

On the longitudinal vibration of the railway bogie

Mădălina Dumitriu^{1,*} and Marius Alin Gheți¹

¹University Politehnica of Bucharest, Department of Railway Vehicles, 313 Splaiul Independenței, Bucharest, Romania

Abstract. This paper examines the influence of the rebound longitudinal motion of the bogie upon the vibration behaviour in the bogie. The analysis is based on the results of the numerical simulations developed on the basis of a general model with a nine-degree freedom of the bogie-track system, which takes into account the rigid vibration modes of the bogie – bounce, pitch and rebound, vertical and longitudinal displacements of the wheels, vertical displacements of the rails, and the wheel-rail contact elasticity, and a simplified model that does not consider the bogie longitudinal vibrations of rebound. The bogie vibration behaviour is assessed based on the power spectral density of the vertical acceleration and root mean square of vertical acceleration. It is highlighted the increase of the vibration level of the bogie due to the coupling effect between the pitch and rebound longitudinal vibrations, which, however, does not exceed 3-4% in terms of acceleration.

1 Introduction

The railway vehicle is a complex oscillating system, which presents a vibration behaviour with specific characteristics, mainly generated by irregularities of the track and defects of the wheel/rail rolling surfaces [1 - 6]. The oscillating motions of the railway vehicle develop as translation and rotation motions, independent or coupled among them. The oscillating movements of the railway vehicle result from overlapping of the rigid vibration modes (simple modes) of the suspended masses of the vehicle and the structural vibration modes (complex modes) [7 - 9]. The rigid vibration modes of the suspended masses of the vehicle in the vertical plan are: bounce (vertical translation motion), pitch (vertical rotation motion) and rebound (longitudinal translation motion).

Generally, in the studies from the speciality literature, only the bounce and pitch vertical vibrations of the bogie are considered to study the behaviour vibration of the bogie [10 - 12]. Due to the fact that the primary suspension corresponding to each axle normally uses identical elastic and damping components, the bounce and pitch vertical vibrations of the bogie are decoupled [13]. However, the pitch and the rebound vibrations of the bogie are coupled each other. On the other hand, the pitch and the rebound of the bogie are coupled

* Corresponding author: madalinadumitriu@yahoo.com

with the bending of the carbody, and this fact causes the increase of the vibration level of the carbody with potential unfavourable effects upon the ride quality and ride comfort.

In this paper, the influence of the rebound longitudinal motion of the bogie upon the vibration behaviour of the bogie is analysed. The analysis is based on the results of the numerical simulations developed on the basis of a general model with nine-degree of freedom of the bogie-track system, that takes into account the three rigid vibration modes of the bogie, vertical and longitudinal displacements of the wheels, vertical displacements of the rails, and the wheel-rail contact elasticity, and simplified model that does not consider the bogie longitudinal vibrations. The bogie vibration behaviour is assessed based on the dynamic response of the bogie, expressed as the power spectral density of the vertical acceleration and root mean square of vertical acceleration.

2 The mechanical model of the bogie-track

To analyze the influence of the longitudinal vibrations upon the behaviour of vibrations in a bogie which moving at a constant velocity V on a track with vertical irregularities, the bogie-track system model shown in the Figure 1 is considered.

The bogie model includes 3 rigid bodies that help with modelling the bogie chassis and the two axles connected between them by Kelvin-Voigt type systems that model the primary suspension corresponding to each axle. The primary suspension is modelled via two Kelvin-Voigt systems for translation on the vertical and longitudinal direction, respectively. The elastic element of the vertical primary suspension has the constant $2k_{zb}$ and the damping element has the constant $2c_{zb}$. The Kelvin-Voigt longitudinal system, located in the axles plan, at dimension h_b from the centre of gravity of the bogie chassis, with the elastic constant $2k_{xb}$ and damping constant $2c_{xb}$, is modelling the system of the elastic steering of the axles.

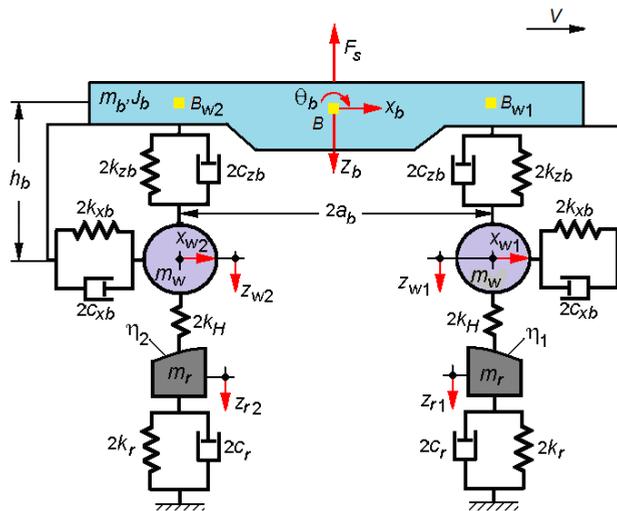


Fig. 1. The mechanical model of the bogie-track system.

The rigid vibration modes of the railway bogie in the vertical plan are: bounce (vertical translation motion) z_b , pitch (vertical rotation motion) θ_b , and rebound (longitudinal translation motion) x_b . The bogie parameters are mass of the bogie (m_b), wheelbase ($2a_b$) and moment of inertia (J_b). The wheelsets of mass m_w have two degrees of freedom, thus generating a vertical translation motion $z_{w1,2}$, and a longitudinal translation motion $x_{w1,2}$.

Against each axle, the track is represented as an oscillating system with one degree of freedom that can travel on the vertical direction, with the displacement $z_{r1,2}$. The equivalent model of the track has mass m_r , stiffness $2k_r$, and the damping coefficient $2c_r$. The vertical irregularities of the track are described against each axle via the functions $\eta_{1,2}$.

The equations of the motion of bounce, pitch and rebound are as below:

$$m_b \ddot{z}_b = \sum_{i=1}^2 F_{bi} - F_s ; \quad (1)$$

$$J_b \ddot{\theta}_b = a_b \sum_{i=1}^2 (-1)^{i+1} F_{zbi} - h_b \sum_{i=1}^2 F_{xbi} , \quad (2)$$

$$m_b \ddot{x}_b = \sum_{i=1}^2 F_{xbi} , \quad (3)$$

where F_s represents the force due to the secondary suspension of the railway vehicle, F_{zbi} are the forces from the primary suspension of the axles i , (for $i = 1, 2$), and F_{xbi} the forces derived from the system of the axles elastic guidance,

$$F_{zbi,2} = -2c_{zb1,2}(\dot{z}_b \pm a_b \dot{\theta}_b - \dot{z}_{w1,2}) - 2k_{zb}(z_b \pm a_b \theta_b - z_{w1,2}) , \quad (4)$$

$$F_{xbi,2} = -2c_{xb}(\dot{x}_b - h_b \dot{\theta}_b - \dot{x}_{w1,2}) - 2k_{xb}(x_b - h_b \theta_b - x_{w1,2}) . \quad (5)$$

The equations of vertical translation motion and longitudinal translation motion for the wheelsets are:

$$m_w \ddot{z}_{w1,2} = 2Q_{d1,2} - F_{b1,2} , \quad (6)$$

$$m_o \ddot{x}_{o1,2} = -F_{xbi} , \quad (7)$$

where $Q_{d1,2}$ are the dynamic contact forces; it is assumed that the dynamic contact forces on both wheels of a wheelset are equal.

For determining the dynamic contact forces, the hypothesis of the linear Hertzian theory is applied to the wheel-rail contact:

$$Q_{d1,2} = -k_H [z_{w1,2} - z_{r1,2} - \eta_{1,2}] , \quad (8)$$

where k_H represents the stiffness of the wheel-rail contact.

The vertical motion of the rails against the axle is described by the equations:

$$m_r \ddot{z}_{r1,2} = F_{r1,2} - 2Q_{d1,2} , \quad (9)$$

with

$$F_{r1,2} = -2c_r \dot{z}_{r1,2} - 2k_r z_{r1,2} . \quad (10)$$

Finally, the equations of motion of the bogie-track system are:

$$m_b \ddot{z}_b + 2c_{zb}[2\ddot{z}_b - (\dot{z}_{w1} + \dot{z}_{w2})] + 2k_{zb}[2z_b - (z_{w1} + z_{w2})] - F_s = 0 \quad (11)$$

$$J_b \ddot{\theta}_b + 2c_{zb} a_b [2a_b \dot{\theta}_b - (\dot{z}_{w1} - \dot{z}_{w2})] + 2k_{zb} a_b [2a_b \theta_b - (z_{w1} - z_{w2})] + 2c_{xb} h_b [2h_b \dot{\theta}_b - 2\dot{x}_b + (\dot{x}_{w1} + \dot{x}_{w2})] + 2k_{xb} h_b [2h_b \theta_b - 2x_b + (x_{w1} + x_{w2})] = 0 \quad (12)$$

$$m_b \ddot{x}_b + 2c_{xb} [2\dot{x}_b - 2h_b \dot{\theta}_b - (\dot{x}_{w1} + \dot{x}_{w2})] + 2k_{xb} [2x_b - 2h_b \theta_b - (x_{w1} + x_{w2})] = 0 \quad (13)$$

$$m_w \ddot{z}_{w1,2} + 2c_{zb} (\dot{z}_{w1,2} - \dot{z}_b \mp a_b \dot{\theta}_b) + 2k_{zb} (z_{w1,2} - z_b \mp a_b \theta_b) + 2k_H (z_{w1,2} - z_{r1,2} - \eta_{1,2}) = 0 \quad (14)$$

$$m_w \ddot{x}_{w1,2} + 2c_{xb} (\dot{x}_{w1,2} - \dot{x}_b + h_b \dot{\theta}_b) + 2k_{xb} (x_{w1,2} - x_b + h_b \theta_b) = 0 \quad (15)$$

$$m_r \ddot{z}_{r1,2} + 2c_r \dot{z}_{r1,2} + 2k_s z_{r1,2} + 2k_H (z_{r1,2} - z_{w1,2} + \eta_{1,2}) = 0. \quad (16)$$

Hence results a 9-equation system with ordinary derivatives, which can be solved numerically using MATLAB code.

3 The frequency response functions of the bogie

The dynamic response of the bogie to the vertical track irregularities is evaluated based on the power spectral density of vertical acceleration (PSD acceleration) and the root mean square of acceleration (RMS acceleration) calculated in three reference points on the bogie.

According to Figure 1, the reference bogie points are thus defined: the point B – at the centre of the bogie chassis, the points $B_{w1,2}$ – above the axles, respectively above the supporting points of the bogie chassis on the primary suspension.

The power spectral density of the vertical acceleration at the reference points is

$$G_B(\omega) = G(\omega) \left| \omega^2 \bar{H}_{z_b}(\omega) \right|^2, \quad (17)$$

$$G_{B_{w1,2}}(\omega) = G(\omega) \left| \omega^2 [\bar{H}_{z_b}(\omega) \pm a_c \bar{H}_{\theta_b}(\omega)] \right|^2, \quad (18)$$

where $\bar{H}_{z_b}(\omega)$ and $\bar{H}_{\theta_b}(\omega)$ are the frequency response functions corresponding to the bounce and pitch (z_c și θ_c) of the bogie, and $G(\omega)$ is the power spectral density of the vertical track irregularities,

$$G(\omega) = \frac{A \Omega_c^2 V^3}{[\omega^2 + (V \Omega_c)^2][\omega^2 + (V \Omega_r)^2]}, \quad (19)$$

where $\Omega_c = 0.8246$ rad/m, $\Omega_r = 0.0206$ rad/m, and $A = 4.032 \cdot 10^{-7}$ radm - for a high level quality track, $A = 1.080 \cdot 10^{-6}$ radm - for a low level quality track [14].

Starting from the power spectral density of the vertical acceleration, RMS acceleration is determined in the reference points of the bogie, namely:

$$a_B = \sqrt{\frac{1}{\pi} \int_0^{\infty} G_B d\omega}, \quad a_{B_{w1,2}} = \sqrt{\frac{1}{\pi} \int_0^{\infty} G_{B_{w1,2}} d\omega}. \quad (20)$$

4 The results of the numerical simulations

This section deals with the results of the numerical simulations regarding the influence of the longitudinal vibrations upon the dynamic response of the bogie at moving on a track with vertical irregularities. To this purpose, a simplified model that does not consider the longitudinal vibrations is derived from the general model of the bogie in section 2. The results thus obtained based on the simplified model of the bogie are compared to the results coming from the general model of the bogie. The parameters of the bogie-track system used in the numerical simulations are included in Table 1.

Table 1. Parameters of numerical simulation

$m_b = 3200$ kg	$2k_{zb} = 2.20$ MN/m
$m_w = 1650$ kg	$2c_{zb} = 26.1$ kNs/m
$m_r = 175$ kg	$2k_{xb} = 70$ MN/m
$2a_b = 2.56$ m	$2c_{xb} = 50$ kNs/m
$h_b = 0.25$ m	$k_r = 70$ MN/m
$J_b = 2.05 \cdot 10^3$ kg·m ²	$c_r = 20$ kNs/m
$F_s = 167$ kN	$k_H = 1500$ MN/m

Figure 2 features the power spectral density of the vertical acceleration calculated in the reference points of the bogie - at the centre of the bogie chassis and above the axles, based on the general model of the bogie (blue line) and on the simplified model of the bogie (red line). The peaks corresponding to the resonance frequencies of the vibration on the vertical direction of the bogie, such as bounce at 5.85 Hz and pitch at 9.22 Hz, are highlighted. Similarly, the level of vibrations in the bogie is noticed to be higher against the axles and lower at the centre of the chassis, as well as the fact that the dynamic response of the bogie above bogie 2 is higher than above bogie 1. The asymmetry of the dynamic response in the bogie against the two axles is due to the damping of the primary suspension [15]. As for the influence of the longitudinal vibrations upon the dynamic response of the bogie, it mainly manifests itself in the resonance frequencies of the bogie, thus being more visible above the axles and less at the centre of the bogie chassis. Despite the small differences, an increase in the level of bogie vibrations can be noticed, coming from the coupling effect between pitch vibrations and the longitudinal vibrations of the bogie.

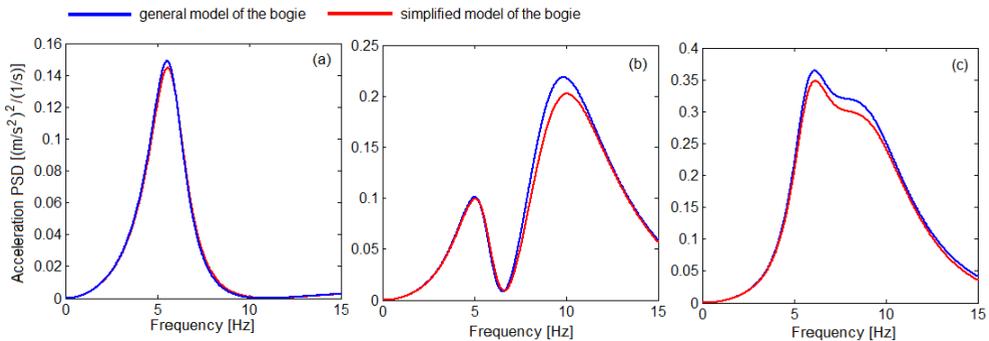


Fig. 2. The power spectral density of the vertical acceleration: (a) at the centre of the bogie chassis; (b) above the axle 1; (c) above the axle 2.

Figure 3 features the power spectral density of the vertical acceleration above the axles, calculated at the resonance frequencies of bounce and pitch in the bogie for various velocities ranging from 50 to 300 km/h. The differences between the results derived from the two models become visible at velocities higher than 150 km/h. For instance, at 200 km/h, at the resonance frequency of the pitch vibrations in the bogie, the bogie longitudinal vibrations trigger an increase in the power spectral density of the acceleration by 12.3% above the axle 1 and by 5.7% above the axle 2. It should be noted that the influence of the longitudinal vibrations operates differently against the two axles, at the bounce resonance frequency. As an example, at 200 km/h, the power spectral density of the vertical acceleration decreases by 9.8% above the axle 1 and increases by 5.4% above the axle 2.

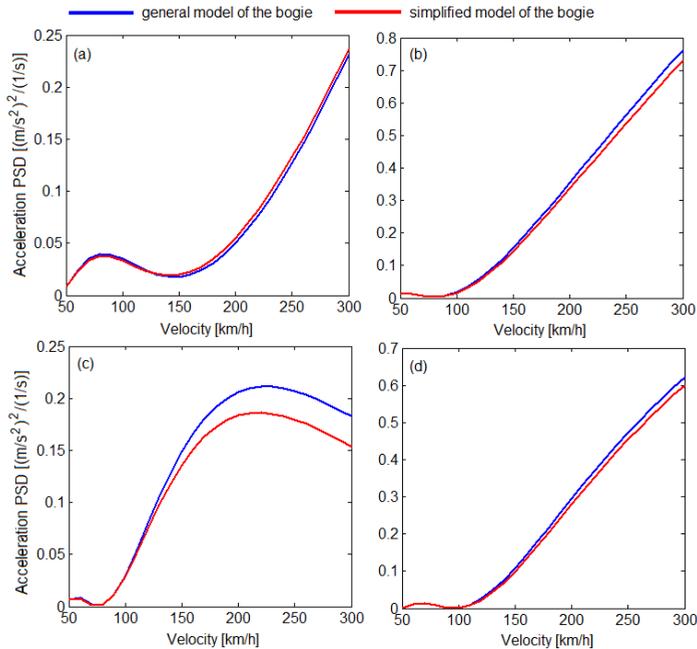


Fig. 3. The power spectral density of the vertical acceleration: (a) above the axle 1 at 5.85 Hz; (b) above the axle 2 at 5.85 Hz; (c) above the axle 1 at 9.22 Hz; (d) above the axle 2 at 9.22 Hz.

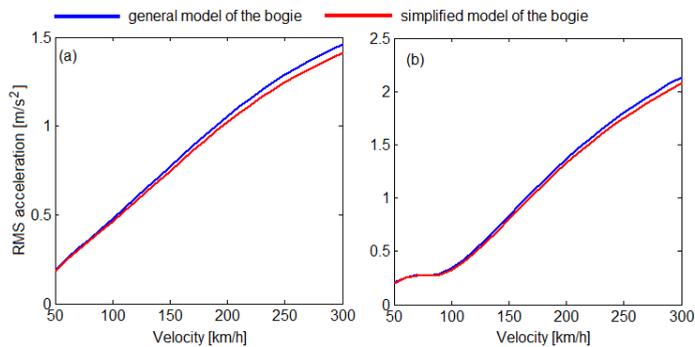


Fig. 4. Root mean square of the vertical acceleration: (a) above the axle 1; (c) above the axle 2.

Figure 4 shows the RMS acceleration calculated in the reference points located above the two axles, for the velocity interval of 50 – 200 km/h. The RMS acceleration keeps the general characteristic of the power spectral density of the acceleration, i.e. it goes up due to the longitudinal vibrations of the bogie and this increase is more visible at high velocities. For instance, the RMS acceleration grows by circa 3% at the velocity of 200 km/h.

5 Conclusions

The paper introduces a numerical study regarding the influence of the longitudinal vibrations upon the vibrations of the bogie generated during circulation on a track with vertical irregularities. The evaluation of the bogie behaviour of vibrations is conducted based on the power spectral density of the vertical acceleration and the RMS vertical acceleration. The results from the numerical simulations have underlined the fact that the bogie vibrations amplify due to the coupling effect between the pitch and the rebound

longitudinal vibrations. The increase of the level of vibrations in the bogie mainly manifests against the two axles, at the resonance frequencies of the bogie bounce and pitch and becomes visible at high velocities. It should be mentioned that the growth of the vertical acceleration does not exceed 3-4% at high velocities.

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