The discrete-time tracking problem with H_∞ model matching approach plus integral control

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Abstract. In this study, the discrete-time H_∞ model matching problem with integral control by using 2 DOF static output feedback is presented. First, the motivation and the problem is stated. After presenting the notation, the two lemmas toward the discrete-time H_∞ model matching problem with integral control are proven. The controller synthesis theorem and the controller design algorithm is elaborated in order to minimize the H_∞ norm of the closed-loop transfer function and to maximize the closed-loop performance by introducing the model transfer matrix. In following, the discrete-time H_∞ MMP via LMI approach is derived as the main result. The controller construction procedure is implemented by using a well-known toolbox to improve the usability of the presented results. Finally, some conclusions are given.

1 Introduction

The model matching problem has attracted a lot of attention in the control theory [13-14]. If $G_m(z)$ and $G(z)$ are the model and the system matrices, respectively, the discrete-time H_∞ model matching problem (MMP) is introduced to derive a controller transfer matrix $R(z)$ that minimizes the H_∞ norm of $G_m(z) - G(z)R(z)$. The model transfer matrix $G_m(z)$ has the desired performance specifications defined by its poles and zeros. Moreover, $G_m(z)$ and $G(z)R(z)$ are stable and proper transfer matrices, that is $G_m(z)$ and $G(z)R(z) \in RH_\infty$. The closed-loop performance $G(z)R(z)$ is considered to be approximated by the desired performance $G_m(z)$ such that,

$$\gamma_{\text{opt}} = \inf_{R(z) \in RH_\infty} \|G_m(z) - G(z)R(z)\|_\infty. \quad (1)$$

H_∞ MMP is elaborated in [5, 8, 9]. In these studies, the dynamic precompensator $R(s)$ is obtained and then it is implemented by dynamic state feedback, [13]. Formerly, continuous-time H_∞ MMP with one degree of freedom (1 DOF) static state feedback is derived in [1], the discrete-time H_∞ MMP with 1 DOF static output feedback and the continuous-time H_∞ MMP with 2 DOF static output feedback is presented in [2-3], respectively. On the other hand, the integral control structure subject to the existence of state feedback is firstly used in [4].

In this paper, the discrete-time H_∞ MMP with integral control is proposed by using a 2 DOF static output feedback. Both the solution of the discrete-time static H_∞ optimal control problem (OCP) and discrete-time H_∞ MMP is revisited toward the solution of our presented problem, whereas discrete-time H_∞ MMP can be completely solved by the LMI-based numerical optimization.

This paper is organized as follows: In Section 2, a special formulation for the discrete-time H_∞ MMP by a 2 DOF static output feedback with integral control in linear matrix inequalities (LMIs) is elaborated. In Section 3, the main result is given by a theorem that provides two existence conditions of the solution. In Section 4, we construct the 2 DOF static output feedback with integral control by using this theorem. Some conclusions are finally given in Section 5.

Notations

$\mathbb{R}$	The set of real numbers.
$\mathbb{C}$	The set of complex numbers.
$\mathbb{R}^{n \times m}$	The set of $n \times m$ real matrices.
$\text{Re}(\alpha)$	The real part of $\alpha \in \mathbb{C}$.
L_∞	The functions bounded on $\text{Re}(s)=0$ including at ∞ .
H_∞	The set of L_∞ functions analytic in $\text{Re}(s)>0$.
I_n	An identity matrix of $n \times n$ dimension.
0_n	A zero matrix of $n \times n$ dimension.
$0_{n \times m}$	A zero matrix of $n \times m$ dimension.
$\text{Ker} M$	The kernel space the linear operator M .
$\text{Im} M$	The image space of the linear operator M .
N^T	The tranpose of the matrix N .
$P > 0$	P positive definite matrix.
$\dim(U)$	The dimension of the linear space U .
$\lambda_{\max}(A)$	The largest eigenvalue of the matrix A .
$\sigma_{\max}(A)$	The largest singular value of the matrix A defined as

$$\sigma_{\max}(A) = \sqrt{\lambda_{\max}(A^T A)}.$$

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$\|G(z)\|_\infty$ The norm of the transfer matrix $G(z)$ defined as

$$\|G(z)\|_\infty = \sup_{\omega \in [0, 2\pi]} \sigma_{\max}[G(e^{j\omega})].$$

In order to present a synthesis theorem on the LMI-based characterization of the discrete-time H_∞ model matching problem with integral control, the following lemmas are given. The first lemma is **The Bounded Real Lemma** and it is used to turn the discrete-time H_∞ optimal control problem into an linear matrix inequality (LMI):

Lemma 1.1 Consider a discrete transfer matrix $T(z)$ of (not necessarily minimal) realization $T(z)=D+C(zI-A)^{-1}B$. The following statements are equivalent:

- i) $\|T(z)=D+C(zI-A)^{-1}B\|_\infty < \gamma$ and the matrix A is Schur ($|\lambda_i(A)| < 1, i=1, \dots, n$).
- ii) There is a solution $X > 0$ to the LMI:

$$\begin{bmatrix} -X^{-1} & A & B & 0 \\ A^T & -X & 0 & C^T \\ B^T & 0 & -\gamma I & D^T \\ 0 & C & D & -\gamma I \end{bmatrix} < 0. \quad (2)$$

Proof: See [10]. ■

Lemma 1.2 Suppose P, Q and H are matrices and that H is symmetric. The matrices N_P and N_Q are full rank matrices satisfying $\text{Im}N_P = \text{Ker}P$ and $\text{Im}N_Q = \text{Ker}Q$. Then there is a matrix J such that,

$$H + P^T J^T Q + Q^T J P < 0 \quad (3)$$

if and only is the inequalities

$$N_P^T H N_P < 0 \quad \text{and} \quad N_Q^T H N_Q < 0 \quad (4)$$

are both satisfied.

Proof: See [10]. ■

Lemma 1.3 The block matrix

$$\begin{bmatrix} P & M \\ M^T & N \end{bmatrix} < 0 \quad (5)$$

if and only if

$$N < 0 \quad \text{and} \quad P - M N^{-1} M^T < 0 \quad (6)$$

In the sequel, $P - M N^{-1} M^T$ is referred to as the **Schur complement** of N .

Proof: See [6]. ■

2 The discrete-time H_∞ mmp by 2 dof static output feedback with integral control in lmi optimization

Toward the solution of the discrete-time H_∞ MMP via LMI approach, the problem should be reformulated as standard discrete-time H_∞ OCP. The state-space representation of the system $G(z)$ and the model system $G_m(z)$ is given:

$$G(z) : \begin{cases} x(k+1) = Ax(k) + Bv(k) \\ y_s(k) = Cx(k) \end{cases} \quad (7)$$

$$G_m(z) : \begin{cases} q(k+1) = Fq(k) + Gw(k) \\ y_m(k) = Hq(k) + Jw(k) \end{cases} \quad (9)$$

where $x(k) \in \mathbb{R}^n$ is the system state, $q(k) \in \mathbb{R}^{n_m}$ is the model state, $v(k), w(k), y_s(k)$ and $y_m(k) \in \mathbb{R}^m$. We take that the given system is strictly proper in order to simplify the solution of the problem. The integral control is modelled by a serie integrator:

$$\hat{x}(k+1) = w(k) - y_s(k) \quad (11)$$

$$v(k) = \hat{x}(k) + u(k) \quad (12)$$

where $\hat{x}(k)$ and $u(k) \in \mathbb{R}^m$. The control input $u(k)$ is generated by a two degrees of freedom static output feedback controller:

$$u(k) = L y_s(k) + M w(k) = \begin{bmatrix} L & M \end{bmatrix} \begin{bmatrix} y_s(k) \\ w(k) \end{bmatrix}. \quad (13)$$

The block diagram of a discrete-time H_∞ MMP by a static output feedback with integral control is illustrated in Figure 1. In this formulation, the steady-output value $y_s(k)$ will follow a step function input with zero error. We will use a 2 DOF feedback control structure which is defined in the control theory, [12]:

Fig. 1. The block diagram of model matching system with 2 DOF static output feedback in the integral control.

The generalized plant $P(z)$ shown in Figure 1 can be modelled as,

$$\begin{bmatrix} \hat{x}(k+1) \\ \hat{x}(k+1) \\ q(k+1) \end{bmatrix} = \begin{bmatrix} A & B & 0 \\ -T.C & I_m & 0 \\ 0 & 0 & F \end{bmatrix} \begin{bmatrix} \hat{x}(k) \\ \hat{x}(k) \\ q(k) \end{bmatrix} + \begin{bmatrix} 0 \\ T.I_m \\ G \end{bmatrix} w(k) + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u(k) \quad (14)$$

$$z(k) = \begin{bmatrix} -C & 0 & H \end{bmatrix} \begin{bmatrix} \hat{x}(k) \\ \hat{x}(k) \\ q(k) \end{bmatrix} + Jw(k) \quad (15)$$

$$y(k) = \begin{bmatrix} y_s(k) \\ w(k) \end{bmatrix} = \begin{bmatrix} C & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}(k) \\ \hat{x}(k) \\ q(k) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} w(k). \quad (16)$$

Matrices are defined as follows :

$$\underline{A} = \begin{bmatrix} A & B & 0 \\ -T.C & I_m & 0 \\ 0 & 0 & F \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ T.I_m \\ G \end{bmatrix} \quad B_2 = \begin{bmatrix} B \\ 0_m \\ 0_{n_m \times m} \end{bmatrix} \quad (17)$$

$$C_1 = \begin{bmatrix} -C & 0_m & H \end{bmatrix} \quad C_2 = \begin{bmatrix} C & 0_m & 0_{m \times n_m} \\ 0_{m \times n_s} & 0_m & 0_{m \times n_m} \end{bmatrix} \quad (18)$$

$$D_1 = J \quad D_2 = \begin{bmatrix} 0_m \\ I_m \end{bmatrix}. \quad (19)$$

The above formulation concludes that the discrete-time H_∞ model matching problem plus integral control with the two degrees of freedom static output feedback is equivalent to the discrete-time H_∞ optimal control problem. This equivalency is drawn in Figure 2:

Fig. 2. The block diagram of the general form of H_∞ OCP with a static controller.

The closed-loop transfer matrix from $w(k)$ to $z(k)$ is derived by

$$T_{zw}(z) = D_{cl} + C_{cl}(zI - A_{cl})^{-1}B_{cl} \quad (20)$$

where

$$A_{cl} = \underline{A} + B_2 K C_2 \quad (21)$$

$$B_{cl} = B_1 + B_2 K D_2 \quad (22)$$

$$C_{cl} = C_1 \quad (23)$$

$$D_{cl} = D_1. \quad (24)$$

A synthesis theorem on the LMI-based solution of the problem is presented in the following section.

3 Main result

We can now present a synthesis theorem on the LMI-based solution of the discrete H_∞ model matching problem with integral control by two degrees of freedom static output feedback:

Theorem 3.1 A 2 DOF static output feedback controller $K=[L \ M] \in \mathbb{R}^{m \times 2m}$ exists for the discrete-time H_∞ MMP with integral control and the closed-loop system is internally stable if and only if there is a matrix $X > 0$ such that,

$$\begin{bmatrix} N_o & 0 & 0 & 0 \\ 0 & I_m & 0 & 0 \\ 0 & 0 & I_{n_m} & 0 \end{bmatrix}^T \left\{ \begin{bmatrix} A & B & 0 \\ -T.C & I_m & 0 \\ 0 & 0 & F \end{bmatrix}^T X \begin{bmatrix} A & B & 0 \\ -T.C & I_m & 0 \\ 0 & 0 & F \end{bmatrix} - X + \frac{1}{\gamma} \begin{bmatrix} C^T C & 0 & -C^T H \\ 0 & 0 & 0 \\ -H^T C & 0 & H^T H \end{bmatrix} \right\} \begin{bmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{bmatrix} < 0 \quad (25)$$

$$\begin{bmatrix} \begin{bmatrix} N_c & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{bmatrix} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T \left\{ \begin{bmatrix} A & B & 0 \\ -T.C & I_m & 0 \\ 0 & 0 & F \end{bmatrix}^T X^{-1} \begin{bmatrix} A & B & 0 \\ -T.C & I_m & 0 \\ 0 & 0 & F \end{bmatrix} - X^{-1} \begin{bmatrix} C^T C & 0 & -C^T H \\ 0 & 0 & 0 \\ -H^T C & 0 & H^T H \end{bmatrix} \right\} \begin{bmatrix} 0_{n_m} \\ T.I_m \\ G \end{bmatrix} \begin{bmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{bmatrix} < 0 \quad (26)$$

where T is the sampling period and N_c and N_o are full rank matrices with

$$\text{Im } N_c = \text{Ker } B^T \quad (27)$$

$$\text{Im } N_o = \text{Ker } C. \quad (28)$$

Proof: From the Bounded Real Lemma, $K=[L \ M] \in \mathbb{R}^{m \times 2m}$ is the two degrees of freedom static output feedback controller in Figure 2 if and only if the LMI

$$\begin{bmatrix} -X^{-1} & A_{cl} & B_{cl} & 0 \\ A_{cl}^T & -X & 0 & C_{cl}^T \\ B_{cl}^T & 0 & -\gamma I & D_{cl}^T \\ 0 & C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0 \quad (29)$$

holds for some $\gamma > 0$ in $\mathbb{R}^{(ns+nm+m) \times (ns+nm+m)}$. Using the expressions A_{cl} , B_{cl} , C_{cl} and D_{cl} in (21), (22), (23) and (24), this LMI can also be written as,

$$H_X + P^T K Q + Q^T K^T P < 0 \quad (30)$$

where

$$H_X = \begin{bmatrix} -X^{-1} & \underline{A} & B_1 & 0 \\ \underline{A}^T & -X & 0 & C_1^T \\ B_1^T & 0 & -\gamma I_m & D_1^T \\ 0 & C_1 & D_1 & -\gamma I_m \end{bmatrix} \quad (31)$$

$$P = \begin{bmatrix} B_2^T & 0_{m \times (n_s + n_m + m)} & 0_m & 0_m \end{bmatrix} \quad (32)$$

$$Q = \begin{bmatrix} 0_{2m \times (n_s + n_m + m)} & C_2 & D_2 & 0_{2m \times m} \end{bmatrix} \quad (33)$$

We can use Lemma 1.2 to eliminate the matrix K in (30). Therefore, the linear matrix inequality (30) holds for some K if and only if

$$N_P^T H_X N_P < 0 \quad \text{and} \quad N_Q^T H_X N_Q < 0 \quad (34)$$

where

$$\text{Im} N_P = \text{Ker} P \quad (35)$$

$$\text{Im} N_Q = \text{Ker} Q \quad (36)$$

$$X > 0. \quad (37)$$

Meanwhile, from (32) the bases of KerP are obtained:

$$N_P = \begin{bmatrix} \begin{pmatrix} N_c & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 & 0 & 0 \\ 0 & I_{n_s + n_m + m} & 0 & 0 \\ 0 & 0 & I_m & 0 \\ 0 & 0 & 0 & I_m \end{bmatrix} \quad (38)$$

Here, the matrix N_c is any basis of the null space of B^T . Thus the inequality $N_P^T H_X N_P < 0$ can be reduced to

$$\begin{bmatrix} \begin{pmatrix} N_c & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 & 0 & 0 \\ 0 & I_{n_s + n_m + m} & 0 & 0 \\ 0 & 0 & I_m & 0 \\ 0 & 0 & 0 & I_m \end{bmatrix}^T \begin{bmatrix} -X^{-1} & \underline{A} & B_1 & 0 \\ \underline{A}^T & -X & 0 & C_1^T \\ B_1^T & 0 & -\gamma I_m & D_1^T \\ 0 & C_1 & D_1 & -\gamma I_m \end{bmatrix} \begin{bmatrix} \begin{pmatrix} N_c & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 & 0 & 0 \\ 0 & I_{n_s + n_m + m} & 0 & 0 \\ 0 & 0 & I_m & 0 \\ 0 & 0 & 0 & I_m \end{bmatrix} < 0 \quad (39)$$

and

$$\begin{bmatrix} \begin{pmatrix} N_c & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 & 0 & 0 \\ 0 & I_{n_s + n_m + m} & 0 & 0 \\ 0 & 0 & I_m & 0 \\ 0 & 0 & 0 & I_m \end{bmatrix}^T \begin{bmatrix} \underline{A} X^{-1} \underline{A}^T - X^{-1} & B_1 & \underline{A} X^{-1} C_1^T \\ \underline{A}^T & -X & 0 \\ C_1 X^{-1} \underline{A}^T & -\gamma I_m & -\gamma I_m + C_1 X^{-1} C_1^T \\ 0 & C_1 & D_1 & -\gamma I_m \end{bmatrix} \begin{bmatrix} \begin{pmatrix} N_c & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 & 0 & 0 \\ 0 & I_{n_s + n_m + m} & 0 & 0 \\ 0 & 0 & I_m & 0 \\ 0 & 0 & 0 & I_m \end{bmatrix} < 0 \quad (40)$$

or

$$\begin{bmatrix} \begin{pmatrix} N_c & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 & 0 & 0 \\ 0 & I_{n_s + n_m + m} & 0 & 0 \\ 0 & 0 & I_m & 0 \\ 0 & 0 & 0 & I_m \end{bmatrix}^T \begin{bmatrix} \underline{A} X^{-1} \underline{A}^T - X^{-1} & \underline{A} X^{-1} C_1^T & B_1 \\ \underline{A}^T & -X & 0 \\ C_1 X^{-1} \underline{A}^T & -\gamma I_m + C_1 X^{-1} C_1^T & D_1 \\ B_1^T & 0 & -\gamma I_m \end{bmatrix} \begin{bmatrix} \begin{pmatrix} N_c & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 & 0 & 0 \\ 0 & I_{n_s + n_m + m} & 0 & 0 \\ 0 & 0 & I_m & 0 \\ 0 & 0 & 0 & I_m \end{bmatrix} < 0 \quad (41)$$

The condition 26 is found when the relations (17), (18) and (19) are used (41). On the other hand,

$$\begin{aligned} \text{Im} N_Q &= \text{Ker} \begin{bmatrix} 0_{2m \times (n_s + n_m + m)} & C_2 & D_2 & 0_{2m \times m} \end{bmatrix} \\ &= \text{Ker} \begin{bmatrix} 0_{m \times (n_s + n_m + m)} & C & 0_m & 0_{m \times n_m} & 0_m & 0_m \\ 0_{m \times (n_s + n_m + m)} & 0_{m \times n_s} & 0_m & 0_{m \times n_m} & I_m & 0_m \end{bmatrix} \end{aligned} \quad (42)$$

and

$$N_Q = \begin{bmatrix} I_{n_s + n_m + m} & 0 & 0 \\ 0 & \begin{pmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 \\ 0 & 0 & 0_m \\ 0 & 0 & I_m \end{bmatrix} \quad (43)$$

Where the matrix N_o is any basis of the null space C. Therefore, the condition $N_Q^T H_X N_Q < 0$ is equivalent to

$$\begin{bmatrix} I_{n_s + n_m + m} & 0 & 0 \\ 0 & \begin{pmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 \\ 0 & 0 & 0_m \\ 0 & 0 & I_m \end{bmatrix}^T \begin{bmatrix} -X^{-1} & \underline{A} & B_1 & 0 \\ \underline{A}^T & -X & 0 & C_1^T \\ B_1^T & 0 & -\gamma I_m & D_1^T \\ 0 & C_1 & D_1 & -\gamma I_m \end{bmatrix} \begin{bmatrix} I_{n_s + n_m + m} & 0 & 0 \\ 0 & \begin{pmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 \\ 0 & 0 & 0_m \\ 0 & 0 & I_m \end{bmatrix} < 0 \quad (44)$$

or

$$\begin{bmatrix} \begin{pmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 \\ 0 & I_m \end{bmatrix}^T \begin{bmatrix} \underline{A}^T X \underline{A} - X & \underline{A}^T X B_1 & C_1^T \\ B_1^T X \underline{A} & -\gamma I_m + B_1^T X B_1 & D_1^T \\ C_1 & D_1 & -\gamma I_m \end{bmatrix} \begin{bmatrix} \begin{pmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 \\ 0 & 0 & 0_m \\ 0 & 0 & I_m \end{bmatrix} < 0 \quad (45)$$

and

$$\begin{bmatrix} \begin{pmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 \\ 0 & I_m \end{bmatrix}^T \begin{bmatrix} \underline{A}^T X \underline{A} - X & C_1^T \\ C_1 & -\gamma I_m \end{bmatrix} \begin{bmatrix} \begin{pmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{pmatrix} & 0 \\ 0 & 0 & I_m \end{bmatrix} < 0 \quad (46)$$

thus we have,

$$\begin{bmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{bmatrix}^T \begin{bmatrix} \underline{A}^T X \underline{A} - X + \frac{1}{\gamma} C_1^T C_1 \\ C_1 & -\gamma I_m \end{bmatrix} \begin{bmatrix} N_o & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_{n_m} \end{bmatrix} < 0. \quad (47)$$

To complete the proof, it suffices to use the relations (17), (18) and (19) into (47). ■

4 CONTROLLER CONSTRUCTION

Although Theorem 3.1 is about the solvability conditions of the discrete-time H_∞ MMP by the 2 DOF static output feedback with integral control, it also provides a controller construction procedure. Moreover The MATLAB LMI Control Toolbox [9] can be used to solve LMIs. The controller construction procedure can be summarized as follows:

Step 1: Find a solution $X > 0$ to the LMIs (25) and (26) for γ_{opt} which is the minimal of γ .

Step 2: Solve a 2 DOF static output feedback control law $K = [L \ M] \in \mathbb{R}^{m \times 2m}$ from LMI

$$H_X + Q^T K^T P + P^T K Q < 0 \quad (48)$$

where

$$P = \begin{bmatrix} B^T & 0_m & 0_{m \times n_m} & 0_{m \times (n_s + n_m + m)} & 0_m & 0_m \end{bmatrix} \quad (49)$$

$$Q = \begin{bmatrix} 0_{m \times (n_s + n_m + m)} & C & 0_m & 0_{m \times n_m} & 0_m & 0_m \\ 0_{m \times (n_s + n_m + m)} & 0_{m \times n_s} & 0_m & 0_{m \times n_m} & I_m & 0_m \end{bmatrix} \quad (50)$$

$$H_X = \begin{bmatrix} -X^{-1} & \begin{pmatrix} A & B & 0 \\ -T.C & I_m & 0 \\ 0 & 0 & F \end{pmatrix} & \begin{pmatrix} 0 \\ T.I_m \\ G \end{pmatrix} & 0 \\ \begin{pmatrix} A & B & 0 \\ -T.C & I_m & 0 \\ 0 & 0 & F \end{pmatrix}^T & -X & 0 & (-C \ 0 \ H)^T \\ \begin{pmatrix} 0 \\ T.I_m \\ G \\ 0 \end{pmatrix}^T & 0 & -\gamma I_m & J^T \\ (-C \ 0 \ H) & J & -\gamma I_m & 0 \end{bmatrix} \quad (51)$$

6 Conclusions

In this paper, we have studied the discrete-time H_∞ model matching problem with two degrees of freedom static output feedback. We have induced integral controller to this classical problem. The introduction of integral type of controller to this configuration naturally forces the steady-output error to zero. Moreover, the nearly proposed block diagram reduces the problem to an H_∞ optimal control problem and a theorem is proposed which provides a procedure to design the controller. However, we suppose that the two LMI conditions provided in the Theorem 3.1 can be simplified in future works.

The authors gratefully acknowledge the support of Galatasaray University, scientific research support program under grant # 19.401.003

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