Reliability Evaluation of Power Supply for More-Electric-Aircraft Based on Information Entropy

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Abstract. In order to overcome the shortcomings of traditional power supply reliability evaluation model of multi-electric aircraft in the process of multi-component system research, this paper introduces the approximate probability distribution of multi-component system by cross-entropy, it proposes a Monte Carlo method based on information entropy to evaluate the power supply reliability of multi-electric aircraft, and obtains the approximate probability distribution by differential evolution to make the reliability evaluation. The estimated variance is approximately zero. Finally, taking an aircraft power supply system as an example, the convergence and accuracy of several reliability analysis methods are compared and analyzed. The results show the superiority of this method.

1 Introduction

Multi-power aircraft technology integrates the power generation, power distribution and power consumption of aircraft into a unified system, and implements unified planning, unified management and centralized control of power generation, distribution and power consumption systems. Due to the increase in the number and variety of finished parts of multi-electric aircraft, the reliability model of the multi-electric aircraft becomes a key problem. Therefore, it is very important to establish an efficient reliability evaluation model for multi-electric aircraft power supply systems.

Authors in [1-2] introduce the reliability analysis model of multi-electric aircraft power system, and explains the importance of calculating the reliability of multi-electric aircraft power supply from various aspects. The traditional analysis method is to analyze the reliability of the aircraft power supply system by using the fault tree [3-6], and theoretically provide the aircraft reliability analysis results. However, with the increase in the number of components of the aircraft power supply system, the analysis of the fault tree method is more complicated, and it is difficult to evaluate the reliability of the entire power supply system. In order to improve the efficiency of the fault tree, the neighboring matrix algorithm [7-9] is used to look for the minimal segmental sets of the fault tree, which has the advantage of strong versatility and compensates for the deficiency of the fault tree to some extent. However, the increase in the number of components in the aircraft power supply system makes the analysis process of the algorithm complicated and the calculation time is long. Due to the improvement of computer computing power, the probability analysis method-Monte Carlo method [10-12] is gradually applied to the reliability evaluation of complex systems, which is flexible and simple to implement. Due to the large number of system components and low failure rate, if the traditional Monte Carlo method is used to evaluate the reliability of the entire multi-electric aircraft power supply system, a large number of samples are needed, which will greatly reduce the efficiency of the algorithm.

In view of the above problems, the approximate probability distribution of the multi-element system is introduced by cross entropy [13-18]. The approximate probability distribution can make the reliability evaluation variance zero, which improves the sampling efficiency. Finally, a case-by-case example is used to verify the superiority of the Monte Carlo method based on information entropy in the reliability evaluation of multi-electric aircraft power supply system.

2 Power supply system and reliability model

The power supply modes of multi-electric aircraft mainly include main power supply system and reserve power supply system. The main power supply system also contains two subsystems that are mutually hot standby. In Fig.1, S_1 and S_2 are parallel main power supply systems that are alternate with each other. No matter which subsystem is faulty, the other system will switch to the load supply by itself. S_3 is a reserve power supply system. When both S_1 and S_2 fail, switch to S_3 to supply power to the load. Z_1, Z_2, Z_3, Z_4, and Z_5 represent the secondary exciter,exciter,generator,inverter and controller, respectively. The protection devices in each power supply mode are C_1, C_2, and C_3. To ensure the reliability of the switching system power supply mode, the switching devices between the power generation systems are B_1 and B_2.

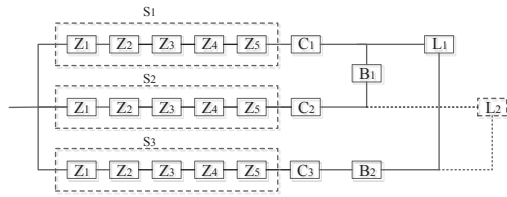* Corresponding author: hfut_qxx@163.com

Fig.1 Aircraft power supply system structure

The reference value of the component probability density distribution is given in [9]. Since Z_1, Z_2, and Z_3 are related to the motor, it is assumed that their failure probability density function obeys a normal distribution. Z_4, Z_5 are fundamentally electronic devices, so they are assumed to follow an exponential distribution. Circuit breakers C_1, C_2 and C_3, contactors B_1 and B_2 are assumed to also follow an exponential distribution. To simplify the description of this model, each load in the system can be grouped into a load. This paper assumes that the set of all loads in the system is L_1. The aircraft power supply system has many components and its state model is different. For the convenience of modeling, this paper has done a simplification process, assuming that the state model of each component in the system is a two-state model.

The fault probability density function of the component is $f_i(t)$, $i = 1, 2...10$. The probability density function of the component is integrated to derive the probability of failure of the component, and the reliability of the component is determined.

$$R_i(t) = 1 - F_i(t) = 1 - \int_0^t f_i(t)dt \qquad (1)$$

According to the reliability calculation criterion, the reliability of the series system is the reliability accumulation of each series element, so the reliability function expression of each subsystem is obtained as follows.

$$R_{S_n}(t) = \prod_{i=1}^{5} R_i(t) \quad n = 1, 2, 3 \qquad (2)$$

Since the three systems are connected in parallel, the failure probability of the parallel system is the cumulative probability of failure of the three subsystems. First, obtain the probability of system failure, and then use one to get the reliability model of the whole system.

$$R(t) = 1 - \prod_{i=1}^{3}(1 - Rs_i(t)) \qquad (3)$$

Through the above derivation, the analytical expression of the system reliability of the simplified aircraft power supply system model can be obtained. Since the probability distribution of each component is different, the formula (2) is accumulated after integration, so the complexity is high. To further introduce system reliability, the complexity of equations (3) is even higher. Formula (3) is applicable to the simplified model of aircraft power supply system in this paper. When the number of system components increases and the system structure is not simply series-parallel, it will be difficult to find an analytical expression such as equation (3). Therefore, this paper will avoid the analytical method to calculate system reliability. Monte Carlo is used to sample aircraft system

fault conditions for the current short time range. At the same time, the method of information entropy is introduced to optimize the sampling probability density function of the sample. The probability density function can effectively solve the problem of low efficiency of traditional Monte Carlo method.

3 Establishment of multi-electric aircraft estimation model

Because of the high reliability of the aircraft power supply system, it is difficult to sample the system fault components during Monte Carlo sampling, so the calculated system reliability is generally 1. This is obviously not what this article needs. In order to improve the sampling efficiency, the distribution of the probability $f(x,u)$ of the original system is improved to the optimal probability distribution $g(x)$, so that the coefficient of variance can be theoretically zero. But $g(x)$ is often difficult to find, or even impossible to solve. For the reason that, this paper introduces the cross entropy method, which is generally used to measure the gap between the target distribution and the predicted value distribution. Therefore, this paper constructs a probability distribution close to the $g(x)$ distribution by the method of cross entropy.

The reliability evaluation expression of the aircraft power supply system is as follows:

$$l = \int I_{s(x)<r} f(x,u)dx \approx \frac{1}{N}\sum_{i=1}^{N} I_{S(X)<r} \qquad (4)$$

In formula (4), $f(x,u)$ represents the probability density function of the state of the system component. In order to improve the sampling efficiency, it is necessary to deform the formula (4). The probability density after deformation can make the reliability variance coefficient zero.

$$l = \int I_{s(x)<r} \frac{f(x,u)}{g(x)}g(x)dx = \int L(x)g(x)dx \qquad (5)$$

In formula (5), $L(x)$ is the ratio between the original probability density function of system components and the improved probability density function, which is often called likelihood ratio. $g(x)$ is an arbitrary probability density function. The Monte Carlo estimate for equation (5) is:

$$\hat{l} = \frac{1}{N}\sum_{i=1}^{N} I_{S(X_i)<r} \frac{f(X_i,u)}{g(X_i)} \qquad (6)$$

In equation (6), X_i is the state of the system component obtained by sampling one time under the probability density function $g(x)$.

When $g(x)$ is equal to $g_{opt}(x)$, the system reliability estimation variance of l is zero. The expression of $g_{opt}(x)$ is as in formula (7).

$$g_{opt}(x) = I_{S(x)<r} \frac{f(x,u)}{l} \qquad (7)$$

It can be seen that the original probability density function divided by l can make the variance of the reliability estimate be zero. Therefore, when Monte Carlo sampling, N samples can be randomly sampled

under the $g_{opt}(x)$ probability density function. However, it can be seen from equation (10) that $g_{opt}(x)$ is related to l, but l is the reliability to be determined. So it is unrealistic to use the $g_{opt}(x)$ probability density function directly.

This paper introduces the probability density function $f(x,v)$ as an approximate probability density function of $g_{opt}(x)$. In order to measure the closeness of $f(x,v)$ and $g_{opt}(x)$, this paper uses cross entropy to measure the distance between the target distribution and the approximate probability distribution. As shown in formula (8).

$$D = \int g(x)\ln(g(x))dx - \int g(x)\ln(f(x,v))\,dx \qquad (8)$$

Minimizing the closeness of $f(x,v)$ and $g_{opt}(x)$ is to find the minimum value of D, that is, to find the minimum value of $-\int g_{opt}(x)\ln(f(x,v))dx$. Finally, it is equivalent to the maximum value of the formula (9).

$$\max_v \int g_{opt}(x)\ln(f(x,v))dx \qquad (9)$$

Substitute $g_{opt}(x)$ in formula (7) into formula (9) to get formula (10).

$$\max_v \int \frac{I_{\{S(x)\leq r\}}f(x,u)}{l}\ln(f(x,v))dx \qquad (10)$$

Since l is constant for known systems. Therefore, the equivalent estimates of equation (10) are as follows:

$$\max_v E_U\left(I_{\{S(X)\leq r\}}\ln(f(X,v))\right) \qquad (11)$$

The above derivation is based on the probability density function. Since the variables in this paper are discrete, it is necessary to convert the probability density function into a probability distribution function. For example, $f(x,u)$ is converted to $F(x,u)$, $f(x,v)$ is converted to $F(x,v)$, and the integral becomes a summation.

X is a sample of the system sampled from the probability distribution function $F(X,u)$. Since the sampling of $F(x,u)$ yields only a small number of valid samples, the important sampling method is used to replace $F(x, u)$ with $F(x,w)$. Therefore, the formula (11) is transformed into the formula (12).

$$\max_v E_w I_{\{S(X)\leq r\}}W(X;u,w)\ln(F(X,v)) \qquad (12)$$

X is a sample of the system sampled from the probability distribution function $F(X,w)$. Where $W(X;u,w) = F(x,u)/F(x,w)$. The optimal probability distribution function $F(x,v)$ of the system can be obtained by solving the maximum value of the formula (12). It can be seen that the maximum solution of formula (12) is a multi-variable maximum optimization process. In this paper, differential evolution [19-21] is introduced to iteratively solve $F(x,v)$. Then the samples are sampled under the new probability distribution function, and the system reliability index is solved according to the formula (6).

In order to further improve the sampling efficiency, one-time sampling using dual sampling [22-24] produces random numbers that are negatively correlated with each other. This feature of the negative correlation between related points is used to quickly reduce the variance of the estimated value.

4 Simulation and Verification

4.1 Instantiated System Estimation Model

Table 1 shows the distribution types and parameters of the various components of the power supply system under the two system models. The remaining components not given are considered to be very reliable.

In this paper, the test function $IS(X)<r$ represents the system fault condition. When $S(X)<r$ is zero, it represents fault, and $S(X)>r$ is one for operation. $S(X)$ represents the generating capacity of aircraft power supply system, and r represents aircraft load. The number of system components is n. It is specified that X_k represents the kth value in the system state space vector $\mathbf{X}$, $X_k = [x_1, x_2, x_3....x_n]$. The system component is a two-state model and u_i is the probability of failure of component i. The specific value of u_i is sampled according to the failure probability distribution of the component. The probability distribution function $F(x,u)$ of the system is derived from the above assumptions.

$$F(x,u) = \prod_{i=1}^{n}(1-u_i)^{1-x_i}(u_i)^{x_i} \qquad (13)$$

Table 1 Failure distribution parameters

unit	distribution pattern	A system parameters	B system parameters
Z1	Normal	$u_1=1500;\sigma_1=120$	$u_1=3750;\sigma_1=600$
Z2	Normal	$u_2=1500;\sigma_2=150$	$u_2=3750;\sigma_2=750$
Z3	Normal	$u_3=2800;\sigma_3=500$	$u_3=7000;\sigma_3=1250$
Z4	exponential	$\lambda_4=1.25\times10^{-3}$	$\lambda_4=5\times10^{-4}$
Z5	exponential	$\lambda_4=1.25\times10^{-3}$	$\lambda_4=5\times10^{-4}$

Since $S(X)<r$ is a small probability event, the probability distribution function $F(x,u)$ is changed to $F(x,v)$ by cross entropy. The system reliability estimate under the improved probability distribution function $F(x,v)$ is shown in equation (14).

$$l = \frac{1}{N}\sum_{i=1}^{N} I_{S(X)<r}\, W(X,u,v) \qquad (14)$$

Where $W(X,u,v)$ is the correction factor. The specific form is as follows:

$$W(X,u,v) = \frac{\prod_{i=1}^{n}(1-u_i)^{1-x_i}(u_i)^{x_i}}{\prod_{i=1}^{n}(1-v_i)^{1-x_i}(v_i)^{x_i}} \qquad (15)$$

The change in state space probability is determined by the parameter vector $\mathbf{v}=[v_1, v_2, v_3....v_n]$. The problem now is to look for the optimal $\mathbf{v}$ to find $F(x,v)$. In this paper, the differential evolution algorithm is used to solve the optimal value $\mathbf{v}$. Differential evolution is characterized by simple structure, superior performance, and parallelism of algorithms, which can be searched collaboratively. The objective function can be optimized quickly and concisely.

4.2 Implementation process

4.2.1 Parameter initialization

Initialize parameter vector **v**, optimize process sample size N, system component failure rate u_i, failure rate scaling factor k and differential evolution iteration number N_s. The sampling sample size of the dual variable method N_d, the variance convergence coefficient β.

4.2.2 Determine the optimization objective function

N samples are randomly sampled according to the system component failure rate u_i and the failure rate scaling factor k. The optimization objective function is determined according to the formula (12).

4.2.3 Variations

The i_{th} individual variation in a population. Individual variation is realized by difference strategy.

4.2.4 Cross

The purpose of the crossover operation is to randomly select individuals and determine whether to accept the variation results of the previous step.

4.2.5 Screening

The greedy selection strategy is adopted, which is to select the better individual as the new individual according to the value of the objective function.

4.2.6 Conditional judgment

Determine whether the number of differential evolution iterations reaches N_s. If it reaches the next step, the system probability distribution function is $F(x,v)$, otherwise it jumps back to step 4.2.2.

4.2.7 Dual variable sampling

Generate n sample arrays in the interval [0,1], and subtract the array from 1 to get its dual array. The state of the components of the sampling system is obtained by the formula (4).

4.2.8 Calculating the reliability index

Calculate the reliability indicator according to formula (12).

4.2.9 Conditional judgment

Determine whether the variance coefficient satisfies the convergence condition. If it is satisfied, jump out of the loop and output related information and graphics, otherwise skip to step 4.2.7.

4.3 Analysis of simulation results

In order to verify the superiority of the algorithm in this paper, the traditional monte carlo method, analytical method and reliability evaluation method based on information entropy were used to simulate the calculation of A and B systems.

In order to verify the convergence characteristics of the proposed algorithm, this paper compares the convergence characteristics of the variance coefficients of the Monte Carlo method and the information entropy-based reliability evaluation method. This stage simulates the system reliability at *1000h*. At the same time, in order to ensure the accuracy of the calculation results, this paper carried out three reliability evaluations for the A system. Then take the average of *3* results as the final result. The Fig.below is the result of simulation with MATLAB.
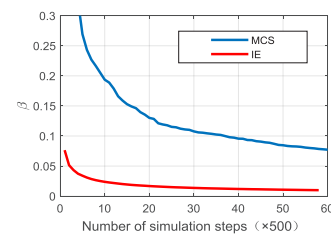

Fig.2 Convergence coefficient convergence graph

It can be seen from Fig.2 that the reliability evaluation method based on information entropy has very good convergence characteristics when evaluating the reliability of the aircraft power supply system. This method can achieve good convergence stability when the number of simulation times is *10000,* while the traditional sampling method can achieve good convergence stability when the number of simulation times is *30000*. In comparison, the algorithm proposed in this paper improves the convergence by 2 times.
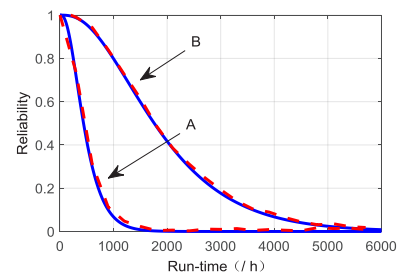

Fig.3 reliability of power system

In order to verify the accuracy of the proposed algorithm, this paper analyzes the reliability and time-varying characteristics of the analytical method and the reliability evaluation method based on information entropy.

Fig.3 shows the distribution of power system reliability over time. The red line represents the method of this article and the blue line represents the traditional analytical method. Fig. 4 is the error distribution diagram between the reliability evaluation method based on information entropy and the traditional analytical method in a simulation. For the A power system, the maximum error of the method proposed in this paper is 0.02

compared with the traditional method, and the maximum proportion is 2%. For the B power system, the maximum error of the method in this paper compared with the traditional method is 0.018, and the maximum proportion is 1.8%.
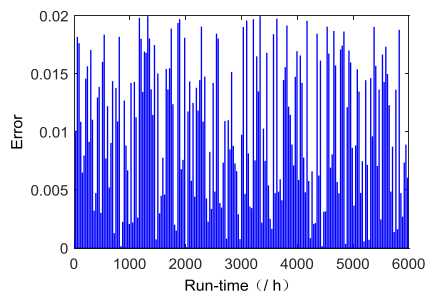

Fig.4 error distribution

From the above analysis, it can be concluded that for the reliability analysis of the power system, the analysis results of the reliability estimation model obtained by the information entropy-based reliability evaluation method are more convergent than the reliability analysis results directly obtained by Monte Carlo simulation. Well, compared with the reliability analysis results obtained by the traditional analytical method, the accuracy of the calculation results is relatively close, and the computational model is less complex. Therefore, the model can meet the requirements of aviation power system reliability estimation.

5 Conclusion

This paper uses information entropy-based reliability assessment to assess the reliability of complex systems. And the probability density distribution is introduced by using cross entropy, so that the reliability variance is theoretically zero. Then it uses the differential evolution approximation probability function. Finally, under the approximate probability distribution, combined with the dual sampling method, the reliability of the whole system is evaluated. At the end of the paper, an example is used to evaluate the reliability of the method using the analytical method, the Monte Carlo method and the information entropy-based reliability evaluation method. From the convergence analysis, the convergence of this method is more than 2 times compared with the reliability analysis results obtained by direct Monte Carlo simulation. From the calculation accuracy analysis, compared with the reliability analysis results obtained by the traditional analytical method, the accuracy of the calculation results is relatively close, but the computational model is less complex. Therefore, the comprehensive analysis of this method is very suitable for the more complex reliability evaluation of aircraft power system.

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