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Analysis of the operation process of a training aircraft-Cessna 150

Marta Woch[1,*], *Justyna* Tomaszewska[2,**], *Grzegorz* Kamiński[3,***], *Kinga* Kościak[2,****], *Tomasz* Zienkiewicz[2,†], *Mariusz* Michalski[2,‡], and *Mariusz* Zieja[1,§]

[1]Air Force Institute of Technology, Warsaw, Poland
[2]Faculty of Aviation, Polish Air Force Academy, Dęblin, Poland
[3]Institute of Aeronautics and Applied Mechanics, Warsaw University of Technology, Warsaw, Poland

Abstract. The method of the stochastic Markov process used for the analysis of operation of a training aircraft-Cessna 150 was presented in the article. In this article, the time changing probability of readiness was determined on the basis of knowledge of the current aircraft operation. Markov stochastic processes have been used as a model to determine aircraft-Cessna 150 readiness for specific tasks. In order to determine the readiness, the probability of being in one of the investigated states is determined. The analysed states included: standby, pre-flight service, flight, interstate service, after-flight service and hangar service.

1 Introduction

The method of the stochastic Markov process used for the analysis of operation of a training aircraft-Cessna 150 was presented in the article. The Cessna 150 aircraft is a two-seat training and tourist aircraft. Production began at the Cessna Aircraft Company's factory in Wichita, Kansas in 1958. On the basis of it, a number of models were developed, which differ in terms of equipment. Due to its ease of piloting, Cessna 150 is still very popular plane used for tourist licenses training in Poland and the United States [1, 2].

The Cessna 150 aircraft is a high-wing aircraft with a fully metal, semi-corup structure. The plane is equipped with a three-wheel fixed undercarriage. Controlled front undercarriage is equipped with a hydraulic-air cushioned shank. What distinguishes this plane from others is the window on the rear side around the rear window [3–5].

Description and characteristics of selected dimensions:

- Wingspan: 10.11m
- Maximum length: 7.24 m
- Maximum height: 2.63 m including signal lamp and bent chassis
- Chassis spacing: 2.31 m
- Fixed pitch propeller diameter: 1.752 m
- Wing area: 14.8 m^2 [6, 7].

*e-mail: marta.woch@itwl.pl
**e-mail: j.tomaszewska@law.mil.pl
***e-mail: gkaminski@meil.pw.edu.pl
****e-mail: kinga.kosciak07@gmail.com
†e-mail: t.zienkiewicz@law.mil.pl
‡e-mail: m.michalski@law.mil.pl
§e-mail: mariusz.zieja@itwl.pl

2 Calculation methodology

2.1 Markov chains

Among analytical methods based on the analysis of random processes, also called state space methods, the most frequently used are the methods of the Markov chains and processes, and recently the semi-Markov processes. They are based on the assumption that the tested object of random process fulfilling the property of the Markov process [8, 9].

The random process is called the Markov process, when for any finite sequence of moments and any real numbers there is an equality [10]:

$$\begin{aligned} &P\left[X(t_n) < x_n | X(t_{n-1}) = x_{n-1}, \ldots, X(t_1) = x_1\right] = \\ &= P\left[X(t_n) < x_n | X(t_{n-1}) = x_{n-1}\right] \end{aligned} \tag{1}$$

This relationship means that the conditional probability distribution of a random variable $X(t_n)$ depends solely on the probability distribution of one of the random variables $X(t_{n-1})$. Properties of the Markov process at the moment of t_n do not depend on the values that the process assumed at moments $t_1, t_2, \ldots, t_{n-2}$. The Markov process is therefore fully characterized by the conditional distribution [11–13]:

$$F(s, t, x, y) = P\left[X(t) < x | X(s) = y\right], s < t \tag{2}$$

or the total distribution of random vector $(X(s), X(t))$ with initial distribution $F(s, y) = P[X(s) < y]$.

In the analysis of Markov's processes, a function called the probability of transition is essential, which is defined for any moment t, state s and for any number of real y and any Borel set $\mathbf{B}$, in the following way [14, 15]:

$$P(s, t, \mathrm{B}, y) = P\left[X(t) \in \mathrm{B} | X(s) = y\right] \tag{3}$$

In the case of the Markov process, the probability distributions of time in the states must be exponential. The exception is the calculation of asymptotic reliability indicates. In some cases it is also possible to transform the state space in such a way that non-explanatory probability distributions will be replaced by a sequence of exponential distributions.

2.2 Smoluchowski-Chapman-Kolmogorov equation

In practical applications, especially in reliability considerations, the most important part is performed by Markov's point processes defined on the range $T = \langle t_0, \infty)$ with the state space $S = 0, 1, 2, \dots$ [16, 17]. The realizations of the Markov point process are functions of fixed intervals and their graphs are stair line [18].

For a point process Markov probability of transition [19, 20]:

$$p_{ij}(s,t) = P\left[X(t) = j | X(s) = i\right], t \geq s; i, j = 0, 1, 2, \dots \quad (4)$$

satisfy the relationships:

$$p_{ij}(s,t) = \sum_{k=0}^{\infty} p_{ik}(s,t_1) p_{kj}(t_1,t), (s < t_1 < t) \quad (5)$$

known as Smoluchowski-Chapman-Kolmogorov equation. Moreover, for each i $(i = 0, 1, 2, \dots)$ there is an equation:

$$\sum_{j=0}^{\infty} p_{ij}(s,t) = 1 \quad (6)$$

Introducing functions $\lambda_{ij}(t)$ known as process transition intensities or transition rates:

$$\lambda_{ij}(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} p_{ij}(t, t + \Delta t); i, j = 0, 1, 2, \dots; \ i \neq j \quad (7)$$

the system of differential equations with variable coefficients is obtained [21, 22]:

$$\begin{aligned} &\forall i \in S : \tfrac{dP_i(t)}{dt} = \lambda_{ii}(t) P_i(t) + \sum_{\substack{j \in S \\ i \neq j}} \lambda_{ji}(t) P_j(t) \\ &\lambda_{ii}(t) = - \sum_{\substack{j \in S \\ i \neq j}} \lambda_{ij}(t) \end{aligned} \quad (8)$$

where:

$P_i(t)$ - the unconditional probability of the process remaining at time t at state i [5, 23],

$\lambda_{ij}(t)$ - the transition rate of the process at t from state i to state j.

When the Mark process is homogeneous, the transition rate is independent of the time $\lambda_{ij}(t) = \lambda_{ij} = const.$, $i \neq j$ and a system of differential equations with constant coefficients is obtained:

$$\forall i \in S : \frac{dP_i(t)}{dt} = \lambda_{ii} P_i(t) + \sum_{\substack{j \in S \\ i \neq j}} \lambda_{ji} P_j(t) \quad (9)$$

for which knowledge of initial probabilities $P_i(0)$, $i \in S$ is needed [24].

The above system can be written in vector form, as [25–27]:

$$\frac{d}{dt}\mathbf{P}(t) = \mathbf{\Lambda}^T \mathbf{P}(t) \quad (10)$$

where:

$\mathbf{P}(t) = [P_1(t), P_2(t), \dots, P_m(t)]_{m \times 1}$ - column vector of probabilities of process presence in particular states,

$$\mathbf{\Lambda} = \begin{bmatrix} -\sum\limits_{j=2}^{m} \lambda_{1j} & \lambda_{12} & \dots & \lambda_{1m} \\ \lambda_{21} & -\sum\limits_{\substack{j=1 \\ j \neq 2}}^{m} \lambda_{2j} & \dots & \lambda_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{m1} & \lambda_{m2} & \dots & -\sum\limits_{j=1}^{m-1} \lambda_{mj} \end{bmatrix} -$$

transition rate matrix,

m - number of sets S (number of process states).

The elements of the $\mathbf{\Lambda} = \left[\lambda_{ij}\right]; \ i, j \in S$ transition rate matrix the following probabilistic interpretation may be given [28–30]:

$$\lambda_{ij} \cdot \Delta t + o(\Delta t) = p_{ij}(\Delta t) \quad (11)$$

$$1 - \lambda_i \cdot \Delta t + o(\Delta t) = p_{ii}(\Delta t) \quad (12)$$

where $\frac{o(\Delta t)}{\Delta t} \xrightarrow[\Delta t \to 0]{} 0$.

The expected value of the random variable T_{ij} can be understood as the average time of stay in the state S_i before the state S_j. From equation (11), the estimator of parameter λ from the sample can be calculated from the formula [31–33]:

$$\hat{\lambda}_{ij} = \frac{1}{E[T_i]} \cdot \hat{p}_{ij} = \frac{n_i}{\sum\limits_{k=1}^{N} t_i{}^{(k)}} \frac{n_{ij}}{n_i} \quad (13)$$

$$\hat{p}_{ij} = \frac{n_{ij}}{n_i} \quad (14)$$

where:

n_{ij} – number of transitions from the state S_i to the state S_j,

n_i – number of outputs from the S_i state ,

$t_i{}^{(k)}$ - time of the object being in the S_i state for observation number k from the sample.

3 The operational state model

The operation process is the transition of aircraft from one operating state to another [34]. The transition from state to state of the aircraft under operation can be illustrated, by a direct graph or as a zero-one matrix.

An aircraft may be in one of the states in the operation process:

- S_1 - standby state;
- S_2 - pre-flight service state;
- S_3 - flight state;
- S_4 - interstate service state;
- S_5 - after-flight service state;
- S_6 - hangar service state.

The states S_1, S_2, S_3 and S_4 are classified as readiness states.

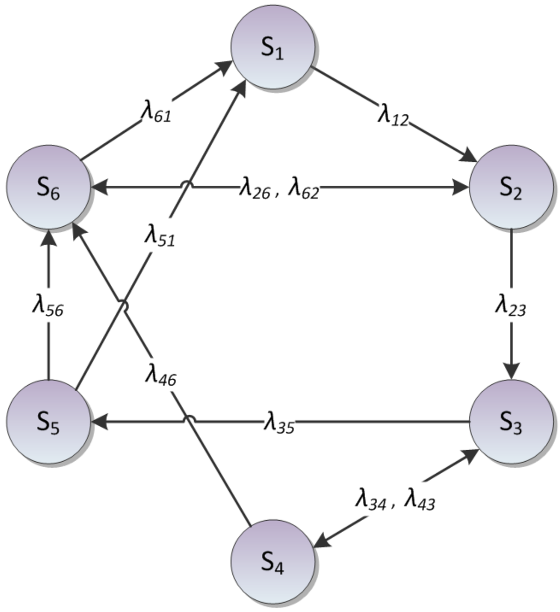

Figure 1. Directed graph of operating states of the Cessna 150 aircraft

Table 1. Number of flights performed at intervals assumed

Aircraft type	Day	Week	Month	Year
Cessna 150	7	43	172	2009

Table 2. Average residence time of the aircraft in each state

No.	State	Time
1	Standby	8h
2	Pre-flight service	40 min
3	Flight	90 min
4	Interstate service	22 min
5	After-flight service	35 min
6	Hangar service	8h – 24h

Table 3. The probability of transition between individual operating states

p_{ij}	S_1	S_2	S_3	S_4	S_5	S_6
S_1	0.000	0.975	0.000	0.000	0.000	0.025
S_2	0.000	0.000	0.975	0.000	0.000	0.025
S_3	0.000	0.000	0.000	0.699	0.301	0.000
S_4	0.000	0.000	0.996	0.000	0.000	0.004
S_5	0.975	0.000	0.000	0.000	0.000	0.025
S_6	1.000	0.000	0.000	0.000	0.000	0.000

4 Results

On the basis of the collected data, the exploitation graph presented in Figure 1 was developed.

The number of flights performed at intervals assumed are presented in table 1.

The average residence time of the aircraft in each state are presented in table 2.

Table 4. The transition rates between individual operating states

λ_{ij}	S_1	S_2	S_3	S_4	S_5	S_6
S_1	–0.090	0.088	0.000	0.000	0.000	0.002
S_2	0.000	–1.907	1.859	0.000	0.000	0, 048
S_3	0.000	0.000	–0.100	0.077	0.033	0.000
S_4	0.000	0.000	0.505	–0.507	0.000	0.002
S_5	1.767	0.000	0.000	0.000	–1.813	0.046
S_6	1.170	0.000	0.000	0.000	0.000	–1.170

The system shown in Figure 1 can be described by a system of differential equations [10]:

$$
\begin{aligned}
\frac{dP_1(t)}{dt} &= -\lambda_{12}P_1(t) + \lambda_{51}P_5(t) + \lambda_{61}P_6(t) \\
\frac{dP_2(t)}{dt} &= -(\lambda_{23} + \lambda_{26})\,P_2(t) + \lambda_{12}P_1(t) + \lambda_{62}P_6(t) \\
\frac{dP_3(t)}{dt} &= -(\lambda_{34} + \lambda_{35})\,P_3(t) + \lambda_{23}P_2(t) + \lambda_{43}P_4(t) \\
\frac{dP_4(t)}{dt} &= -(\lambda_{43} + \lambda_{46})\,P_4(t) + \lambda_{34}P_3(t) \\
\frac{dP_5(t)}{dt} &= -(\lambda_{51} + \lambda_{56})\,P_5(t) + \lambda_{35}P_3(t) \\
\frac{dP_6(t)}{dt} &= -(\lambda_{61} + \lambda_{62})\,P_6(t) + \lambda_{26}P_2(t) + \lambda_{46}P_4(t) \\
&\quad + \lambda_{56}P_5(t)
\end{aligned}
\tag{15}
$$

where:

$P_1(t)$ - the probability that the system is in a standby state;

$P_2(t)$ - the probability that the system is in a pre-flight service state;

$P_3(t)$ - the probability that the system is in a flight state;

$P_4(t)$ - the probability that the system is in a interstate service state;

$P_5(t)$ - the probability that the system is in a after-flight service state;

$P_6(t)$ - the probability that the system is in a hangar service state.

These probabilities can be determined using the program Wolfram Mathematica.

4.1 Analysis of the readiness of a Cessna 150 aircraft to perform a training task

The probabilities of transition between individual operating states are presented in the table 3.

The transition rates between particular operating states are presented in the table 4.

Figure 2 shows the probabilities of being in one of the analysed operating states as a function of time, assuming that the initial state was the standby state.

On this basis, it can be observed that the probability that an aircraft of type Cessna 150 will remain in the standby state in the initial phase is 100% and decreases over time. After approximately 30 days, this probability reaches a constant level (known as the threshold probability) of approximately 23%. The probability of an aircraft being in a flight condition increases within 30 days and has

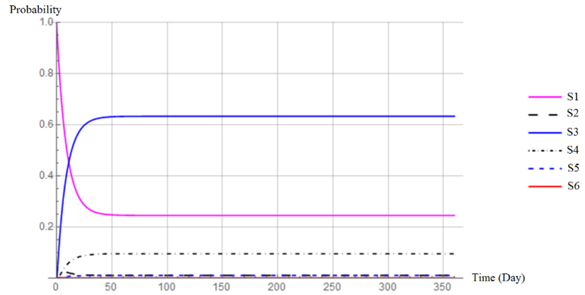

Figure 2. The transition rates between individual operating states

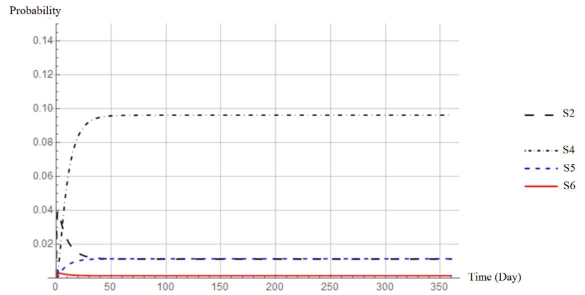

Figure 3. The transition rates between individual operating states

remained stable since then, at approximately 62%. Probabilities for states with a lower value are shown in Figure 3. The probability of being in the interstate service state of approximately 9% has been obtained. It shows that the probability of the aircraft in the pre-flight service state is decreasing, while the probability of the aircraft in the after-flight service state is increasing. After approximately 30 days, these two probabilities converge and remain stable at 1%. The probability that an aircraft is in the hangar service state is just over 0%.

On the basis of the calculations, it can be seen that the results for the different initial states differ in the initial phase and are the same in the final phase. The presented probabilities reach a constant level of probability after 30 days, known as the limit probability.

The Cessna 150 aircraft rarely fails, provided that the proper maintenance is carried out regularly and that it is carried out in accordance with the manufacturer's instructions for use.

5 Conclusions

On the basis of the presented results, approximate probabilities of a Cessna 150 aircraft in the assumed operating states were determined. The results obtained differ in the initial phase depending on the assumed initial state. After about 30 days, the probabilities shown in the probability diagrams, reach constant levels called limit probabilities. It has been assumed that hangar service operations are performed regularly in accordance with the guidelines described by the manufacturer in the operating instructions.

In addition, it is assumed that operations are carried out correctly by appropriately trained personnel. It can be concluded that Cessna 150 aircraft rarely fail.

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