \bar{C}_{m,k}^{(i)}D_{m,k}$, where

$$\bar{C}_{m,k} = \text{diag}(H_m \ H_{m+1} \ \dots \ H_k), \quad (14)$$

$$\bar{C}_{m,k}^{(i)} = \text{diag}(H_m^{(i)} \ H_{m+1}^{(i)} \ \dots \ H_k^{(i)}), \quad (15)$$

$$\mathcal{F}_r^g = \begin{cases} F_r F_{r-1} \dots F_g, & g < r+1 \\ I, & g = r+1 \\ 0, & g > r+1 \end{cases}. \quad (16)$$

Referring to [19], equation (5) can now be rewritten as

$$\hat{x}_k^c = K_{m,k}Y_{m,k} + J\lambda_k^{\text{opt}}K_{m,k}Y_{m,k} - J\lambda_k^{\text{opt}}K_{m,k}^{(i)}Y_{m,k}^{(i)}. \quad (17)$$

As we are interested in a robust UFIR filter that ignores the initial values, the unbiasedness condition must hold for the distributed, centralized and individual estimates,

$$E\{\hat{x}_k^c\} = E\{\hat{x}_k\} = E\{\hat{x}_k^{(i)}\} = E\{x_k\} \quad (18)$$

where

$$x_k = \mathcal{F}_k^{m+1}x_m + \bar{D}_{m,k}W_{m,k} \quad (19)$$

with $\bar{D}_{m,k} = [\mathcal{F}_k^{m+1}B_m \ \mathcal{F}_k^{m+2}B_{m+1} \ \dots \ F_k B_{k-1} \ B_k]$. The corresponding gains are defined by

$$K_{m,k} = G_k C_{m,k}^T, \quad (20)$$

$$K_{m,k}^{(i)} = G_k^{(i)} C_{m,k}^{(i)T}, \quad (21)$$

where $G_k = (C_{m,k}^T C_{m,k})^{-1}$ and $G_k^{(i)} = (C_{m,k}^{(i)T} C_{m,k}^{(i)})^{-1}$.

In real world applications, the nodes of the WSN may be unable to implement equation (17), due to large-dimension matrices and operations involved into the limited memory resources of the smart sensors. Therefore, below we develop an iterative form of (17) which fits better with the WSNs resources.

4.2 Optimum λ_k

Consider the error covariance $P_k = E\{\varepsilon_k \varepsilon_k^T\}$ as function of λ_k , with the estimation error $\varepsilon_k = x_k - \hat{x}_k^c$, to be

$$\begin{aligned} P_k = & (\bar{D}_{m,k} - K_{m,k}M_{m,k})Q_{m,k}(\bar{D}_{m,k} - K_{m,k}M_{m,k})^T \\ & + J\lambda_k(K_{m,k}\bar{R}_{m,k}K_{m,k}^T - K_{m,k}^{(i)}\bar{R}_{m,k}^{(i)}K_{m,k}^T) \\ & + J[\lambda_k(K_{m,k}\bar{R}_{m,k}K_{m,k}^T - K_{m,k}^{(i)}\bar{R}_{m,k}^{(i)}K_{m,k}^T)]^T \\ & + J^2\lambda_k(K_{m,k}\bar{R}_{m,k}K_{m,k}^T - K_{m,k}^{(i)}\bar{R}_{m,k}^{(i)}K_{m,k}^T)\lambda_k^T \\ & + J^2\lambda_k(K_{m,k}^{(i)}\bar{R}_{m,k}^{(i)}K_{m,k}^T + K_{m,k}^{(i)}\bar{R}_{m,k}^{(i)}K_{m,k}^T)\lambda_k^T, \end{aligned} \quad (22)$$

where

$$\bar{R}_{m,k} = E\{v_{m,k}v_{m,k}^T\} = \text{diag}(R_m \dots R_k),$$

$$\bar{R}_{m,k}^{(i)} = E\{v_{m,k}^{(i)}v_{m,k}^{(i)T}\} = \text{diag}(R_m^{(i)} \dots R_k^{(i)}),$$

$$\bar{R}_{m,k}^{(i)} = E\{v_{m,k}v_{m,k}^{(i)T}\} = \text{diag}(\bar{R}_m^{(i)} \dots \bar{R}_k^{(i)}).$$

We next apply the derivative with respect to λ_k to the trace of (22) by using the identities $\frac{\partial}{\partial X} \text{tr}(X^T B X) = B X + B^T X$, and $\frac{\partial}{\partial X} \text{tr}(X A) = A^T$. Putting the derivative to zero yields

$$\begin{aligned} \lambda_k^{\text{opt}} = & -\frac{1}{J}(K_{m,k}\bar{R}_{m,k}K_{m,k}^T - K_{m,k}\bar{R}_{m,k}^{(i)}K_{m,k}^{(i)T}) \times \\ & (K_{m,k}\bar{R}_{m,k}K_{m,k}^T - K_{m,k}\bar{R}_{m,k}^{(i)}K_{m,k}^{(i)T}) \\ & - (K_{m,k}\bar{R}_{m,k}^{(i)}K_{m,k}^{(i)T})^T + K_{m,k}^{(i)}\bar{R}_{m,k}^{(i)}K_{m,k}^{(i)T} \end{aligned} \quad (23)$$

and using the identities

$$K_{m,k}\bar{R}_{m,k}^{(i)}K_{m,k}^{(i)T} = (K_{m,k}\bar{R}_{m,k}^{(i)}K_{m,k}^{(i)T})^T,$$

$$K_{m,k}\bar{R}_{m,k}^{(i)}K_{m,k}^{(i)T} = G_k G_k^{(i)-1} K_{m,k}^{(i)}\bar{R}_{m,k}^{(i)}K_{m,k}^{(i)T}$$

we obtain the final form of (24) as

$$\begin{aligned} \lambda_k^{\text{opt}} = & -\frac{1}{J}(K_{m,k}\bar{R}_{m,k}K_{m,k}^T - G_k G_k^{(i)-1} K_{m,k}^{(i)} \\ & \times \bar{R}_{m,k}^{(i)} K_{m,k}^{(i)T})(K_{m,k}\bar{R}_{m,k}K_{m,k}^T - 2G_k G_k^{(i)-1} \\ & \times K_{m,k}^{(i)}\bar{R}_{m,k}^{(i)}K_{m,k}^{(i)T} + K_{m,k}^{(i)}\bar{R}_{m,k}^{(i)}K_{m,k}^{(i)T})^{-1}. \end{aligned} \quad (24)$$

If, for some particular application, the network and the process dynamics are both time invariant, λ_k^{opt} is also time invariant, to mean that equation (24) can be computed beforehand and embedded into the nodes.

4.3 Iterative Distributed UFIR filter

An iterative algorithm for the centralized estimates $\hat{x}_k$ can be derived following the procedure described in [31], including a variable l that starts at $l = k - N + K + 1$ and ending in $l = k$. The recursions are given by

$$G_l = [H_l^T H_l + (A_l G_{l-1} A_l^T)^{-1}]^{-1}, \quad (25)$$

$$\hat{x}_l = A_l \hat{x}_{l-1}, \quad (26)$$

$$\hat{x}_l = \hat{x}_l^- + G_l H_l^T (y_l - H_l \hat{x}_l^-). \quad (27)$$

The initial values G_{l-1} and $\hat{x}_{l-1}$ are computed at $s = k - N + K$ in batch forms as

$$G_s = (C_{m,s}^T C_{m,s})^{-1}, \quad (28)$$

$$\hat{x}_s^c = G_s C_{m,s}^T Y_{m,s}. \quad (29)$$

The individual estimates $\hat{x}_k^{(i)}$ are provided by

$$G_l^{(i)} = [H_l^{(i)T} H_l^{(i)} + (A_l G_{l-1}^{(i)} A_l^T)^{-1}]^{-1}, \quad (30)$$

$$\hat{x}_l^{(i)-} = A_l \hat{x}_{l-1}^{(i)}, \quad (31)$$

$$\hat{x}_l^{(i)} = \hat{x}_l^{(i)-} + G_l^{(i)} H_l^{(i)T} (y_l^{(i)} - H_l^{(i)} \hat{x}_l^{(i)-}) \quad (32)$$

with the initial values

$$G_s^{(i)} = (C_{m,s}^{(i)T} C_{m,s}^{(i)})^{-1}, \quad (33)$$

$$\hat{x}_s^{(i)} = G_s^{(i)} C_{m,s}^{(i)T} Y_{m,s}^{(i)}. \quad (34)$$

A pseudo code of the designed predictive iterative dU-FIR algorithm with consensus on estimates is listed as Algorithm 2.

Algorithm 2: Iterative dUFIR Filtering Algorithm

Data: $y_k, R_k^{(i)}, R_k, N, \lambda_k^{opt}$

Result: $\hat{x}_k$

```

1 begin
2   for  $k = N - 1 : \infty$  do
3      $m = k - N + 1, \quad s = m + K - 1;$ 
4      $G_s = (\mathcal{H}_{m,s}^T \mathcal{H}_{m,s})^{-1};$ 
5      $G_s^{(i)} = (\mathcal{H}_{m,s}^{(i)T} \mathcal{H}_{m,s}^{(i)})^{-1};$ 
6     if  $\gamma_k = 0$  then
7        $y_k^{(j)} = H_k^{(j)} F_k \hat{x}_{k-1}^{(j)};$ 
8     end if
9      $\tilde{x}_s = G_s \mathcal{H}_{m,s}^T Y_{m,s};$ 
10     $\tilde{x}_s^{(i)} = G_s^{(i)} \mathcal{H}_{m,s}^{(i)T} Y_{m,s}^{(i)};$ 
11    for  $l = s + 1 : k$  do
12       $\hat{x}_l^- = F_l \hat{x}_{l-1};$ 
13       $\hat{x}_l^{(i)-} = F_l \hat{x}_{l-1}^{(i)};$ 
14       $G_l = [H_l^T \hat{H}_l + (F_l G_{l-1}^{(i)} F_l^T)^{-1}]^{-1};$ 
15       $G_l^{(i)} = [H_l^{(i)T} \hat{H}_l^{(i)} + (F_l G_{l-1}^{(i)} F_l^T)^{-1}]^{-1};$ 
16       $\hat{x}_l = \hat{x}_l^- + G_l H_l^T (y_l - H_l \hat{x}_l^-);$ 
17       $\hat{x}_l^{(i)} = \hat{x}_l^{(i)-} + G_l^{(i)} H_l^{(i)T} (y_l^{(i)} - H_l^{(i)} \hat{x}_l^{(i)-});$ 
18    end for
19     $\hat{x}_k^c = (I + J \lambda_k^{opt}) \tilde{x}_k - J \lambda_k^{opt} \tilde{x}_k^{(i)};$ 
20  end for
21 end
22 † First data  $y_0, y_1, \dots, y_{N-1}$  must be available.
```

5 Simulations

The following scenario is considered to test Algorithm 1 and Algorithm 2 for robustness against missing data. We consider the ground truth trajectory of a robot available for free from the MagPIE project dataset [32]. The coordinates x and y are measured by eight nodes of a WSN,

Table 1. Nodes sorted by the Number of Available Links with Neighbors

	Number of links			
	2	4	5	6
Node	1	2,7,8	3,6	4,5

Figure 1. A simulated WSN covering the ground truth trajectory of a mobile robot.

whose connections are depicted in Fig. 1 and stated in table 1. Measurements are simulated by adding white Gaussian noise to the ground truth data with different variances for each sensor as $\sigma_v^{(i)} = 0.25 + \phi$, where ϕ is uniformly distributed as $\phi = U(0.5, -0.5)$. Several data points are removed from measurements by following the binomial distribution with $p = 0.99$, where only a few data points are lost (Fig. 2 a) and $p = 0.6$, where many data points are lost (Fig. 2 b)).

The process dynamics of the moving robot is described in state space with

$$A = \begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H^{(i)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} \tau/2 & 0 \\ 1 & 0 \\ 0 & \tau/2 \\ 0 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_w^2 \end{bmatrix},$$

where $\sigma_w = 0.76$ m/s. For dUFIR, the optimal horizon N_{opt} was found at a test stage to be 57 in average.

As stated by (24), the optimal factor λ_k^{opt} depends on the appropriate knowledge of the noise statistics. In order to analyze the robustness of the algorithms, we let $Q_k \leftarrow (0.1)^2 Q_k$ and $R_k^{(i)} \leftarrow (2)^2 R_k^{(i)}$.

The RMSE computation for each node reveals remarkable results in terms of robustness as it is represented in Fig. 3. While it is true that both filters require knowledge of the measurement noise statistics, the performance of dKF falls behind the dUFIR performance due to its dependency on errors in the process noise statistics. Also, we observe that an unstable network ($p = 0.66$) has a more dramatic effect on dKF, as error in the dKF increase regardless of the number of links, while in the dUFIR an increase in the errors is more contained. This behavior is better observed in Fig. 4, where the bias errors in dUFIR are practically unaltered by the missing data.

The effect of the missing measurements is better appreciated in the overall trajectory estimation as shown in

Figure 2. Missing measurements corresponding to node 5: a) $p = 0.99$ and b) $p = 0.6$.

Figure 3. The RMSE produced by the dUFIR filter and dKF of nodes 1, 2, 3 and 4.

Figure 4. Estimation error produced along the coordinate x of node 5 by: a) dKF and b) dUFIR filter.

Figure 5. The ground truth and a trajectory estimated by the dKF and dUFIR algorithms.

Fig. 5, where the estimates obtained with Algorithm 1 demonstrate lesser accuracy than the estimates obtained with Algorithm 2.

6 Conclusion

In this paper, we have introduced a mathematical expression for λ_k^{opt} that allows the dUFIR filter to produce estimates with a minimum possible RMSE for consensus on estimates. The performance of the dUFIR filter was shown to have a better robustness than of the dKF under the missing data. This inference was shown to hold even for very unstable networks. An overall conclusion that has been made is that the dUFIR is a better choice than the dKF for distributed WSNs.

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