Three-phase power transformer modelling in AC/DC traction substations

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Abstract. Three-phase power transformer is one of the most important elements in the electric power systems, and it plays a significant role in terms of energy savings. Since the efficiency standards can be expressed in terms of electrical efficiency, in an attempt to improve the transformer efficiency, in this study an enhancement of three-phase power transformer modelling with space phasors is presented. There are established the equations with space phasors of the three-phase transformer with symmetrical compact core. This equations system can be used to analyze the dynamic regimes of three-phase transformers. In this paper have been analyzed some aspects of three-phase power transformer operation in a AC/DC traction substation.

1 Introduction

Nowadays, because of economic and business growth, standards of life and development of civilization are too often interpreted in correlation with the use of electricity, and the demand of electricity is constantly rising. Three-phase power transformer is one of the most important elements in the electric power systems, and it plays a significant role in terms of energy savings [1-4]. The transmission and distribution of electricity through different voltage levels are possible due to the use of power transformers. The efficiency and sustainability of power transformers are in correlation with the reliability of the whole network, and could have considerable economic and environmental impact.

Forecast based on mathematical models enlarges our beliefs on the world functionality [5]. Although the mathematical modeling is a complex process and entails a large element of compromise the interacting systems in the real world can be studied identifying the most important interrelations of the systems [2-7]. Since the efficiency standards can be expressed in terms of electrical efficiency depending on load characteristics, in an attempt to improve the transformer efficiency, below an enhancement of three-phase power transformer modelling with space phasors is presented [3-4].

2 Three-phase transformer modelling with space phasors

Basically, three-phase transformers are widely used since three phase power is the common way to produce, transmit and use the electrical energy. A three-phase transformer transfers electric power from the three-phase primary winding through inductively coupled three-phase secondary winding, changing values of three-phase RMS voltage and current [3-4]. Most common, the transformers windings are wound around a ferromagnetic core.

In this study we take into consideration a three-phase transformer with a non-saturated magnetic core, and in a symmetrical construction, as depicted in Fig.1.

![Fig.1. Three-phase power transformer](image)

The primary phase windings (A-X), (B-Y) and (C-Z) are identical, each of them having \( w_1 \) turns and the electric resistance \( R_1 \). Similarly, the secondary phase windings (a-x), (b-y) and (c-z) are identical each of them having \( w_2 \) turns and the electric resistance \( R_2 \). Moreover, the three-phase primary winding is connected in star (Y) being supplied by the RST power network, while the three-phase secondary winding is connected in star (y) and is supplying the three-phase load connected in star, with the parameters \( R_3L_3C_3 \) on each phase, as shown in Fig.1.

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Varying currents flowing in the primary winding (due to the varying phase voltages \(u_a, u_b\) and \(u_c\)) create a varying magnetic flux in the transformer core, and thus a varying magnetic field through the secondary winding. This varying magnetic field induces a varying electromotive force in the secondary winding. Since a three-phase electric load is connected to the secondary winding, electrical energy will be transferred from the primary circuit through the transformer to the load [1-4, 6-7].

In this paper the three-phase electromagnetic phenomena will be described into the space phasors theory [3-4,8].

In case of three-phase transformer all variables (\(u \equiv \) primary voltage; \(i \equiv \) primary current; \(u \equiv \) secondary voltage and \(i \equiv \) secondary current) are not real but complex mathematical quantities. In this context, in the study of the three-phase transformer one can use the space phasors method, highlighting that the time axes \(t_a=t_a, t_b=t_b\) and \(t_c=t_c\) are physically associated at the axes of the three-phase primary and secondary windings, which are symmetrically disposed in space.

In this framework we obtain the voltage equations of the primary phase windings as follows:

\[
\begin{align*}
\vec{u}_A &= R_1 \cdot \vec{i}_A + \frac{d}{dt} \Psi_A \\
\vec{u}_B &= R_1 \cdot \vec{i}_B + \frac{d}{dt} \Psi_B \\
\vec{u}_C &= R_1 \cdot \vec{i}_C + \frac{d}{dt} \Psi_C
\end{align*}
\]  

(1)

By amplifying the equations (1) with \(2/3, 2a/3, 2a^2/3\) and subsequently summing them will result the equation with space phasors as below:

\[
\vec{u}_d = R_1 \cdot \vec{i}_d + \frac{d}{dt} \Psi_d
\]  

(2)

where:

\[
\begin{align*}
\vec{u}_d &= \frac{2}{3} (\vec{u}_A + \vec{a} \cdot \vec{u}_B + \vec{a}^2 \cdot \vec{u}_C) \\
\vec{i}_d &= \frac{2}{3} (\vec{i}_A + \vec{a} \cdot \vec{i}_B + \vec{a}^2 \cdot \vec{i}_C) \\
\Psi_d &= \frac{2}{3} (\Psi_A + \vec{a} \cdot \Psi_B + \vec{a}^2 \cdot \Psi_C)
\end{align*}
\]

are the space phasors of voltages \(u\), currents \(i\) and total fluxes \(\Psi\) corresponding to primary phase windings of three-phase transformer.

Similarly, we obtain the voltage equations of the secondary phase windings as below:

\[
\begin{align*}
\vec{u}_a &= -R_2 \cdot \vec{i}_a - \frac{d}{dt} \Psi_a \\
\vec{u}_b &= -R_2 \cdot \vec{i}_b - \frac{d}{dt} \Psi_b \\
\vec{u}_c &= -R_2 \cdot \vec{i}_c - \frac{d}{dt} \Psi_c
\end{align*}
\]  

(4)

By amplifying the equations (4) with \(2/3, 2a/3, 2a^2/3\) and subsequently summing them will result the voltage equation with space phasors as follows:

\[
\vec{u}_d = -R_2 \cdot \vec{i}_d - \frac{d}{dt} \Psi_d
\]  

(5)

In equation (5) \(u_d\) and \(i_d\) denote, respectively, the space phasors of voltages, currents and fluxes, corresponding to secondary phase windings of three-phase transformer.

One could note that the symmetrical three-phase transformer with compact ferromagnetic core has the phase windings magnetically coupled. Consequently, the total magnetic fluxes will be determined based on superposition principle. As example below there are presented the relationships for the flux through the total turns surface of windings A-X and a-x that are wound around the same column of ferromagnetic core.

\[
\begin{align*}
\Psi_{a} &= \Psi_{ad} + \Psi_{ab} + \Psi_{ac} + \Psi_{bd} + \Psi_{bc} + \Psi_{cd} \\
\Psi_a &= \Psi_a + \Psi_{ba} + \Psi_{ca} + \Psi_{db} + \Psi_{cb} + \Psi_{dc}
\end{align*}
\]  

(6)

Taking into consideration the magnetic core symmetry and in correlation with the positive sense of useful fascicular fluxes the relationships for the total magnetic coupling fluxes result as below:

\[
\begin{align*}
\Psi_{ad} &= \frac{1}{2} \Psi_{ab} \cdot \Psi_{cd} = \frac{1}{2} \Psi_a \\
\Psi_{ad} &= \frac{1}{2} \Psi_{ab} \cdot \Psi_{cd} = \frac{1}{2} \Psi_a
\end{align*}
\]  

(7)

Subsequently the expressions (6) can be rewritten as:

\[
\begin{align*}
\Psi_a &= \Psi_{ad} + \Psi_{ab} - \frac{1}{2}(-\Psi_{ab} + \Psi_{ac}) + \frac{1}{2} \left( \Psi_{ab} - \frac{1}{2} \Psi_{ac} \right) \\
\Psi_a &= \Psi_{ad} + \Psi_{ab} - \frac{1}{2}(-\Psi_{ab} + \Psi_{ac}) + \frac{1}{2} \left( \Psi_{ab} - \frac{1}{2} \Psi_{ac} \right)
\end{align*}
\]  

(8)

Since the useful fascicular fluxes verify the equations:

\[
\Phi_{ad} + \Phi_{ab} + \Phi_{ac} = 0; \Phi_{ad} + \Phi_{ab} + \Phi_{ac} = 0
\]  

by amplifying the turn numbers will result the relations for the total useful magnetic fluxes as follows:

\[
\begin{align*}
\Psi_{ab} + \Psi_{ac} &= -\Psi_{ad} \cdot \Psi_{ab} + \Psi_{ac} = \Psi_{ad}
\end{align*}
\]  

(9)

Based on expressions (10) the relationships (8) will become:

\[
\begin{align*}
\Psi_a &= \Psi_{ad} + \frac{3}{2} \Psi_{ab} + \frac{3}{2} \Psi_{ac} \\
\Psi_a &= \Psi_{ad} + \frac{3}{2} \Psi_{ab} + \frac{3}{2} \Psi_{ac}
\end{align*}
\]  

(11)

One could note that in the case of three-phase compact core, due to the windings’ magnetic coupling the useful fascicular flux corresponding to each phase winding is increased by \(3/2\) in comparison with the ferromagnetic core with free fluxes.

Since \(\Psi_{ad} = \vec{w} \cdot \phi_{ad}\), respectively \(\Psi_{ad} = \vec{w} \cdot \phi_{ad}\),
respectively

\[ i_{\lambda} = \frac{w_1}{w_2} \cdot i_2 \]

where: \( L = \frac{3}{2} L_{u1} \) denotes the cyclic inductance of primary and \( i_{\lambda} \) denotes the space phasor of magnetization currents.

With respect to the three-phase load connected at the transformer secondary terminals, the phase voltage equations are as follows:

\[ u_a = R_s \cdot i_a + L_s \cdot \frac{d i_a}{dt} + \frac{1}{C_s} \cdot \int i_a \, dt \]
\[ u_b = R_s \cdot i_b + L_s \cdot \frac{d i_b}{dt} + \frac{1}{C_s} \cdot \int i_b \, dt \]
\[ u_c = R_s \cdot i_c + L_s \cdot \frac{d i_c}{dt} + \frac{1}{C_s} \cdot \int i_c \, dt \]

By amplifying the equations (17) with \( 2/3, 2a/3, 2a^2/3 \) respectively, and subsequently summing them will result the voltage equation with space phasors of the three-phase load circuit, as below:

\[ u_s = R_s \cdot i_s + L_s \cdot \frac{d i_s}{dt} + \frac{1}{C_s} \cdot \int i_s \, dt \]

Subsequently, by introducing the space phasors of the electromotive forces induced in primary and secondary windings, the equations of the three-phase transformer with symmetrical compact core can be ordered in the following space phasors system:

\[ u_1 = -e_1 + R_s \cdot i_1 + L_s \cdot \frac{d i_1}{dt} \]
\[ u_s = e_s - R_s \cdot i_s - L_s \cdot \frac{d i_s}{dt} \]
\[ \Psi_{u1} = L \cdot i_{\lambda} \cdot \Psi_{u2} = \frac{w_2}{w_1} \cdot L \cdot i_{\lambda} \]

(16)

In relationships (15) have been introduced the notations:

\[ \Psi_{u1} = L \cdot i_{\lambda} \cdot \Psi_{u2} = \frac{w_2}{w_1} \cdot L \cdot i_{\lambda} \]

(17)

\[ i_{\lambda} = \frac{w_1}{w_2} \cdot i_2 \]

(18)

Moreover, one could proceed to secondary reported to primary, with the secondary space phasors:

\[ i_{s2} = \frac{w_1}{w_2} \cdot i_2 \]
\[ R_s = (\frac{w_1}{w_2})^2 \cdot R_s \]
\[ L = \frac{3}{2} \cdot L_{u1} \]

(19)

(20)
further obtaining the equations of three-phase transformer with secondary reduced to primary:

\[
\begin{align*}
\mathbf{u}_3 &= -\varepsilon_1 + R_1 \cdot \mathbf{i}_1 + L_{\sigma_1} \cdot \frac{d}{dt} \mathbf{i}_1 \\
\mathbf{u}_2 &= \varepsilon_2' - R_2' \cdot \mathbf{i}_2' - L_{\sigma_2} \cdot \frac{d}{dt} \mathbf{i}_2' \\
\mathbf{i}_1 + \mathbf{i}_2' &= \mathbf{i}_{1\mu} \\
\varepsilon_1 &= \varepsilon_2' - \mathbf{d}_2 \cdot \mathbf{u}_1 \\
\mathbf{u}_1 &= -L \cdot \mathbf{i}_1 + L_3 \cdot \frac{d}{dt} \mathbf{i}_1 + \frac{1}{C_s} \int \mathbf{i}_1' \, dt \\
\mathbf{u}_2 &= R_s' \cdot \mathbf{i}_2' + L_3' \cdot \frac{d}{dt} \mathbf{i}_2' + \frac{1}{C_s} \int \mathbf{i}_2' \, dt
\end{align*}
\]

The equations system (21) are on the whole conclusive. Particularly, in a permanent harmonic regime all space phasors of the three-phase symmetrical systems of sinusoidal quantities take the form \( v = SQRT \cdot e^{-i\omega t} \). Taking into account the space phasors derivation and integration relationships:

\[
\frac{d}{dt} v = j \cdot \omega \cdot v \quad \int v \, dt = \frac{1}{j \cdot \omega} \cdot v
\]

one could find the space phasors equations of system (21) rewritten in the classic form:

\[
\begin{align*}
\mathbf{u}_3 &= -\varepsilon_1 + R_1 \cdot \mathbf{i}_1 + j \cdot X_{\sigma_1} \cdot \mathbf{i}_1 \\
\mathbf{u}_2 &= \varepsilon_2' - R_2' \cdot \mathbf{i}_2' - j \cdot X_{\sigma_2} \cdot \mathbf{i}_2' \\
\mathbf{i}_1 + \mathbf{i}_2' &= \mathbf{i}_{1\mu} \\
\varepsilon_1 &= -j \cdot \omega \cdot \mathbf{u}_1 \\
\mathbf{u}_1 &= -\varepsilon_1 + R_1 \cdot \mathbf{i}_1 + j \cdot X_{\sigma_1} \cdot \mathbf{i}_1 \\
\mathbf{u}_2 &= R_s' \cdot \mathbf{i}_2' + j X_s \cdot \mathbf{i}_2'
\end{align*}
\]

As spreading, the DC substations are used both in urban (surface and underground) electrical traction and in DC electrified railway traction. As a location, they are “indoor” installations, most of the equipment being arranged in a “cellular” structure (in sideboards).

The basics of the DC traction substations are the AC-DC conversion groups. Over time, the AC-DC conversion groups have made significant progress in terms of performance, efficiency, maintenance and reliability [8-9].

Nowadays, all substations are equipped with static rectifiers with diodes [9-11].

Optionally, reversible DC substations (with anti-parallel transformer- thyristorized inverter groups) can also be used to recover electrical energy in case of electric vehicle recuperative braking.

In principle, any DC traction substation consists of a high-voltage alternating current system (comprising: the three-phase primary line, three-phase high-voltage bars, the tripolar protective circuit breakers of the transformer rectifier group and the power transformers) and a DC power system with \( U_{DC} \) rated voltage (consisting of rectifier bridges, DC breakers and ultrafast DC switches).

The AC-DC conversion groups are made up of:

- a three-phase transformer in order to reduce the actual voltage value (from \( U_1 \) of the three-phase primary line to the \( U_2 \) for supplying the rectifier) in close correlation with the continuous voltage \( U_{AC} \) magnitude across the contact line, and
- a three-phase rectifier, usually with diodes (connected in three-phase bridge and mounted in “cabinets”).

The basic structure of a rectifier system in a DC traction substation consists of a three-phase bridge of type Graetz bridge [9-10], depicted in Fig.2.

![Fig.2. Rectifier with Graetz three-phase bridge](image)

This bridge is powered from the secondary of a three-phase transformer (T), usually with a Delta-Star (Dy) connection scheme, having the transformation ratio \( K \) (of the line voltages) given as follows:

\[
K = \frac{U_{1L}}{U_{2L \ast}} = \frac{I_{1}}{\sqrt{3} \cdot w_1} \cdot \frac{w_1}{w_2} \tag{24}
\]

where:

- \( w_1 \) = the phase turns number of the primary winding (connected in Δ), and
- \( w_2 \) = the phase turns number of the secondary winding (connected in star).

3 Power transformer in AC/DC traction substation

The DC electrical traction substations are those fixed traction installations that receive electricity (in three-phase AC) from the national power system (at high voltage), reduce the voltage level and modify the current type (from AC to DC) and, finally, distributes the electric power to contact line sections in order to supply the non-autonomous electric railway vehicles [9-11].
3.1 The rectified voltage (idealized)

In order to study the idealized operation of the three-phase rectifier bridge, the following hypotheses are accepted [9]:

1. The inductance \( L_d \) (of the DC circuit) can be considered as infinitely high \((L_d\rightarrow\infty)\); consequently the DC current \( i_d \) will be perfectly smooth, and constant over time \( i_d=I_d \).

2. It is considered perfect magnetic coupling between the rectifier transformer windings. This means neglecting the transformer leakages \((L_k=0)\) and consequently the neglect of the inductance \( L_k \) of the switching circuit \((L_k\rightarrow0)\). Therefore, sudden variations in currents are admitted, which is equivalent to neglecting the natural switching phenomenon.

3. There are neglected the ohmic resistance (primary \( R_1\rightarrow0 \) and secondary \( R_2\rightarrow0 \)) of the rectifier transformer windings.

Under these conditions, the three-phase transformer \( T \) (fed into the primary) and seen on the secondary terminals will appear as a three-phase (ideal) source with sinusoidal phase voltages \( e_{ab}, e_{bc} \) and \( e_{ca} \) (symmetric, by direct sequence, with effective values \( E_{ab} \)) so that the composed voltages (line voltages) can preserve their effective value \( U_{20}=2E_{ab} \). Accordingly one can write:

\[
U_{20} = \frac{U_{20}}{K} \quad \text{and} \quad E_{20} = \frac{1}{\sqrt{3}} U_{20} \tag{25}
\]

If the three-phase (diode) bridge is fed from the ideal three-phase source (equivalent to the transformer) with the symmetrical sinusoidal phase voltages:

\[
e_{ab}=\sqrt{2} E_{20} \sin \omega t \\
e_{bc}=\sqrt{2} E_{20} \sin (\omega t - \frac{2\pi}{3}) \\
e_{ca}=\sqrt{2} E_{20} \sin (\omega t - \frac{4\pi}{3}) \tag{26}
\]

then, at any time moment \( \omega t>0 \), there will be only two diodes in conduction, namely:

a) only the diode in the cathode group \((1, 3, 5)\) with the anode connected to that phase of the source with the highest positive instantaneous phase voltage, and

b) only the diode in the anode group \((2, 4, 6)\) with the cathode connected to that phase of the source with the lowest negative instantaneous phase voltage.

All other diodes being momentarily subjected to inverse voltages are locked.

As an example, when the temporary origin \( \omega t = 0 \) (in the phase voltages diagram) when \( e_{ab}=e_{d0} \) (see Fig.4, pos.a), left it is noted that in the interval \([0,\pi/3]\) only lead diodes 1 and 6 and the rectified voltage \( u_{d0} \) results as:

\[
u_{d0}=e_{d0}-e_{b0}-2 \cdot U_D = u_{ab}-2 \cdot U_D \tag{27}
\]

Here \( u_{ab}=e_{a}-e_{b} \) represents the line voltage, and \( U_D \) is the direct voltage drop at the terminals of any diode in conduction.

The situation analyzed above is repeated (with other pairs of diodes) six times in each T period. The sequence of conduction intervals of the three-phase bridge diodes is shown in Fig.3, pos.b). In addition, if direct voltage drops are also neglected on diodes temporarily in conduction (meaning if \( U_D=0 \)), then the rectified voltage \( u_{d} \) will be given (in each period \( T=1/6 \)) only by the „positive elevations” of line voltages, exactly as is depicted in Fig.3, pos.c) (in left side, where the thickened curve represents the diagram \( u_{d0}=f(\omega t) \)).

Consequently, the rectified voltage \( u_{d} \) is not constant over time (since it has \( p=6 \) "elevations") but is is periodical, with the main period \( T=\pi/p \) or, in angular magnitude, with the angular period \( \beta_i \) as below:

\[
\beta_i = \frac{2\pi}{p} = \frac{2\pi}{s \cdot q} = \frac{\beta}{s} = \frac{\pi}{3} \tag{28}
\]

The average value \( U_{d0} \) of the rectified voltage \( u_{d} \) calculated on the interval of a main period \( \beta_i \) when the voltage \( u_{d} = u_{bc} \) (see Fig.3, pos.c)) has the analytical expression:

\[
u_{d}(\alpha t) = u_{d} \approx u_{bc} = \sqrt{2} U_{20} \cdot \cos(\alpha t) \tag{29}
\]

is determined (according to the first theorem of average) with the formula:

\[
u_{d0} = \frac{1}{\beta_1} \int_0^{\beta_1} u_{d}(\alpha t) \, d(\alpha t) = \frac{1}{\beta_1} \int_0^{\beta_1} \sqrt{2} U_{20} \cdot \cos(\alpha t) \, d(\alpha t) \tag{30}
\]

Concretely, for \( \beta_i=\pi/3 \) the average value \( U_{d0} \) (of the rectified voltage \( u_{d} \)) becomes:

\[
u_{d0} = \sqrt{2} U_{20} \cdot \frac{\sin \frac{\pi}{6} + \frac{\pi}{2}}{\frac{\pi}{6}} = 3 \sqrt{2} U_{20} \approx 1.35 \cdot U_{20} \tag{31}
\]

3.2 Diagrams of currents

As previously assumed, the inductance \( L_d \) (of the DC circuit) can be considered as infinitely high \((L_d\rightarrow\infty)\).
Consequently the DC current \( i_d \) will be perfectly smooth, and constant over time \( i_d = I_d \). Subsequently, under these assumptions will be determined the diagrams of currents.

### 3.2.1 Currents through diodes

If the diode switching phenomenon (in each switching group) is neglected, it can be admitted that through each semiconductor diode (of the three-phase bridge) will flow the constant current:

\[
I_D = I_d = I_d
\]

(32)
during each conduction interval \( \beta = 2\pi/3 \) of each variation period \( \omega T = 2\pi \) of the supply voltage.

Outside of the conduction interval, the current through the respective diode is null \( (i_d = 0) \).

Taking into consideration the sequence of the conduction intervals (see Fig.3, pos.b)), in Fig. 4 there are depicted (through "rectangular blocks") the currents \( i_{d1}, i_{d3} \) and \( i_{d5} \), and respectively \( i_{d2}, i_{d4} \) and \( i_{d6} \) corresponding to the valves (diodes) of the two switching groups of the three-phase bridge.

The average value (on a period interval \( \omega T = 2\pi \)) of the currents through the three-phase bridge is calculated with the formula:

\[
I_{Dmed} = \frac{I_D}{\sqrt{2}} = \frac{1}{2\pi} \int_0^{2\pi} I_d \cdot d(\omega t) = \frac{1}{2\pi} \int_0^{2\pi} I_d \cdot d(\omega t) = \frac{I_d}{\sqrt{3}}
\]

(33)

The effective value of the currents through the diodes \( I_d \) is given by:

\[
I_D = \sqrt{\frac{2\pi}{\omega T}} \int_0^{2\pi} I_d^2 \cdot d(\omega t) = \sqrt{\frac{2\pi}{\omega T}} \int_0^{2\pi} I_d^2 \cdot d(\omega t) = \frac{I_d}{\sqrt{3}}
\]

(34)

### 3.2.2 Currents through secondary windings

To the star connection (Y) of the secondary phase windings, the currents \( i_a, i_b \) and \( i_c \) in the three secondary phases of the rectifier transformer (see Fig.2) result as follows:

\[
i_a = i_{d1} - i_{d4}
\]

\[
i_b = i_{d3} - i_{d6}
\]

\[
i_c = i_{d5} - i_{d2}
\]

(35)

Graphically, the diagrams of secondary currents \( i_a, i_b \) and \( i_c \) depending on \( \omega t \) are depicted in Fig.4. In the neglect of the switching, they are formed (on each phase) of "rectangular blocks" of amplitude \( \pm I_d \) and duration \( 2\pi/3 \) separated by pauses (of null value) of duration \( \pi/3 \).

The average value of these alternating currents (non-sinusoidal) is null. Instead the effective value \( I_2 \) of the secondary phase currents is given by:

\[
I_2 = \sqrt{\frac{2\pi}{\omega T}} \int_0^{2\pi} I_2^2 \cdot d(\omega t) = \sqrt{\frac{2\pi}{\omega T}} \int_0^{2\pi} I_2^2 \cdot d(\omega t) = \sqrt{\frac{2\pi}{\omega T}} = \sqrt{\frac{2\pi}{3}} \cdot I_d
\]

(36)

#### 3.2.3 Currents through primary windings

Let \( i_a, i_b \) and \( i_c \) be the three-phase system of the currents passing through the transformer primary phase windings (see Fig.2). If \( w_1 \) and \( w_2 \) represent the phase turns numbers of the primary and secondary, respectively from the condition of neglecting the magnetization currents in the total currents equations corresponding to each column of the core of the three-phase transformer (so in the hypothesis \( \mu_{lc} \to \infty \)) we obtain the expressions of the primary currents \( i_{la}, i_{lb} \) and \( i_{lc} \):

\[
w_1 \cdot i_A + w_2 \cdot i_a = 0 \quad i_a = \frac{-w_2}{w_1} \cdot i_A
\]

\[
w_1 \cdot i_B + w_2 \cdot i_b = 0 \quad i_b = \frac{-w_2}{w_1} \cdot i_B
\]

\[
w_1 \cdot i_C + w_2 \cdot i_c = 0 \quad i_c = \frac{-w_2}{w_1} \cdot i_C
\]

(37)

Consequently, the primary phase currents \( i_{la}, i_{lb} \) and \( i_{lc} \) vary (with the time) vary proportionately (being in phase opposition and having the amplitudes \( w_2/w_1 \) times increased) with the secondary phase currents \( i_a, i_b \) and \( i_c \). So their effective values \( I_2 \) will be proportional with \( I_2 \).

\[
I_1 = \frac{I_2}{\sqrt{3}} \int_0^{2\pi} I_2 \cdot d(\alpha) = \frac{w_2^2}{w_1^2} \cdot \frac{2\pi}{3} \cdot I_2
\]

(38)

#### 3.2.4 Line currents in the primary

To the delta connection (Δ) of the primary phase windings of the power transformer (see Fig.2), the three-
phase system of the line currents $i_{LA}$, $i_{LB}$ and $i_{LC}$ is determined with the relationships:

$$i_{LA} = i_A + i_C = \frac{w^2}{w_1} (i_c - i_d)$$

$$i_{LB} = i_B - i_A = \frac{w^2}{w_1} (i_d - i_b)$$

$$i_{LC} = i_C - i_B = \frac{w^2}{w_1} (i_b - i_c)$$

(39)

Graphically, the diagrams of line currents $i_{LA}$, $i_{LB}$ and $i_{LC}$ (depending on $\omega t$) are depicted in the bottom side of Fig.4 [9]. The effective value $I_{L1}$ of the line currents is given by:

$$I_{L1} = \left[ \frac{1}{2\pi} \int_{0}^{2\pi} I_{LA} \cdot d(\omega t) \right]^{\frac{1}{2}} = \left[ \frac{2}{\pi} \int_{0}^{2\pi} I_{LA} \cdot d(\omega t) \right]^{\frac{1}{2}}$$

(40)

$$= \frac{1}{\sqrt{2\pi}} \sqrt{\frac{w^2}{w_1} \int_{-2\pi}^{2\pi} \left( i_{c} \cdot \frac{\pi}{3} + \frac{\pi}{3} (i_d) \cdot \frac{\pi}{3} \right)} = \frac{w^2}{\sqrt{2\pi}} \sqrt{\frac{1}{w_1}}$$

One can highlight that although the line and phase voltages vary sinusoid over time, both primary currents and secondary currents (phase and line) vary non-sinusoid over time. This way results explicitly in the deforming (non-sinusoidal) regime in which the rectifier transformer of traction substation is operating.

4 Discussion and Conclusion

The transmission and distribution of electricity through different voltage levels are possible due to the use of power transformers. The efficiency and sustainability of power transformers are in correlation with the reliability of the whole network, and could have considerable economic and environmental impact. Forecast based on mathematical models enlarges our beliefs on the world functionality. Although the mathematical modeling is a complex process and entails a large element of compromise the interacting systems in the real world can be studied identifying the most important interrelations of the systems. Since the efficiency standards can be expressed in terms of electrical efficiency, in order to enhance the transformer efficiency, in this study it is carried out the three-phase power transformer modelling with space phasors. The equations system obtained with space phasors can be used to analyze the dynamic regimes of three-phase transformers, being successfully applied, for instance, in the method of structural diagrams for the power transformer operation. Subsequently we have analyzed some aspects of three-phase power transformer in a AC/DC traction substation, concluding that although the line and phase voltages vary sinusoid over time, both primary currents and secondary phase and line currents vary non-sinusoid over time. This way results explicitly in the deforming (non-sinusoidal) regime in which the rectifier transformer of traction substation is operating. Looking forward the authors of this study intend to analyze the currents’ harmonics in the three-phase power transformer of a AC/DC traction substation.

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