

# Nonlinear inversion method based on dynamics model modification

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**Abstract.** In the fields of modern aviation system, subgrade vehicle system and complex mechanical system, there is a problem that parameters of most dynamic models are inaccurate. This problem results in a large difference between the model results and the experimental results. In order to solve this problem, this paper build a nonlinear inversion method based on dynamics model modification (NIDM). Firstly, the error relationship was obtained by integrating the experimental data with the simulation results of the forward modelling model by the cost function and penalty function. Then, the problem of error function minimization was solved by using the parameter iteration generated by particle swarm optimization algorithm, and the corrected parameters of the forward modelling model were obtained. Finally, the method was tested by building a vehicle suspension vibration model and a pavement excitation model as test samples. The test results show that the fitting degree between the simulation results and the experimental results can be effectively improved by modifying the parameters of the dynamic model based on the NIDM method.

## 1 Introduction

In the design and development of roadbed mobile platforms, aerospace vehicles, and complex mechanical equipment, in order to reduce the use of physical prototypes so that to reduce the test cost and shorten the development cycle, it is necessary to construct a dynamic model to simulate the products under development. In general, dynamic models tend to be less accurate at the beginning of modeling and the model parameters need to be modified. The simulation error of the model is often caused by the following several reasons: the system is simplified to a certain extent when the model is built; the parameters caused by the error of empirical determination or equivalent analysis are inaccurate, and the model is nonlinear and difficult to be solved accurately. In order to improve the accuracy of the model simulation results, the parameters of the dynamic model need to be modified according to the limited number of physical prototype test results.

In the domestic and foreign related studies, scholars use different techniques to modify the model parameters, and the accuracy of the models have been improved to different degrees. By constructing response surface models of different types, writers in [1] realized the modification of engine models and multi-objective modification of machine tool models, and the accuracy of the modified models were improved. Writers in [2] use Kriging agent model to modify the finite element models of the building structure and verify the feasibility of the method. Writers in [3] used simulated annealing algorithm to optimize the multi-objective global optimization of rocket nozzle model and improve the accuracy and efficiency of correction. Writers in [4]

apply multi-level theory to define the weight coefficients of complex structures and perform dynamic model corrections. However, the methods for constructing the response surface model in the existing methods have problems of over-fitting and under-fitting; genetic algorithm, simulated annealing algorithm, etc. are not fast enough, and the algorithm itself is more complicated; the method of defining weight coefficients by multi-level theory is not universal.

In this paper, the nonlinear inversion method is chosen. Firstly, the test data is combined with the simulation results of the forward model to obtain the error relation function. Secondly, the error function minimization problem is solved iteratively. Then the vehicle chassis suspension model and the road spectrum excitation model are constructed. Finally, the parameters of the chassis suspension model constructed in this paper are modified by the method mentioned. The result of the sample demonstrates the effectiveness of the method. Compared with the traditional method, the forward model does not need to obey the specific form when using this method, and the solution speed is fast, and it can cover most of the solution space, which has certain universality.

## 2 Nonlinear inversion method based on dynamics model modification

The nonlinear inversion method is used to construct the framework of the NIDM method. The cost function and the penalty function are combined to construct the error function of the simulation results of model and the test data. The error function is iteratively solved by particle

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swarm optimization with time-varying constrict factors (PSOTCF), and the optimal parameters of the model are obtained. The framework of the optimization process is shown in Fig. 1. Firstly, determine the termination conditions of the optimization algorithm, which is the minimum threshold of the objective function or the maximum number of iterations. Then, the value range of the optimized variable is taken as a constraint condition to construct the objective function and the relevant parameters of the particle swarm optimization algorithm are set. Finally, the PSOTCF algorithm is implemented, the output that are the optimal value of the variables which can minimize the error between the simulation result and the test result will be obtained by iterative solution.

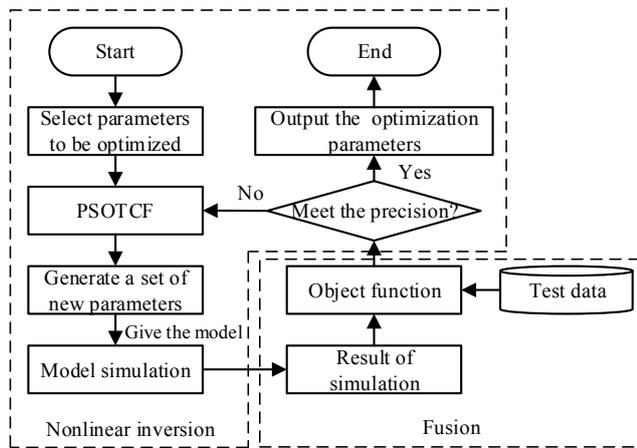


Fig. 1. Operation scheme of NIDM.

### 2.1 Particle swarm optimization with time-varying constrict factors algorithm

A set of parameters are generated for the forward model in each cycle by the generation rules of the PSOTCF. Particle swarm optimization (PSO) imitates the behaviour of birds, fish and other social creatures in search of food to optimize the problem. Individuals in a group use the group's shared information to dynamically adjust their state so that the individuals can constantly approach the target. Therefore, the update equation for speed and position of the population is the core of the algorithm, as in

$$\begin{cases} v_{id}(t+1) = \omega \times v_{id}(t) + c_1 \times r_1 \times (p_{id}(t) - x_{id}(t)) \\ \quad \quad \quad + c_2 \times r_2 \times (p_{gd}(t) - x_{id}(t)) \\ x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \end{cases} \quad (1)$$

Where,  $i$  represents the  $i$ -th particle, and  $d$  represents the  $d$ -th attribute assigned to this particle;  $v_{id}$  is the speed of the  $d$ -th attribute when the algorithm iterates  $t$  times for the  $i$ -th particle;  $x_{id}(t)$  represents the value of the  $d$ -th property for the  $i$ -th particle when the algorithm iterates  $t$  times;  $p_{id}(t)$  represents the value of the  $d$ -th attribute of the optimal position  $p_i$  of the  $i$ -th particle itself when the algorithm performs  $t$  iterations;  $p_{gd}(t)$  represents the  $d$ -th attribute value of the optimal

position  $p_g$  of the group when the algorithm iterates  $t$  times;  $\omega$  is the inertia weight;  $c_1$  is the cognitive coefficient of the population;  $c_2$  is the social coefficient of the population;  $r_1$  and  $r_2$  are the random numbers.

The inertia weight  $\omega$  of the particle swarm optimization algorithm determines whether the particle can search for the solution space and the speed of the search. Its value is usually determined by Shi and Eberhart's setting method in [5], which decreases from 0.9 to 0.4 as the iteration progresses.

The cognitive factors and social factors in the particle swarm optimization algorithm also determine whether the particle can search for the solution space and the speed of the search. In this paper, the Particle swarm optimization with time-varying constrict factors (PSOTCF) algorithm proposed by Ratnaweera in [6] is presented. The dynamic equations are shown in (2) and (3). Where  $c_{1i}$ ,  $c_{1f}$ ,  $c_{2i}$  and  $c_{2f}$  are constants. It is found that when the cognitive factor  $c_1$  decreased from 2.5 to 0.5, and the social factor  $c_2$  increased from 0.5 to 2.5, the algorithm performed best.

$$c_1 = (c_{1f} - c_{1i}) \frac{t}{t_{\max}} + c_{1i} \quad (2)$$

$$c_2 = (c_{2f} - c_{2i}) \frac{t}{t_{\max}} + c_{2i} \quad (3)$$

### 2.2 Construction of objective function based on cost function and penalty function

The optimization problem is actually a minimization optimization problem. The goal of the optimization problem is to minimize the error between the simulation results of the dynamic model and the test data. Therefore, the problem can be transformed into the minimum problem of the objective function and the constraint problem can be transformed into the unconstrained problem by constructing the objective function of the optimization method by combining the cost function and the penalty function.

The coefficient of determination is an indicator that reflects the degree of correlation between a set of dependent variables and a set of independent variables [7]. The determination coefficient is used to reflect the degree of fitting between the simulation results of the system dynamics model and the test data, as in

$$R^2 = \frac{SSR}{SST} = 1 - \frac{\sum_{i=1}^m (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^m (y_i - \bar{y})^2} \quad (4)$$

Where,  $y_i$  is the test data at the sample point,  $\hat{y}_i$  is the simulation value at the sample point,  $\bar{y}$  is the average value of the test data, and  $m$  is the number of sample points. The interval of the result of fitting degree

value  $a$  is  $[0,1]$ . The larger the complex correlation coefficient  $R^2$  is, the closer the linear correlation between the simulation results and the test data will be. Therefore, we require  $R^2$  to be greater than 0.9.

The objective function is converted into the minimization form, as in

$$\begin{cases} f(k, c) = \min R^2 \\ R^2 = 1 - \left(1 - \frac{\sum_{i=1}^m (\bar{y} - y_i)^2}{\sum_{i=1}^m (y_i - \bar{y})^2}\right) = \frac{\sum_{i=1}^m (a_i - \hat{a}_i)^2}{\sum_{i=1}^m (a_i - \bar{a})^2} \end{cases} \quad (5)$$

The inequality constraint mathematical model can be transformed into the unconstrained mathematical model by using the penalty function, as in

$$F(x, M) = f(x) + M \sum_{i=1}^m (\max\{0, g_i(x)\})^2 \quad (6)$$

Where  $M$  is a large positive integer. The equation  $f(x)$  is the cost function and the equation  $g_i(x)$  is constraint conditions of the problem.

### 3 Test case: parameter modification of chassis suspension vibration model

The vibration model of vehicle chassis suspension can reflect the state change of each component when the vehicle is subjected to forced vibration under a given excitation mathematically. In this vibration model, the road roughness and speed are taken as the input of the model, and the acceleration of the sprung mass is taken as the output. The elastic element, damping element and mass body of the wheel/car body in the model are used as the intermediate parts to convert input into output [8].

#### 3.1 Part of the vibration model

The vibration model of automobile chassis suspension is shown in Fig. 2, where  $q$  is the mathematical expression of road surface excitation on tires;  $z_1$  is the vertical displacement of the unsprung mass;  $z_2$  is the vertical displacement of the sprung mass,  $m_1$  is the mass under the spring,  $m_2$  is the mass on the spring,  $k$  is the suspension stiffness coefficient,  $k_t$  is the tire stiffness coefficient,  $c$  is the suspension damping coefficient,  $c_t$  is the tire damping coefficient. The equation of state of the model is as (7). MATLAB is used to convert the mathematical model of chassis suspension vibration model into code.

$$\begin{cases} m_1 \ddot{z}_1 = k_t(q - z_1) + c_t(\dot{q} - \dot{z}_1) - k(z_1 - z_2) \\ \quad \quad \quad - c(\dot{z}_1 - \dot{z}_2) \\ m_2 \ddot{z}_2 = k(z_1 - z_2) + c(\dot{z}_1 - \dot{z}_2) \end{cases} \quad (7)$$

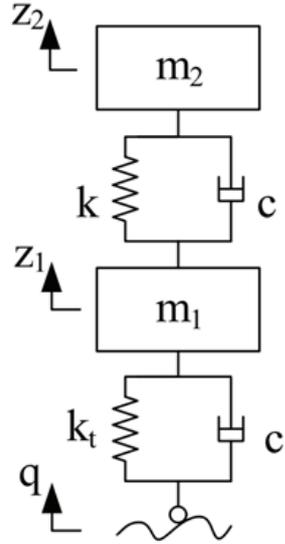


Fig. 2. The suspension vibration model.

#### 3.2 Part of the road surface input

By using the white noise module contained in MATLAB/SIMULINK, the time-domain model of pavement is established [9]. The mathematical model of road surface excitation on tires is in

$$\dot{q}(t) = -0.111[vq(t) - 92.58\sqrt{G_q(n_0)}v\omega_0(t)] \quad (8)$$

Where,  $q(t)$  is the displacement of road surface unevenness,  $v$  is the speed,  $G_q(n_0)$  is the geometric average value of the coefficient of unevenness specified in the 8-level classification standard of road surface unevenness, and  $\omega_0(t)$  is the white noise with zero mean value[10].

According to (8), white-noise limiter module, gain module, integral tool and RMS root mean square tool and oscilloscope in MATLAB SIMULINK toolbox are called.

The time-domain simulation model of road roughness constructed by combining these modules.

In order to make the covariance value of the generated white Noise be  $1m^2/s$ , the Sample time and Noise power of the white-noise-limited module are set to 0.01s and 0.01m<sup>2</sup> respectively. The speed gain in the model is set as 10m/s, and the calculation formula of parameter gain is in

$$\beta = 92.58\sqrt{G_q(n_0)}v \quad (9)$$

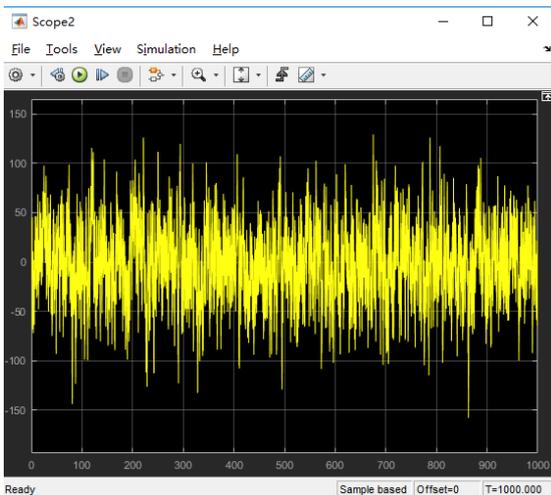
The geometric mean value of the coefficient of unevenness specified in the 8-level classification standard of road surface unevenness was put into the model for calculation, and the root mean square value of 8 groups of road surface unevenness was obtained, and the root mean square value was compared with the standard value. According to TABLE I, it can be seen that the RMS value of the road roughness simulated by the road roughness model constructed in this paper differs very

little from the RMS value of the road roughness coefficient specified in the 8-level classification standard of road roughness, indicating that the road spectrum model has a high accuracy.

**Table 1.** Comparison table of simulation results and standards.

The road level	Root mean square value	
	Pavement grading standard	Simulation result
A	3.81	3.81
B	7.61	7.62
C	15.23	15.24
D	30.45	30.49
E	60.90	60.98
F	121.80	121.96
G	243.61	243.92
H	487.22	487.83

Class C pavement was selected as the road spectral excitation of the suspension model, and the output waveform was shown in Fig. 3.



**Fig. 3.** The waveform of class C pavement spectral.

### 3.3 Construction of constraint conditions

**Stiffness constraint:** by introducing the range of static deflection of suspension from 60mm to 130mm into the expression of static deflection of suspension  $f_s = m_2g / k$ , the constraint range of suspension stiffness can be obtained as in

$$\frac{m_2g}{0.13} < k < \frac{m_2g}{0.06} \quad (10)$$

**Damping constraint:** the damping ratio range of suspension is selected as 0.2 ~ 0.4, and this range and the range of static deflection of suspension 60mm ~ 130mm are taken into the suspension damping expression

$c = 2\psi\sqrt{km_2}$  to obtain the damping coefficient constraint range as in

$$0.4m_2\sqrt{\frac{g}{0.13}} < c < 0.8m_2\sqrt{\frac{g}{0.06}} \quad (11)$$

**Dynamic deflection constraint:** with the increase of dynamic deflection, the probability of suspension hitting limit block will also increase, and the ride comfort of the car will become worse [10]. Therefore, the root means square value  $\sigma_{f_d}$  and allowable value  $[f_d]$  generally meet the following requirements:  $\sigma_{f_d} \leq [f_d]/4$ .

According to the constraints mentioned above, the objective function of this case is constructed as in

$$f(k, c) = \min \left\{ \begin{array}{l} R^2 \\ +M[\max(4\sigma_{f_d} - [f_d], 0)] \\ +\max(2\sigma_{F_d/G} - 1, 0) \end{array} \right\} \quad (12)$$

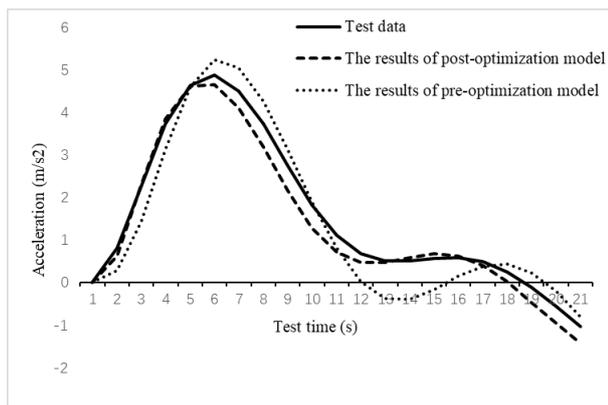
### 3.4 Result and dissuasion

MATLAB is used to make the method NIDM which mentioned in this paper into a program to modify the parameters of the chassis suspension vibration model constructed above. The vibration model of the chassis suspension can be optimized with the stiffness coefficient  $k$  and the damping coefficient  $c$ , according to the description of parameter constraints in chapter 3 of this paper, the value range of  $k$  and  $c$  is determined to be  $k \in [271380, 588000]$  and  $c \in [12503, 36807]$ . Firstly, input this range into the program, then, set the algorithm population number to 30 and the maximum number of iterations to 200, finally, enter the test data and execute the model optimization program.

After the decimal point is retained, the optimized results obtained are  $k=476226.28$  and  $c=30424.97$ . After model parameters optimizing, the simulation fitness of the model decreased from 0.083 to 0.012. The parameters of the NIDM method and the simulation results of the pre-optimization models and the post-optimization models are shown in TABLE II.

It can be seen from table II that the deviation degree index between the simulation results of the pre-optimization model and the post-optimization model decreased from 0.083 to 0.012. The test data, the simulation results of the pre-optimization model and the post-optimization model are drawn in a two-dimensional diagram. By comparing the three curves in Fig. 4, it can be proved that the deviation between the simulation results of the dynamic model and the test data can be solved well by the NIDM method. The simulation results of the dynamic model are more fitting with the test data, and the model can reflect the state change of the system more realistically.

## 4 Conclusion



**Fig. 4.** Comparison of results.

**Table 2.** Parameter selection and results.

Input of the method			
Factors	Value		
Interval of k	[271380,588000]		
Interval of c	[12503,36807]		
Iterations	200		
Speed factor	0.30		
Particle number	30		
Parameters and fitness			
	k	c	Fitness
Pre-optimization			0.083
Post-optimization	476226.28	30424.97	0.012

This paper firstly proposed a dynamic model modification technology based on nonlinear inversion (NIDM), then built the vehicle chassis suspension model and road spectrum incentive model, finally used the NIDM method to correct parameters of the chassis suspension model built in this paper. The comparison of results between test data, results of the pre-optimization model and results of the post-optimization model proved the effectiveness of the method.

The constitution of system dynamics model is usually complex, and the number of variables to be optimized for each subsystem is often very large. Meanwhile, the absolute constraint of each variable and the relative constraint between each variable need to be taken into account. All of the above problems are difficult to solve by the NIDM method. Sensitivity analysis may be introduced in future studies to solve the problem of low efficiency of optimization solution due to too many variables to be optimized. The agent model or response surface model may be introduced to solve the problem that the mathematical model is too complex to solve. The methods of Multi-source data fusion may be introduced to fuse the different groups of test data by assigning weights to them so as to improve the accuracy of test data and reduce the uncertainty of data, and finally make the model optimization results more accurate.

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