

A Novel Robust Adaptive Backstepping Method Combined with SMC on Strict-Feedback Nonlinear Systems Using Neural Networks

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Abstract. Adaptive backstepping methodology is a powerful tool for nonlinear systems, especially for strict-feedback ones, but its robustness still needs improvements. In this paper, combined with sliding mode control (SMC), a new backstepping design method is proposed to guarantee the robustness. In this method, based on the novel combining method, the auxiliary controller is introduced only in the final step of the real controller, unlike traditional methods, which usually all include an auxiliary controller in every de-signing step to guarantee the robustness of the closed-loop systems. The novel combining methods can avoid calculating multiple and high-order derivatives of the auxiliary controllers in the intermediate steps, lowering the computational burden in evaluating the controller. The effectiveness of the proposed approach is illustrated from simulation results.

1 Introduction

In recent years, numerous adaptive and robust controller design approaches have been presented for nonlinear control systems. Among them, adaptive backstepping has become one of the famous design methodologies [1]-[3]. The backstepping design is a recursive procedure. Firstly, the system must be or be transformed into a special form, in which states are selected recursively and represent the input for lower dimension subsystems of the overall system. Then, you can select a function of state variables as a pseudo-controller, and let the output of the next subsystem to track it. Following this procedure step by step, the true feedback controller will result. Usually, a candidate sub-Lyapunov function can be achieved at each individual design stage, and the final Lyapunov function can be formed by summing them [1].

The backstepping design provides a systematic framework suitable for a large class of state feedback linearizable nonlinear systems. Besides the systematic approach, it has another important feature of ability to shape performance [1], [4]. So, a great quantity of research and applications on backstepping have emerged.

The early studies deal with systems in which the functions must be “linear in the unknown parameters” [5], [6], which limits applicability only to a limited set of systems. Furthermore, some very tedious analysis is needed to determine “regression matrices” [5], [7]. Impulse to developments in adaptive and robust control techniques is given by the use of neural networks (NNs) [8]-[22] or fuzzy systems [23-26]. Using neural networks and/or fuzzy systems, the linearity-in-the-parameter

assumption of nonlinear functions and the determination of regression matrices can be overcome. In recent years, the backstepping method has been also applied to some special nonlinear systems, such as delay-time, stochastic, saturation and some other systems [27-43], which greatly broadens the application fields of backstepping methodology.

Several papers give more attention about robustness while using backstepping design [23, 44-48]. Method in [23] can guarantee H^∞ performance for the nonlinear function with unknown parameters, even when parameters appear in nonlinear form. The method proposed in [44] just focuses on a special kind of nonlinear systems. For much more general systems, in which there are unknown dynamic disturbances, the relative backstepping methods are proposed [45-48]. However, an additional function is drawn in every virtual controller, and all include hyperbolic tangent function (i.e., \tanh) or similar, which increase the calculating burden of the closed-loop systems. Sliding mode control (SMC) method is one important robust technique for nonlinear systems [49]. Actually, every subsystem can be considered a SMC application in [45-48]. Besides the problem above mentioned, the higher order of the systems will require a much faster response speed for the controllers [46], such that the robustness and control accuracy may not be reached to some extent.

In this paper, a novel backstepping scheme combined with SMC method will be given, in which the auxiliary terms will be part of the final real controller only, so the high-order derivative of the auxiliary terms can be

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avoided and the degree of calculation complexity can be partially lowered.

The paper is organized as follows. In Section 2, the nonlinear system and radial basis function (RBF) neural networks are described briefly. A new adaptive backstepping design and stability of the closed-loop system is given in detail in Section 3, and simulated on a nonlinear system. Section 4 is where the simulation is studied. Finally, conclusions are given in Section 5.

2 Problem formulation

2.1 System Description

The model of many practical systems can be expressed as, or transformed into, the following strict-feedback form,

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + x_{i+1} \quad 1 \leq i \leq n-1 \\ \dot{x}_n &= f_n(\bar{x}_n) + u \quad n \geq 2 \\ y &= x_1 \end{aligned} \quad (1)$$

where,

1) $\bar{x}_i = [x_1, x_2, \dots, x_n]^T \in R^i$, $i = 1, 2, \dots, n$, $u \in R$ and $y \in R$ are state variables, system input and output, respectively; we also suppose that all the state variables can be measured or observed;

2) functions $f_i(\bar{x}_i) \in R$, $i = 1, 2, \dots, n$, are supposed to be unknown but smooth nonlinear functions.

The control objective is to design an adaptive NN controller for system (1) such that,

1) all the signals in the closed-loop remain semi-globally uniformly ultimately bounded;

2) the output y follows a desired trajectory y_c , that is bounded up to the $(m+1)$ th order with its derivatives.

Note that in the following derivation of the adaptive neural controller, NN approximation is only guaranteed with some compact sets. Accordingly, the stability results obtained in this work are semiglobal in the sense that, as long as desired, there exists controllers with sufficient large number of NN nodes such that all the signals in the closed-loop remain bounded.

2.2 Function Approximation Using Neural Networks

The controller design method proposed here does not depend on any specific neural network, which is just an approximator. In other words, you can adopt any suitable network. For convenience, we employ the radial basis function neural networks (RBF NN) in this paper. The key features of RBF NNs are the following:

The RBF NN has the general form $\hat{F}(\cdot) = \theta^T \xi(\cdot)$, where $\theta \in R^p$ is a vector of adjustable weights and $\xi(\cdot) \in R^p$ is a vector of RBF's. Their ability to uniformly approximate smooth functions over compact sets is well documented in the literature (for example [50]). In general, it has been shown that given a smooth function $F: \Omega \rightarrow R$, where Ω is a compact subset of R^m (m is

an appropriate integer), and $\varepsilon > 0$, there exists a RBF vector $\xi: R^m \rightarrow R^p$ and a weight vector $\theta \in R^p$ such that,

$$|F(x) - \theta^{*T} \xi(x)| \leq \varepsilon, \quad \forall x \in \Omega$$

The quantity $F(x) - \theta^{*T} \xi(x) = d_F(x)$ is called the network reconstruction error and obviously $|d_F(x)| \leq \varepsilon$.

The optimal weight vector θ^* defined above is typically chosen as the value of θ that minimizes $d_F(x)$ over Ω , that is

$$\theta^* = \arg \min_{\theta \in R^p} \left\{ \sup_{x \in \Omega} |F(x) - \theta^T \xi(x)| \right\} \quad (2)$$

The Gaussian functions are employed as basis functions, in the same form as in [51], which are located on a regular grid that contains the subset of interest of the state space.

3 Adaptive control

3.1 The New Robust Method Combined with Sliding Mode Control

The detailed design procedure is described in the following steps. For clarity and conciseness, Steps 1 and 2 are described with detailed explanations, while Step i and Step n are simplified, with the relevant equations and the explanations being omitted. Step 1: Let reference signal $x_{1d} = y_d$ and define $e_1 = x_1 - x_{1d}$. Its derivative is

$$\dot{e}_1 = f_1(\bar{x}_1) + x_2 - \dot{x}_{1d} \quad (3)$$

and x_2 is viewed as a virtual control input. We choose the controller as follows,

$$x_{2d} = x_2 = -f_1(\bar{x}_1) + \dot{x}_{1d} - ke_1 \quad (4)$$

where k is constant. Substituting (4) into (3), $\dot{e}_1 = -ke_1$ is obtained. So, it is easy to know that e_1 is asymptotically stable.

However, since the function $f_1(\bar{x}_1)$ is unknown, the desired controller cannot be implemented in practice. Instead, a NN-based virtual controller can be used as follows:

$$x_{2d} = -\theta_1^T \xi_1(\bar{x}_1) + \dot{x}_{1d} - ke_1 \quad (5)$$

where $\theta_1^T \xi_1(\bar{x}_1)$ is RBF NN used to approximate $f_1(\bar{x}_1)$.

Defining $e_2 = x_2 - x_{2d}$, \dot{e}_1 can be obtained as

$$\begin{aligned} \dot{e}_1 &= f_1(\bar{x}_1) + e_2 + x_{2d} - \dot{x}_{1d} \\ &= \theta_1^{*T} \xi_1(\bar{x}_1) - \dot{x}_{1d} + e_2 + x_{2d} + d_1 \end{aligned} \quad (6)$$

where θ_1^* is the optimal weight vector of $f_1(\bar{x}_1)$. The neural reconstruction error $d_1 = f_1(\bar{x}_1) - \theta_1^T \xi_1(\bar{x}_1)$ is bounded, i.e., there exists a constant $\varepsilon_1 > 0$ such that $|d_1| < \varepsilon_1$. Throughout the paper, we introduce $\theta_i^T \xi_i(\bar{x}_i)$ as neural networks and define their reconstruction errors as $d_i = f_i(\bar{x}_i) - \theta_i^T \xi_i(\bar{x}_i)$, where $i = 1, 2, \dots, n$. Like in the case of d_1 , d_i is bounded, i.e., $|d_i| < \varepsilon_i$.

Substituting (5) into (6), we get

$$\dot{e}_1 = \tilde{\theta}_1^T \xi_1(\bar{x}_1) - ke_1 + e_2 + d_1 \quad (7)$$

where $\tilde{\theta}_1 = \theta_1^* - \theta_1$. Throughout this paper, we shall define $\tilde{\theta}_i = \theta_i^* - \theta_i$.

Step 2: This step makes the error between x_2 and x_{2d} as small as possible. Differentiating gives

$$\begin{aligned} \dot{e}_2 &= f_2(\bar{x}_2) + e_3 + x_{3d} - \dot{x}_{2d} \\ &= \theta_2^{*T} \xi_2(\bar{x}_2) - \dot{x}_{2d} + e_3 + x_{3d} + d_2 \end{aligned} \quad (8)$$

Similarly, take x_{3d} as the virtual controller. We let x_{3d} be of the form

$$x_{3d} = -\theta_2^T \xi_2(\bar{x}_2) + \dot{x}_{2d} - (\lambda_1/\lambda_2)e_2 - ke_2 \quad (9)$$

In (9) and the following formulas, $\lambda_i \in R$ ($i=1,2,\dots,n$) are real numbers and we will show how to choose them in the following of this paper. Substituting (9) into (8), we will have

$$\dot{e}_2 = \tilde{\theta}_2^T \xi_2(\bar{x}_2) - (\lambda_1/\lambda_2)e_2 - ke_2 + e_3 + d_2 \quad (10)$$

Step i ($3 \leq i \leq n-1$): In a similar fashion, we can design a virtual controller $x_{(i+1)d}$ to make the error $e_i = x_i - x_{id}$ as small as possible. Differentiating e_i gives

$$\begin{aligned} \dot{e}_i &= f_i(\bar{x}_i) + e_{i+1} + x_{(i+1)d} - \dot{x}_{id} \\ &= \theta_i^{*T} \xi_i(\bar{x}_i) - \dot{x}_{id} + e_{i+1} + x_{(i+1)d} + d_i \end{aligned} \quad (11)$$

where $e_{i+1} = x_{i+1} - x_{(i+1)d}$. Similarly, let the virtual controller $x_{(i+1)d}$ to be of the form

$$x_{(i+1)d} = -\theta_i^T \xi_i(\bar{x}_i) + \dot{x}_{id} - (\lambda_{i-1}/\lambda_i)e_i - ke_i \quad (12)$$

Then we have

$$\dot{e}_i = \tilde{\theta}_i^T \xi_i(\bar{x}_i) - (\lambda_{i-1}/\lambda_i)e_i - ke_i + e_{i+1} + d_i \quad (13)$$

Step n : This is the final step. Differentiating the error $e_n = x_n - x_{nd}$, we will have

$$\begin{aligned} \dot{e}_n &= f_n(\bar{x}_n) + u - \dot{x}_{nd} \\ &= \theta_n^{*T} \xi_n(\bar{x}_n) - \dot{x}_{nd} + u + d_n \end{aligned} \quad (14)$$

Similarly, letting

$$u = -\theta_n^T \xi_n(\bar{x}_n) + \dot{x}_{nd} - (\lambda_{n-1}/\lambda_n)e_n - ke_n + u_q \quad (15)$$

where u_q is the auxiliary controller. Substituting (15) into (14), gives

$$\dot{e}_n = \tilde{\theta}_n^T \xi_n(\bar{x}_n) - (\lambda_{n-1}/\lambda_n)e_n - ke_n + u_q + d_n \quad (16)$$

In which, $\tilde{\theta}_i = \theta_i^* - \theta_i$ ($i=1,2,\dots,n$).

From (7), (10), (13) and (16), we can get

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \vdots \\ \dot{e}_i \\ \vdots \\ \dot{e}_n \end{bmatrix} = \begin{bmatrix} \tilde{\theta}_1^T \xi_1(\bar{x}_1) \\ \tilde{\theta}_2^T \xi_2(\bar{x}_2) \\ \vdots \\ \tilde{\theta}_i^T \xi_i(\bar{x}_i) \\ \vdots \\ \tilde{\theta}_n^T \xi_n(\bar{x}_n) \end{bmatrix} + \begin{bmatrix} e_2 \\ -(\lambda_1/\lambda_2)e_2 + e_3 \\ \vdots \\ -(\lambda_{i-1}/\lambda_i)e_i + e_{i+1} \\ \vdots \\ -(\lambda_{n-1}/\lambda_n)e_n \end{bmatrix}$$

$$-k \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_i \\ \vdots \\ e_n \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_i \\ \vdots \\ d_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ u_q \end{bmatrix} \quad (17)$$

Let

$$S = \sum_{i=1}^n \lambda_i e_i \quad (18)$$

in which $\lambda_i > 0 \in R$ ($i=1,2,\dots,n$) can be chosen suitably such that all the roots of $S=0$ are in the open left-half of the complex plane. So, the derivative of S is

$$\begin{aligned} \dot{S} &= \sum_{i=1}^n \lambda_i \dot{e}_i \\ &= -kS + \sum_{i=1}^n \lambda_i \tilde{\theta}_i^T \xi_i(\bar{x}_i) + \sum_{i=1}^n \lambda_i d_i + \lambda_n u_q \end{aligned} \quad (19)$$

Consider the Lyapunov function candidate

$$V = \frac{1}{2} S^2 + \frac{1}{2} \sum_{i=1}^n \tilde{\theta}_i^T \lambda_i \Gamma_i^{-1} \tilde{\theta}_i \quad (20)$$

where $\Gamma_i = \Gamma_i^T > 0$ ($i=1,2,\dots,n$) are adaptive gain matrices.

The derivative of V is

$$\begin{aligned} \dot{V} &= S\dot{S} - \sum_{i=1}^n \tilde{\theta}_i^T \lambda_i \Gamma_i^{-1} \dot{\tilde{\theta}}_i \\ &= -kS^2 + \sum_{i=1}^n \lambda_i \tilde{\theta}_i^T \left[S \xi_i(\bar{x}_i) - \Gamma_i^{-1} \dot{\tilde{\theta}}_i \right] \\ &\quad + S \left(\lambda_n u_q + \sum_{i=1}^n \lambda_i d_i \right) \end{aligned} \quad (21)$$

Consider the following adaptation laws:

$$\dot{\tilde{\theta}}_i = \Gamma_i \left[S \xi_i(\bar{x}_i) - \sigma_i |S| \tilde{\theta}_i \right] \quad (22)$$

where $\sigma_i > 0$ ($i=1,2,\dots,n$) are small constants.

Formulas (22) are so-called σ -modification, introduced to improve the robustness in the presence of the approximation error d_i and avoid the weight parameters to drift to very large values.

Substituting (22) into (21), we get

$$\begin{aligned} \dot{V} &= -kS^2 + |S| \sum_{i=1}^n \lambda_i \sigma_i \tilde{\theta}_i^T \tilde{\theta}_i + S \lambda_n u_q + S \sum_{i=1}^n \lambda_i d_i \\ &= -kS^2 + |S| \sum_{i=1}^n \lambda_i \sigma_i \tilde{\theta}_i^T (\theta_i^* - \tilde{\theta}_i) + S \lambda_n u_q + S \sum_{i=1}^n \lambda_i d_i \end{aligned} \quad (23)$$

According to Schwartz inequality, we have

$$\tilde{\theta}_i^T (\theta_i^* - \tilde{\theta}_i) \leq \|\tilde{\theta}_i\| \|\theta_i^* - \tilde{\theta}_i\| \quad (24)$$

In which, $\|\cdot\|$ is the Euclidian norm of a vector.

Additionally, if we let

$$u_q = -\sum_{i=1}^n \lambda_i L_i \text{sgn}(S) \quad (25)$$

Where $L_i > 0$ is an undetermined parameter and $\text{sgn}(\cdot)$ is the sign function. Then, substituting (24) and (25) into (23), we have

$$\begin{aligned} \dot{V} &\leq -kS^2 + |S| \sum_{i=1}^n \lambda_i \sigma_i (\|\tilde{\theta}_i\| \|\theta_i^*\| - \|\tilde{\theta}_i\|^2) \\ &\quad - |S| \lambda_n L + |S| \sum_{i=1}^n \lambda_i |d_i| \\ &= -kS^2 - |S| \sum_{i=1}^n \lambda_i \left[\sigma_i (\|\tilde{\theta}_i\| - \|\theta_i^*\|/2)^2 \right. \\ &\quad \left. - \sigma_i \|\theta_i^*\|^2 / 4 - |d_i| + \lambda_n L_i \right] \end{aligned} \quad (26)$$

Obviously, if we let

$$L_i \geq \frac{1}{\lambda_n} \left(\sigma_i \|\theta_i^*\|^2 / 4 + |d_i| \right) \quad (27)$$

then there is $\dot{V} \leq 0$. In other words, parameter S is semiglobally asymptotic stable. Form (18), we can easily know that e_i ($i=1,2,\dots,n$) are semiglobally asymptotic stable, too.

Obviously too, if for any $i=1,2,\dots,n$

$$\|\tilde{\theta}_i\| \geq \|\theta_i^*\|/2 + \sqrt{\|\theta_i^*\|^2 / 4 + |d_i| / \sigma_i} \quad (28)$$

there is $\dot{V} \leq 0$ again. So, according to a standard Lyapunov theorem extension [51], the weights θ_i ($i=1,2,\dots,n$) of the RBF NNs are uniformly ultimately bounded (UUB)[51].

3.2 Remarks

Remark 1: In the stability analysis of the two methods, the term “semi-globally asymptotically stable” means that the NNs can only approximate the nonlinear function in some compact set. If NNs can approximate the nonlinear function in a large enough space the system states, we can say the approaches are both “global” asymptotic stable.

Remark 2: For a much more general system, its subsystem can be written as follows

$$\dot{x}_i = f_i(\bar{x}_i) + x_{i+1} + \Delta_i(x, t)$$

where $\Delta_i(x, t)$ $i=1,2,\dots,n$ are unknown bounded dynamic disturbances of systems:

- 1) if $|\Delta_i(x, t)| \leq \delta_i$, we can let $L_i \leftarrow L_i + \delta_i$ in (27);
- 2) if $|\Delta_i(x, t)| \leq \delta_i |\psi_i(\bar{x}_i)|$, we can let $L_i \leftarrow L_i + \delta_i |\psi_i(\bar{x}_i)|$ in (27), too.

Obviously, with the simple substitution, we can guarantee that the closed-loop systems are also stable, i.e., the methods presented in this paper can suit much more general nonlinear systems with just simple modifications.

Remark 3: Although the SMC method can guarantee robustness, it has the drawback of chattering. However, you can adopt any suitable chattering reduction approaches to resolve it.

4 Simulation

In this paper, the model of the system is given as

$$\begin{aligned} \dot{x}_1 &= 0.1x_1^2 + x_2 \\ \dot{x}_2 &= 0.2e^{-x_2} + x_1 \sin(x_2) + u \\ y &= x_1 \end{aligned} \quad (32)$$

where x_1 and x_2 are states, which can all be observed, and y is the output of the system, respectively. Initial conditions are $\bar{x}_0 = [x_{10}, x_{20}]^T = [1, 0]^T$ and $y_d = \sin(t/2)$ is the desired reference signal. All the basis functions of the NNs in this example are of the form [52]

$$\xi_{ij}(\bar{x}_i) = \exp \left[-\frac{(\bar{x}_i - \bar{u}_j)^T (\bar{x}_i - \bar{u}_j)}{v_j^2} \right] \quad (j=1,2,\dots,l_i) \quad (33)$$

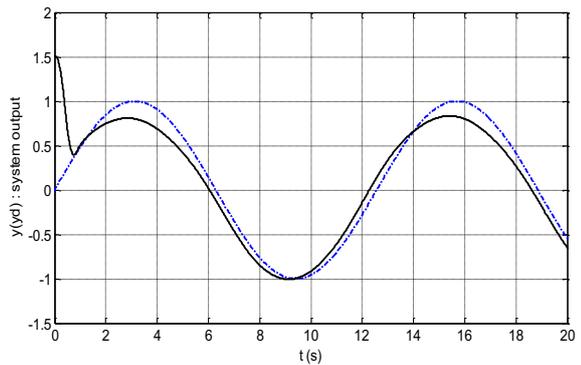
In which, u_j are the centers of the receptive fields and v_j are the widths of the Gaussian function. The NN $\theta_1^T \xi_1(\bar{x}_1)$ includes 13 nodes (i.e., $l_1=13$), whose centers u_j are evenly spaced in $[-6, 6]$, and the NN $\theta_2^T \xi_2(\bar{x}_2)$ contains 169 nodes (i.e., $l_2=169$), with centers u_j evenly spaced in $[-6, 6] \times [-6, 6]$. And all the widths v_j of the NNs are 0.5. The design parameters of the controller are $\Gamma_1 = \Gamma_2 = \text{diag}\{2\}$, $\sigma_1 = \sigma_2 = 0.2$, $\lambda = [4, 1]^T$, $k=2$ and $L_1 = L_2 = 5$. The sign function is replaced by saturation function. The initial weights θ_1 and θ_2 are all given arbitrarily in $[-1, 1]$. Figs. 1 and 2 show the simulation results of applying controller (22), (25) and (29) for tracking desired signal y_d .

We can see that good tracking performance is obtained, especially the time t is large enough. From Fig. 1, we can draw a conclusion that there is a long-term transient response process for the system output to track the desired signal, and the rate of convergence can be improved by selecting the control parameters appropriately. Fig. 2 shows the trajectory of the controller. During simulation, especially due to the finite difference approximation of the derivatives, the input may be huge at the first several steps. To overcome this, you can limit its magnitude. In the simulation of this paper, for example, we let $|u| \leq \pm 8.0$. The fine tuning of the controller is out of scope in the present paper, yet the results show the effectiveness of the structure of the controller proposed.

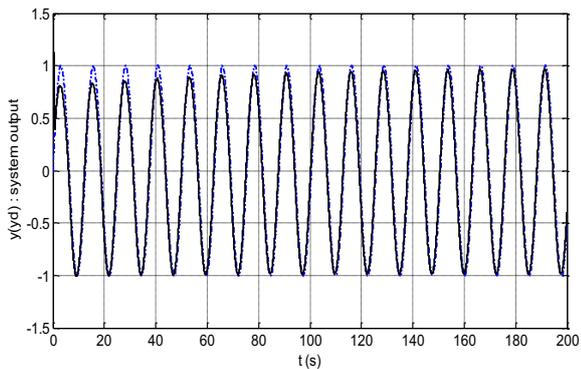
5 Conclusion

In this paper, a adaptive backstepping NN control approaches combined with SMC method are presented for a class of strict-feedback nonlinear systems with unknown nonlinearities. They guarantee the robustness against the unknown bounded dynamic disturbances. Unlike conventional backstepping methods, in the current methods one auxiliary term is introduced only in the final real controller, which can avoid calculating the high-order derivative of the auxiliary item. Therefore, they can both lower the degree of calculation complexity to some

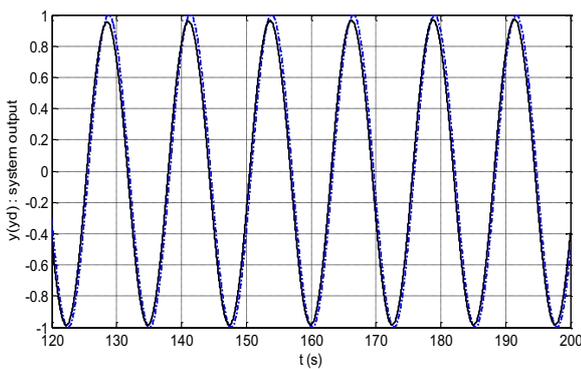
extent. Simulated results demonstrate the effectiveness of the proposed approaches.



a) $t \in [0, 20]$

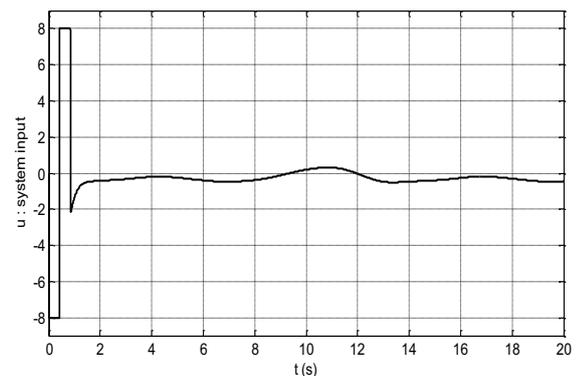


b) $t \in [0, 200]$

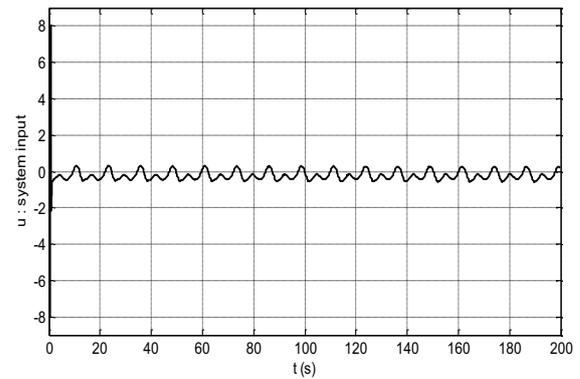


c) $t \in [0, 20]$

Figure 1. The output of the system under control



a) $t \in [0, 20]$



b) $t \in [0, 200]$

Figure 2. The trajectory of system input

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