

# Research on solar radiation potential as a source of renewable energy in the Motru area

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**Abstract.** The continuous development of the human society is in a close correlation with the increased consumption of electricity. In the past few years, the thirst for electricity at a global level, has led both to higher levels of pollution and the depletion of fossil fuels. For this reason, the issue of energy efficiency is a priority of energy strategy at national and EU level. In this context, a solution to the above problems is to increase the rate of production of electricity from renewable energy sources. Thus, the paper presents a study of solar radiation as a source of renewable energy, modelling and simulation of photovoltaic systems and validation of data numerical simulations data with the measured data in the Motru area.

## 1 Introduction

Study, modelling and simulation of a photovoltaic system involves conducting several steps. An essential step in the correct sizing of a photovoltaic system, is to estimate the solar radiation. In this context, the paper aims to highlight the main mathematical models used to estimate the direct sunlight radiation that falls on a horizontal plan, under a clear sky. The mathematical models of sunshine are customized for the city of Motru. The validation of these models shall be based on data provided by Voltcraft PL-110SM Pyranometer.

At the end of the article the program Matlab - Simulink simulation of solar radiation is presented, estimated by using the Haurwitz model.

## 2 Mathematical modelling of solar radiation

For the mathematical modelling of solar radiation, we will define the following angles:

a) the angle of declination ( $\delta$  [degree]) is determined by the following formula:

$$\delta = 23.45^\circ \cdot \sin\left(\frac{360^\circ \cdot (n-81)}{365}\right) \quad (1)$$

where:  $n$  represents the number of days in a year.

b) the hour angle ( $\omega$  [degree]) is calculated by the following formula

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$$\omega = 15^\circ \cdot (T_s - 12) \tag{2}$$

where  $T_s$  is the local solar time.

c) the altitudinal angle ( $\omega$  [degree]), the calculation formula of the altitudinal angle is:

$$\sin(\alpha) = \sin(\delta) \sin(\varphi) + \cos(\delta) \cos(\varphi) \cos(\omega) \tag{3}$$

where  $\varphi$ [degree] is the latitude (the angle between the Equator and the geographical location of interest),  $\delta$ [degree] is the angle of declination and  $\omega$ [degree] the angle of the zone.

Based on the altitudinal angle, the zenith angle can be calculated:

$$\zeta = 90^\circ - \alpha \tag{4}$$

The linking relation between the local solar time and local time is based on the equation of time (EOT). The equation of time is defined by the following relation:

$$EOT(n) = 9.87 \cdot \sin(2 \cdot b(n)) - 7.53 \cdot \cos(b(n)) - 1.5 \cdot \sin(b(n)) \tag{5}$$

where:

$$b(n) = \frac{360^\circ \cdot (n - 81)}{365} \tag{6}$$

where  $n$  represents the number of days in a year.

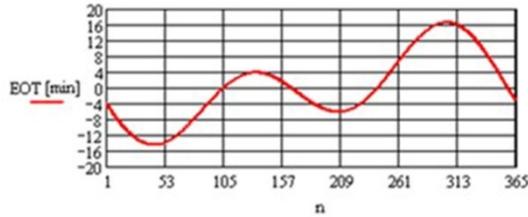


Figure 1 represents the equation of time versus the number of days in 2017.

**Fig. 1.** Changes in relation to the number of days in a year (2017), the equation of time

In this context, the linking relationship between the local solar time ( $T_s$ ) and the local ( $T_L$ ) is:

$$T_s = T_L + \frac{T_C}{60} \tag{7}$$

where:

- local time ( $T_L$ ) is calculated by the following formula:

$$T_L = \frac{\text{hour} \cdot 3600 + \text{min} \cdot 60 + s}{3600} \tag{8}$$

where the "hour", "min" and "s" are hour, minute and second that define the local time.

- the time correction factor ( $T_C$ ) is calculated by the following formula:

$$T_C = 4 \cdot (L_o - L_m) + EOT(n) \tag{9}$$

where  $L_o$  [degree] is the longitude of the place and  $L_m = 15^\circ \cdot \Delta_{GMT}$  is standard local time based on the first meridian, where  $\Delta_{GMT}$  is the difference between local and GMT (Greenwich Mean Time).

The moments at which the sunrise and the sunset are calculated using the following relation:

$$T_R = 12 - \frac{1}{15^\circ} \cdot \arccos(-\text{tg}(\varphi) \cdot \text{tg}(\delta)) - \frac{T_C}{60} \quad (10)$$

$$T_A = 12 + \frac{1}{15^\circ} \cdot \arccos(-\text{tg}(\varphi) \cdot \text{tg}(\delta)) - \frac{T_C}{60} \quad (11)$$

where  $\varphi$ [degree] is the latitude,  $\delta$ [degree] is the angle of declination,  $T_C$  is the factor of time correction,  $T_R$  is the time Sun rises, and  $T_A$  is the time Sun sets.

Considering these, we will present below the most known mathematical models for estimating the solar radiation of the globe, valid under a clear sky [1-3]:

1. the mathematical model Kasten - Czeplak is defined by the following relation:

$$Q = 910 \cdot \sin(\alpha) - 30 \quad (12)$$

where  $\alpha$  is the altitudinal angle.

2. the mathematical model of Adnot is defined by the following relationship:

$$Q = 915.39 \cdot [\sin(\alpha)]^{1.15} \quad (13)$$

where  $\alpha$  is the altitudinal angle.

3. the Haurwitz mathematical model is given by the following equation:

$$Q = 1098 \cdot e^{-\frac{0.057}{\sin(\alpha)}} \cdot \sin(\alpha) \quad (14)$$

where  $\alpha$  is the altitudinal angle.

4. the EIM model (Empirical Irradiance Model) is defined by the following relation:

$$Q = Q_0 \cdot (1 - 0.4645 \cdot e^{-0.69 \cdot \sin(\alpha)}) \cdot e^{-\frac{0.05211}{\sin(\alpha)}} \sin(\alpha) \quad (15)$$

where  $\alpha$  is altitudinal angle and  $Q_0$  is given by the following relation:

$$Q_0 = I_{s0} \cdot \left(1 + 0.033412 \cdot \cos \frac{2\pi \cdot (n-a)}{365}\right) \quad (16)$$

where:  $I_{s0} = 1361$  [W/m<sup>2</sup>] is the sun constant,  $n$  is the number of days in a year, and  $a$  is the day in which the perihelion takes place ( $a=4$ ; January 4).

The model given by relation (16), was developed by Paulescu and Schett (2004) [4] and is based on the data obtained from the meteorological station in Timișoara, under a clear sky [5].

5. the Kasten model is defined by the following relation:

$$Q = 0.84 \cdot I_{s0} \cdot \cos(\zeta) \cdot e^{-0.027 \cdot AM \cdot (f_1 + f_2 \cdot (T_L - 1))} \quad (17)$$

where:  $I_{s0} = 1361$  [W/m<sup>2</sup>] is the solar constant,  $\zeta$  is the zenith angle,  $AM$  is the air mass ratio, [2],  $f_1 = e^{-\frac{\alpha}{8000}}$ ,  $f_2 = e^{-\frac{\alpha}{1250}}$ ,  $\alpha$  is the altitudinal angle, and  $T_L$  is Linke turbidity factor.

6. the Perez model is defined by the following relation:

$$Q = g_1 \cdot I_{s0} \cdot \cos(\zeta) \cdot e^{-g_2 \cdot AM \cdot (f_1 + f_2 \cdot (T_L - 1))} \cdot e^{-0.01 \cdot AM^{1.8}} \quad (18)$$

where:  $g_1 = 5.09 \cdot 10^{-5} \alpha + 0.868$ ,  $g_2 = 3.92 \cdot 10^{-5} \alpha + 0.0387$ .

The most popular formulas of the air mass coefficient (*AM*), which intervenes the relations (17) and (18) are:

- the classic formula that defines the air mass factor:

$$AM = \frac{1}{\cos(\zeta)} = \sec(\zeta) \tag{19}$$

- Kasten and Young formula (1989):

$$AM = [\cos(\zeta) + a \cdot (96.07995 - \zeta)^{-1.6364}]^{-1} \tag{20}$$

where  $a = 0.50572$ .

In the relations (17) and (18), the air mass coefficient is calculated using the formula of Kasten and Young (20). This formula is one of the most used in the literature to determine the air mass coefficient, [6-8].

In the following lines we will customize the shown above mathematical models, for Motru area.

Motru municipality is located in the southwest part of the County of Gorj from Romania, at the dividing line between the Mehedinti and Gorj County. The municipality is located on route DN 67A, at 44 km from Targu-Jiu and at 42 km from Drobeta Turnu Severin. The coordinates of the city are: latitude [degrees] is 44.80 and longitude [degree] is 22.98.

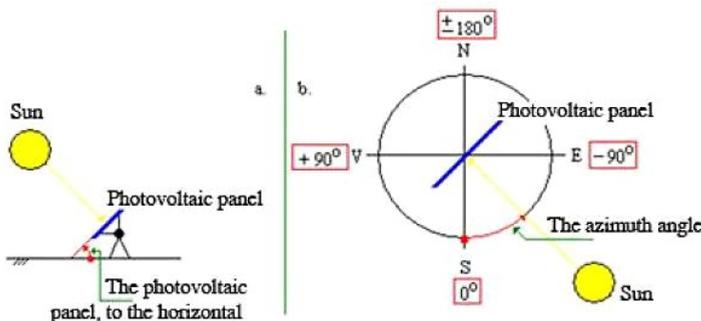
### 3 Simulation of solar radiation

The program developed by Diaconu, Nan and Stoicuta (2017), [2], for Petrosani depression was customized for the Motru area. This program has been optimized to get the maximum efficiency of a photovoltaic panel and is presented below.

Peak electricity what is produced by a photovoltaic module, at a certain moment of time is obtained when the amount of solar radiation absorbed by the photovoltaic panel, is the maximum. In addition to the constructive aspects, this can improve by two methods:

- fixed positioning and optimal compared to the Sun, the photovoltaic panel;
- through a Sun tracking system, the photovoltaic panel.

The optimum position of a photovoltaic module towards the Sun is defined according to two angles: tilt angle of the photovoltaic panel, to the horizontal and the azimuth angle (the angle towards the direction N-S). The two angles above, are highlighted in Figure 2.



**Fig. 2.** The tilt angle (a.) and the azimuth angle (b.)

The azimuth angle  $\psi$  [degree] is calculated based on the following relationships:

$$\psi = \arccos \left[ \frac{\sin(\delta) \cdot \cos(\varphi) - \cos(\delta) \cdot \sin(\varphi) \cdot \cos(\omega)}{\cos(\alpha)} \right] - 180^\circ \tag{21}$$

where:  $\delta$  is the angle of declination,  $\omega$  is the hour angle and  $\alpha$  is the altitudinal angle.

The incidence angle  $\alpha_a$  [degree] is defined as the angle between the perpendicular to the photovoltaic panel plane and the solar ray direction:

$$\cos(\alpha_a) = \sin(\delta) \cdot \sin(\varphi) \cdot \cos(\beta) - \sin(\delta) \cdot \cos(\varphi) \cdot \sin(\beta) \cdot \cos(\psi) + \cos(\delta) \cdot \cos(\varphi) \cdot \cos(\beta) \cdot \cos(\omega) + \cos(\delta) \cdot \sin(\varphi) \cdot \sin(\beta) \cdot \cos(\psi) \cdot \cos(\omega) + \cos(\delta) \cdot \sin(\psi) \cdot \sin(\omega) \cdot \sin(\beta) \tag{22}$$

where:  $\varphi$  is latitude,  $\delta$  the angle of declination,  $\omega$  the hour angle,  $\psi$  is the azimuth angle and  $\beta$  is the tilt angle of the photovoltaic panel from the horizontal.

Direct solar radiation that falls on the solar panel, under a clear sky, it is estimated based on the following relationships:

$$S_p = S \cdot \frac{\cos(\alpha_a)}{\sin(\alpha)} \tag{23}$$

Diffuse solar radiation intercepted by the photovoltaic panel  $D_p$ , is:

$$D_p = D \cdot \frac{1+\cos(\beta)}{2} \tag{24}$$

On the other hand, solar radiation reflected and intercepted by the photovoltaic panel, it can be estimated based on the following expression:

$$R_p = A \cdot (S + D) \cdot \frac{1-\cos(\beta)}{2} \tag{25}$$

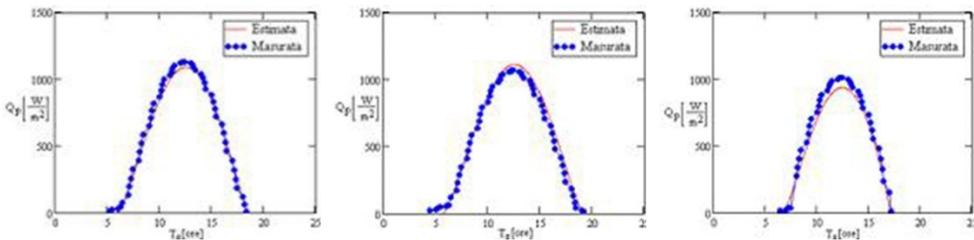
where:  $A$  is albedo it ranging between 0.1 to a paved road and 0.9 to snow.

Based on the above, global solar radiation falls on the photovoltaic panel, under a clear sky, is calculated as follows:

$$Q_p = S_p + D_p + R_p \tag{26}$$

To validate the mathematical model (26), using Voltcraft PL-110SM pyranometer, [9], was achieved global solar radiation intensity measurement from Motru city, which fell on the photovoltaic panel TPS-103, produced by Conrad, from Germany.

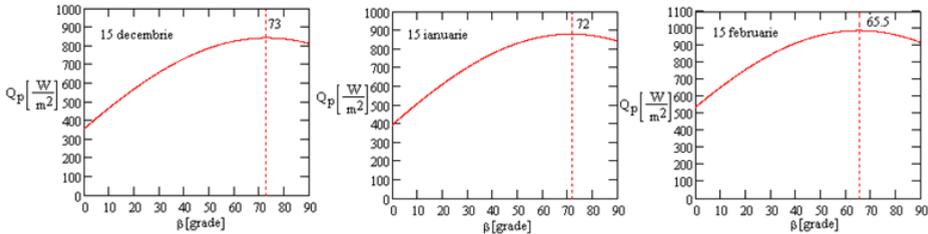
Global solar radiation has been measured, 16 April, 20 July and 15 October 2017, under the following conditions: tilt angle  $\beta = 36$  [degree] and azimuth angle  $\psi = 20$  [degree]. Based on the values within Figure 3, intensity of solar radiation that falls on a global photovoltaic module, measured and assessed, under the conditions outlined above, [10].



**Fig. 3.** Solar radiation measured/estimated – Motru 16 April, 20 July and 15 October 2017

Albedo used in estimating global solar radiation, for April and July was 0.5 and for October by 0.7. To determine the optimal position of the photovoltaic panel towards the Sun, in the following we will plot the variation of solar global radiation (26), in relation to the angle of inclination of the photovoltaic panel, for the city of Motru.

For the winter months, we plot the variation of global solar radiation in relation to the tilt angle, Figure 4, for the following days: December 15, ( $n = 349$ ), January 15, ( $n = 15$ ), and February 15, ( $n = 46$ ). The solar time and the azimuth angle used are:  $T_s = 12$  and  $\psi = 0$  [degree]. Albedo used in estimating global solar radiation during winter months, was 0.8.



**Fig. 4.** The variation of global solar radiation in relation to the tilt angle, in winter months

Similarly, for the spring, summer and autumn months, the overall solar radiation variation in relation to the tilt angle for the following days is graphically analysed: March 15<sup>th</sup>, ( $n = 74$ ), April 15<sup>th</sup>, ( $n = 105$ ), May 15<sup>th</sup>, ( $n = 135$ ), June 15<sup>th</sup>, ( $n = 166$ ), July 15<sup>th</sup>, ( $n = 196$ ), August 15<sup>th</sup>, ( $n = 227$ ), September 15<sup>th</sup>, ( $n = 258$ ), October 15<sup>th</sup>, ( $n = 288$ ) and November 15<sup>th</sup>, ( $n = 319$ ). The conditions under which the tests were performed, are the same as in the winter months, except that albedo used is 0.7 for the months of spring and autumn and during the summer albedo used is 0.5.

From the analysis of extreme points of the functions obtained, abscissas (tilt angles) for producing the maximum value to global solar radiation for all months of the year, are presented in table 1.

**Table 1.** The optimal values of the tilt angle.

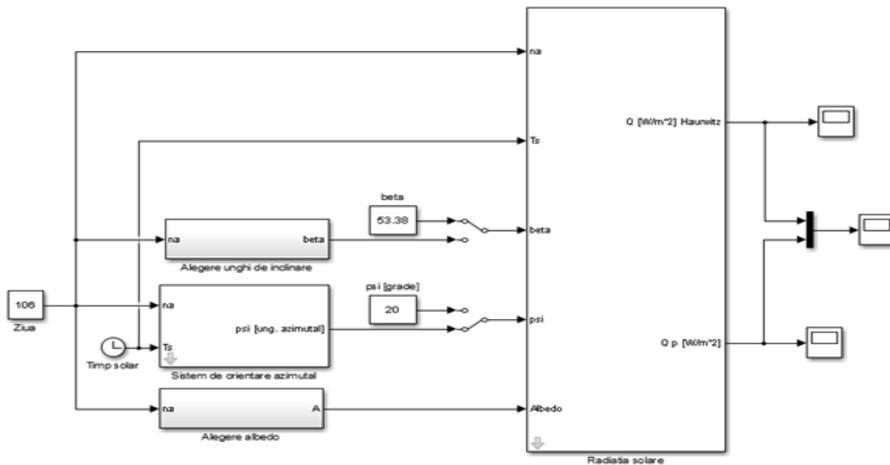
No.	Season	Month	Day	The tilt angles $\beta$ [degree]	The average value for season, the tilt angle [degree]
1	Winter	December	15	73	70.17
2		January		72	
3		February		65.5	
4	Spring	March	15	59	48
5		April		48	
6		May		37	
7	Summer	June	15	28	32
8		July		30	
9		August		38	
10	Autumn	September	15	54	63.33
11		October		64	
12		November		72	
13	The average annual value of the tilt angle [degrees]				53.38

From those shown in table 1, it is observed that the fixed positioning and optimum towards the Sun of a photovoltaic module, in the municipality of Motru, can be done at an angle  $\beta = 53.38^\circ$ .

In order to control the angle of inclination, control systems are used to position the tilt angle of the photovoltaic panel, for which the element of inclination angles provide prescribing, either once a month, either once per season. Items, the most widely used in controlling the angle of inclination of a photovoltaic module, are developed around DC actuators and related drivers [11-13].

For an optimal positioning of the photovoltaic panel towards the Sun, in addition to the adjustment system of the tilt angular position, it is normally used, and a system for adjusting the angular position of bearing of the photovoltaic panel. Runtime element in the system of adjusting the angular position of bearing, has photovoltaic panel provides bearing based on the relationship of angles (21).

Based on the above, a program in Matlab-Simulink, [10], which aims to simulate solar radiation falls in the photovoltaic pane, for the municipality of Motru (see Figure 5). Within Matlab-Simulink, was achieved and simulation system for optimal positioning of the photovoltaic panel. For the simulation of the adjusting system of the azimuth angle was taken in consideration the relationship (21), and for adjusting the tilt angle was to take into account the values in column 6 of Table 1. In the simulation, adjustment systems are considered to be ideal from the point of view of dynamic entry (the size adjustment system, supplied by the prescribing, is identical to the output size, set size adjustment system automatic adjustment system is considered to be an element of proportional representation).

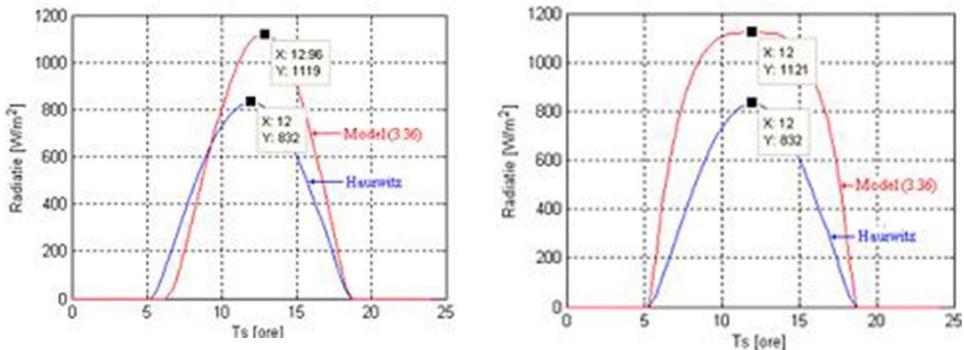


**Fig. 5.** Program Matlab-Simulink to solar radiation that falls on a photovoltaic module

In order to implement the program in Figure 5, the Matlab – Simulink program, the following settings are made:

- The simulation range is  $t \in [t_0, t_f] = [0, 24]$ ;
- The integration method is Dormand - Prince of the order 4/5;
- The relative error of the integration method:  $\varepsilon_r = 10^{-7}$ ;
- The absolute error method of integration:  $\varepsilon_a = 10^{-7}$ ;
- The latitude is set up:  $\phi = 44.80$  [degree];
- The day is fixed (for the program in Figure 5, day is April 16<sup>th</sup>, 2017;  $n = 106$ ).

As a result of running the program in Figure 5, get information on global solar radiation falls in the photovoltaic panel (global solar radiation falls in the horizontal plane - Haurwitz model; and global solar radiation falls in the plane of the photovoltaic panel, defined by the relationship (26). The simulation results are shown in Figure 6.



**Fig. 6.** Global solar radiation. a)  $\beta=53.38$ [degree] ;  $\psi=20$ [degree]; Albedo: 0.7; b)  $\beta=53.38$ [degree],  $\psi$  controlled on the basis of the relationship (21)

Analysing the graphs in Figure 6, there are the following conclusions with respect to the solar radiation on April 16<sup>th</sup>, 2017, from Motru area:

- The maximum value of solar radiation,  $Q=832 \left[ \frac{W}{m^2} \right]$ , estimated by Haurwitz model, is obtained at  $T_s = 12$  [hours].
- Where the azimuth angle is  $\psi = 20$  [degree], the maximum value of solar radiation, estimated by the model (26), cannot be obtained at  $T_s = 12$  [hours], but is obtained  $T_s \cong 13$  [hours]. The maximum amount of solar radiation is  $Q_p=1119 [W/m^2]$ , for at tilt angle  $\beta = 53.38$  [degree]
- Where the azimuth angle is controlled by a control system, based on the relationship (21), it is observed that the maximum amount of solar global radiation,  $Q_p=1121 [W/m^2]$ , is obtained at  $T_s = 12$  [hours], for both the Haurwitz model and the model defined by the relationship (26), for  $\beta = 53.38$  [degree].

## 4 Conclusion

Following the comparative analysis between solar radiation values estimated using a mathematical model, shown above, and the solar radiation values obtained experimentally using Voltcraft PL-110SM pyranometer, it was observed that Haurwitz model, estimates the best global solar radiation, in the city of Motru, that falls on a horizontal plane, under a clear sky.

Peak electricity what is produced by a photovoltaic module, at a certain moment of time is obtained when the amount of solar radiation absorbed by the photovoltaic panel, is the maximum. Global solar radiation intensity at a given moment of time is higher for optimal positioning of a photovoltaic module, only if its use.

With help of the software simulation presented in this article, photovoltaic systems designers, can obtain with an accuracy quite good, the global solar radiation falling on a horizontal plane, under a clear sky, in the city of Motru. Simulation program, shown in Figure 5, can also be used for other localities as well as the simulation of a photovoltaic system.

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