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Multi-criteria optimization of tracked vehicle transmissions

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Abstract. The article is devoted to the problem of computer modeling of rational design of tracked vehicle transmissions with multiple criteria. The problems of finding optimal geometric parameters that satisfy several quality criteria. All the complexity of the layout and the relationship of the parameters make difficult their choice, which is simplified when using approaches of mathematical optimization. Using the famous pseudo-random method *LP*τ-search with the author's modification made it possible to avoid problems associated with the discreteness and the number of parameters. The main optimization criteria for transmission are minimum center distance, minimum length, minimum mass, and maximum uptime probability. To solve this problem, the problem was formulated and design parameters with constraints were specified, criteria were recorded and a transformation from multicriteria to a single criterion was proposed. The approach is based on analysis of test points that obtained using *LP*τ-search, and further processing of the information received. The approach of transformation from many criteria to one is proposed by introducing the scale of importance by the designer and assigning the importance of each of the criteria, finding the desired solution for each trial point of relative offset, which is proposed to be used as a unifying criterion. Basic schemes and flowcharts of the algorithm elements are provided. The implementation of the computer model was carried out in the Delphi 7 environment.

1 The actuality of the task

Wide application in transport engineering has mechanical transmission, which are used to change the torque and rotation speed of driving wheels of tracked vehicles. Their most widespread representatives are cylindrical gearboxes and additional drives. This can be performed both in deployed and in coaxial layouts [1].

The complexity of designing this type of drive is the distribution of gear rations between the stages, and, consequently, the selection of appropriate design parameters. Also, during the design, it is necessary to ensure the equal strength of gearings.

Usually, when designing technical systems, an engineer is confronted with a dilemma, since a wide range of product requirements leads to several quality criteria [2]. Most tasks of optimal design of stepped drives are also multicriteria. From the position of the gearbox design, the following most significant weight and gabarit characteristics are usually used: center distance (coaxial gears) or total center distance; length; mass of the gearbox [3]; integral value, which includes all the calculated safety coefficients of bending and contact stresses.

Simultaneous achievement of the best characteristics is always controversial, complex and subjective process; therefore, it is expedient to use approaches of multi-criteria mathematical optimization at designing.

To solve this problem a pseudo-random search is proposed. It is based on the study of the space of parameters, where the points of the LPτ-sequence [4] are used as test points in the unit multidimensional cube. Also, all the criteria are combined into one.

With such an approach to solving the problem, the following questions are:

- formulation of objective functions according to the criteria;
- development of the approach and algorithm for combining the criteria, that make it possible to reduce the problem to one-criteria;
- computer realization of the proposed algorithm;
- presentation and interpretation of the results.

Thus, the realization of this approach to the rational design of a transmission under several criteria, with their merging into one, and numerical solutions of the problem is actual.

2 Design parameters and design criteria

In order to solve the problem of optimal design, the following design parameters of the gear drive were adopted as variables planning [3, 5]: m_μ – the corresponding modules of gears pairs (μ = 1, 2); $z_{\mu,k}$ – the corresponding number of teeth, k – the number of wheel in the mesh (k = 1 – driving wheel, k = 2 – driven wheel); β_μ – the helix angle of the teeth.

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Let's consider proposed quality criteria for a distribution gearbox.

The objective function of the criterion of minimum center distance for a coaxial arrangement is presented as [3, 5]:

$$F_a = a_{w1} = a_{w2} = 0.5 \cdot m_1 \cdot \left(z_{1,1} + z_{1,2}\right) \cdot \frac{1}{\cos\beta_1} = \\ = 0.5 \cdot m_2 \cdot \left(z_{2,1} + z_{2,2}\right) \cdot \frac{1}{\cos\beta_2}, \; F_a \to \min. \tag{1}$$

Objective function in the case when the criterion of optimality is the minimum length of the drive represented as the sum of the width of the toothed wheels, without other geometric indicators (sizes of gaps, bearings, synchronizers, etc.). This sum of width exactly characterizes the specified criterion, that is:

$$F_L = \sum_{\mu=1}^{s} b_{w\mu}, \quad F_L \to \min. \tag{2}$$

Objective function in the case when the criterion of optimality is the minimum mass of the gear drive. The main mass of the gearbox consists of masses of the following elements: gears, shafts, bearings and crankcase. But for solving the presented problem, it is proposed to evaluate only the masses of the gears. Let's write the objective function in the form:

$$F_M = \sum_{j=1}^{r} M_j, \quad F_M \to \min. \tag{3}$$

The objective function in the case when the criterion of optimality is the probability of failure-free operation (P). It is proposed [6] to represent in the form of product the probabilities of failure-free operation of drive by contact and bending:

$$F_P = p(K_{nH1}) \cdot p(K_{nF11}) \cdot p(K_{nF12}) \times \\ \times p(K_{nH2}) \cdot p(K_{nF21}) \cdot p(K_{nF22}), \\ F_P \to \max. \tag{4}$$

3 Limitations and functional relationships between constructive parameters

1) The center distances of meshes for the coaxial arrangement should be equal, that is:

$$a_{w1} = a_{w2}. \tag{5}$$

2) The teeth of the wheels must have the necessary contact durability:

$$\sigma_{H\mu} \le \sigma_{HP\mu}. \tag{6}$$

3) The teeth of the wheels must have the required bending strength:

$$\sigma_{F\mu,k} \le \sigma_{FP\mu,k}. \tag{7}$$

4) The teeth module is the main parameter of the gearing. They are standardized. We accept the following row for calculations:

$$m_\mu = 1;\ldots;\,8 \text{ mm}. \tag{8}$$

5) The teeth number must accept integers (must be natural – *N*), and also limited to the upper and lower values for manufacturing technology reasons:

$$z_{\mu,k} \in N; \quad z_{\min} \le z_{\mu,k} \le z_{\max}. \tag{9}$$

6) From the requirement of the overall relationship, the gear ration must not exceed a certain value ($u_{\max}$):

$$u_\mu = \frac{\max(z_{\mu,1}, z_{\mu,2})}{\min(z_{\mu,1}, z_{\mu,2})} \le u_{\max}. \tag{10}$$

7) The helix angle of the teeth must be in the range from $\beta_{\min}$ to $\beta_{\max}$:

$$\beta_{\min} \le \beta_\mu \le \beta_{\max}. \tag{11}$$

8) The face width factor is also limited to extreme values:

$$\psi_{bd\,\mu\min} \le \psi_{bd\,\mu} \le \psi_{bd\,\mu\max}. \tag{12}$$

9) The condition for the absence of sharpening tooth tip pointing:

$$s_{a\mu} \ge 0.4 \cdot m_\mu. \tag{13}$$

4 Approach and sequence of problem solving

As is known from [3], the LPτ-search method is based on LPτ-uniformly-distributed sequences and allows operating a large number of parameters (up to 51) and uniformly-distributed test points (up to 2^{20}).

The approach is based on the research positions all possible space of parameters (*W*) by the points of the LPτ-uniformly-distributed (A_i) sequence. This space of parameters is determined by the technical and technological requirements for the type of drive.

Then the points are checked in a certain sequence, which allows to timely manner clear the "inappropriate" points and thus reduce the time of computer solution. From the points that have passed the test, a set is created that satisfies the design conditions (Q, $Q \in W$).

The linear curtailment of criteria for solving multicriterion optimization problems is used quite often. But linear curtailment has a significant drawback – the value of the resulting function has no physical content.

The authors were proposed to depart from linear curtailment [7] and to combine the criteria in the sequence discussed below.

The designer is invited to introduce a **scale of importance** that will be applied to all criteria. In this scale, the importance (α) can vary from 0 to $\alpha_{\max}$ at step 1, the value $\alpha_{\max}$ is also chosen by the designer on its own: $\alpha = 0, 1, 2, ..., \alpha_{\max}$; thus, the designer can independently choose the level of discreteness for the scale of importance. The value $\alpha = 0$ corresponds to the absolute priority of the criterion, the value $\alpha = \alpha_{\max}$ corresponds to the relative unimportance of the criterion. For each of the criteria (F_a, F_L, F_M, F_P), the designer independently assigns the relevant values of **importances** ($\alpha_a, \alpha_L, \alpha_M, \alpha_P$). Importance's can be assigned to values within the accepted scale. The situation of equality of the importance's values for any criteria is possible. That allows implementing hierarchical, binary or any other connections and correlation between the criteria.

After that, the generation of test points of LPτ-uniformly-distributed sequences of the whole possible space of parameters is carried out. Test points are checked for constraints and functional dependencies (5–13), the selected points form a set of solutions Q.

For all points of a set Q, the values of all criteria are calculated separately for the corresponding objective functions (1-4). After, the maximum ($F_{a\max}, F_{L\max}, F_{M\max}, F_{P\max}$) and the minimum ($F_{a\min}, F_{L\min}, F_{M\min}, F_{P\min}$) value for each criterion are determined.

The following is proposed to calculate the **criterion step** [7]:

$$R_u = \frac{F_{u\max} - F_{u\min}}{(\alpha_{\max} + 1)}, u = a, L, M, P. \quad (14)$$

The next step is to determine of **required solution offset** of the relative to the current solutions for each of the criteria [4], for each (s) the point of the set Q:

$$\begin{cases} \mathrm{E}_{a\,s} = \dfrac{\left|\left(F_{a\min} + \alpha_a \cdot R_a\right) - F_{a\,s}\right|}{F_{a\,s}}; \\ \mathrm{E}_{L\,s} = \dfrac{\left|\left(F_{L\min} + \alpha_L \cdot R_L\right) - F_{L\,s}\right|}{F_{L\,s}}; \\ \mathrm{E}_{M\,s} = \dfrac{\left|\left(F_{M\min} + \alpha_M \cdot R_M\right) - F_{M\,s}\right|}{F_{M\,s}}; \\ \mathrm{E}_{P\,s} = \dfrac{\left|\left(F_{P\max} - \alpha_P \cdot R_P\right) - F_{P\,s}\right|}{F_{P\,s}}. \end{cases} \quad (15)$$

Then combine the criteria for the required solution offset relative to the actual as arithmetic mean square-weighted:

$$E_s = \sqrt{\frac{\sum_u \left(E_{u\,s}^2 \cdot \alpha_u\right)}{\sum_u \alpha_u}}, \quad E_s \to \min. \quad (16)$$

Thus, the designer has the possibility to solve multicriteria tasks, turning them into one-criterion, and the introduced criterion has a physical content – the relative approximation of the test point to the desired solution.

5 Software implementations of the optimization algorithm

In view of the presented algorithm [6], the optimal rational design of the gearbox, which combines the given approach, the mathematical model of the problem (objective functions and limits of the variables planning) and rational logical sequences of operations, developed an integrated program complex. Its implementation was carried out in the software Delphi 7, because the program language of this package enables to describe the algorithm qualitatively and rationally, it is quite easy to use.

The received program has a block-procedural structure, therefore it can be easily modified at the request of the designer, which enables to carry out optimally rational design of other types of transmission with fixed parallel shafts.

Also, one of the advantages of Delphi 7 is the ability to create a visual shell program as a standard window. This allows the user to easily enter the necessary input data and easy view the data received.

6 A graphical representation of the calculation results

The authors proposed to present of calculations results in graphical form for a more visual representation.

The calculations results are presented in the form of closed polygonal graph. The graph is based on the axes, the number of which corresponds to the number of criteria.

Axes are locating in the plane of the sheet radially and uniformly. On each axis marked equal segments.

The beginning and end of the segments correspond to the limits of the existence of the criteria solution of within the task.

Figure 1 shows a graphical representation of the solution to the problem of optimally rational design of a distribution gearbox.

Two graphs are shown, with and without a unifying criterion.

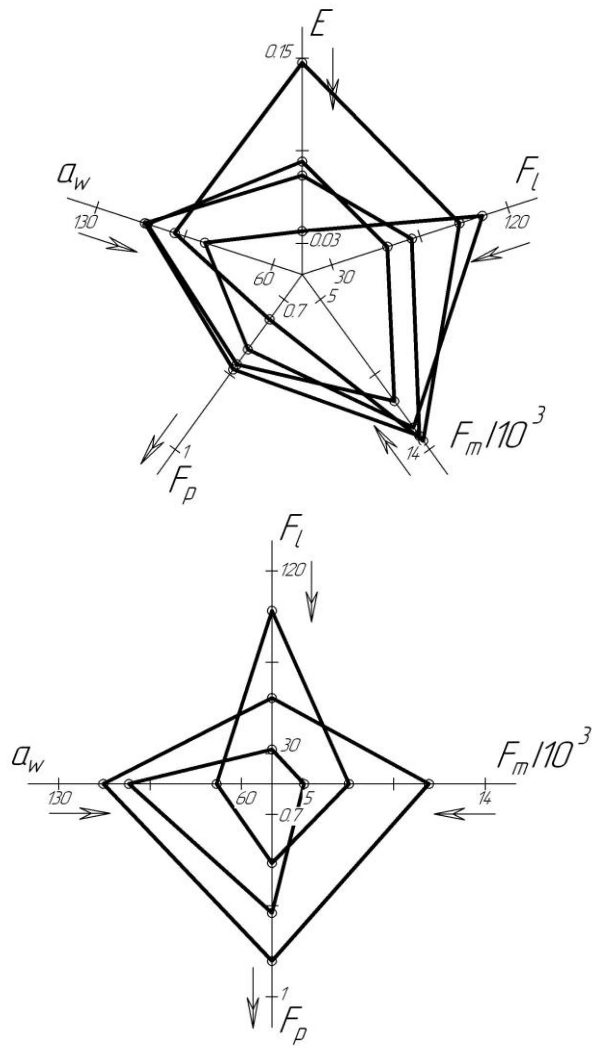

Fig. 1. Polygonal graphs solution.

7 Conclusions

1. The relevance of the task is considered. The necessity to develop an approach to the design of transmission with rational design parameters with several criteria, with their transformation in one is described.

2. Objective functions of the most significant criteria and limits of the variables planning are recorded. The given objective functions allow the designer to choose one or several quality criteria. The structure of the objective functions is logical and concise, and they can be supplemented of necessary refinement applications.

3. A new approach to solving multicriteria problems of a transmission rational design, the essence of which is to combine the criteria into one, is proposed. The approach is based on the analysis of test points obtained using LPτ-search, and further processing of the received information.

4. It is proposed to replace many criteria to one by introducing a scale of importance to the designer and assigning the importance of each criterion. For each test point required solution desired is calculate, which is proposed to be used as a unifying the criteria.

5. A numerical experiments were conducted. Its target is testing the proposed method for unifying the criteria and obtaining adequate output data. The universality and convenience of the proposed software system made it possible to quickly change the concept of a numerical experiment (one-criteria or multi-criteria design) and to comfort vary input parameters.

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