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SIMULATION OF COMPRESSIBLE AND INCOMPRESSIBLE FLOWS IN THE PRESENCE OF SHOCKS

H.BENAKRACH[1*], M. TAHA-JANAN[1] and M.Z. ES-SADEK
1. Laboratory of Applied Mechanics and Technologies
Industrial and Health Science and Technology Research Center (STIS)
ENSET, Mohammed V University in RABAT Avenue des Forces Armées, B.P 6207
Rabat-Instituts, MOROCCO – Corresponding author (*):hind.benakrach@um5s.net.ma

Abstract

The purpose of the present work is to use a finite volume method for solving Euler equations in the presence of shocks and discontinuities, with a generalized equation of state. This last choice allows to treat both compressible and incompressible fluids. The first results of the work are presented. They consist in simulating two-dimensional single-specie flows in the presence of shocks.

The results obtained are compared with the analytical results considered as benchmarks in the domain.

Keywords - *Euler's equations; shocks; discontinuities; generalized equation of state.*

Introduction

Flows with discontinuities can be simulated by different numerical methods. The correct description of this type of flow requires a careful choice of equations to be used. The mathematical model used is based on the resolution of Euler's equations, supplemented by a model that governs the distribution of species.

The presence of discontinuities reveals a particular directional criterion for the propagation of information in this type of flows. This difficulty leads to special provisions such as the use of flow decomposition schemes.

In the context of this work, we are interested in simulating two-dimensional single-specie flows in a cartesian coordinate system. To this end, the model was inspired by the work of Taha-Janan [1], Taha-Janan and El Marjani [2]. Our results will be compared to academic cases. A calculation code in C++ language known for its low calculation cost has been developed.

1. Model description

The governing equations of a non-viscous fluid flow in the presence of shocks are those of Euler’s associated with a species distribution model, which are used to determine the mass or volume concentration of the species.

Solving these equations by finite differences generates two major problems. The first problem is related to the calculation of fractions, while the second one is that of pressure oscillations near interfaces and shocks. To avoid these problems, the choice of the gamma model presented in Shyue's work [3] was considered appropriate for the situations we intend to treat.

Roe's flux difference , aligned with the mesh, was chosen thanks to its performance in various applications [4], [5]. Its association with the "gamma" model, in the general case of a curvilinear coordinate system, has shown its effectiveness for single- and multi-species flows, for compressible or incompressible fluid, in the presence of a shock [1]. This study also demonstrated that the use of the two-parameter general state equation makes it possible to switch from a situation of a compressible fluid to that of an incompressible fluid with a single formulation.

In order to properly treat areas of high gradients, including shocks and discontinuities, the use of the Van Leer limiter [5] was adopted following the comparative study by V. Daru and C. Tenaud [6]. This limiter is used to reduce small oscillations that occur mainly for static pressure, it is based on the TVD (Total Variation Diminishing) criterion [6].

2. Results and discussions

2.1 Validation test - 1D -

A one-dimensional Sod shock tube is used to test the method's ability to capture shocks. This tube contains a fluid that is initially located in two equal regions separated by a diaphragm. The fluid on the left is characterized by $(\rho, u, p)_L=(1., 0, 1.)$, while the fluid in the right of the tube is characterized by $(\rho, u, p)_R=(0.125, 0, 0.1)$. When the diaphragm ruptures, a shock wave and contact discontinuity will propagate to the right, causing the fluid to move downstream. A boundary layer is developed on the bottom wall of the tube. A relaxation wave is also spread to the left [7].

Figures 1, 2 and 3 show the evolution of the pressure, density and velocity of the fluid along the tube. The curves in each figure compare the analytical solution (straight line) with the calculated results. The curves of the latter illustrate the excess diffusion introduced with a shift of the 1st order leading to a smoothing of the discontinuities, and the oscillations caused by the use of a 2^{nd} order spatial precision discretization without limiter. This shows the important role of the limiter used in our model.

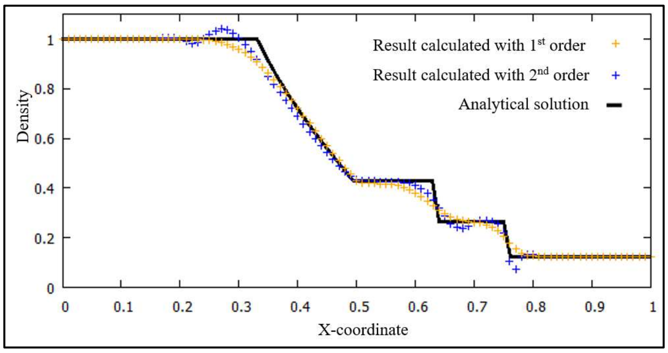

Figure 1- Evolution of the density along the Sod impact tube at t = 0.144.

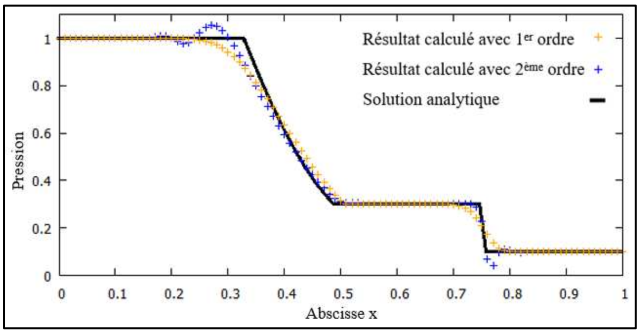

Figure 2 - Pressure evolution along the Sod impact tube at t = 0.144.

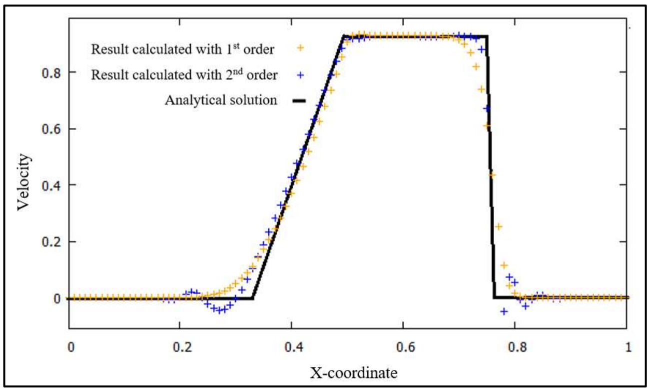

Figure 3 - Evolution of the velocity along the Sod impact tube at t = 0.144.

This test allows us to validate the ability of our model to capture shocks with acuity.

2.2 Validation test - 2D -

A 2D Sod shock tube was tested to validate the used method, so the spatial domain was chosen at [0, 1] in the x-direction and [0, 1] in the y-direction. Therefore the fluid is initially characterized by $(\rho, u, v, p)_L=(1., 0, 0, 1.)$ and $(\rho, u, v, p)_R=(0.125, 0, 0, 0.1)$.

The Bresenham Arc Plotting Algorithm [8] was used to achieve the circular shape, a regular mesh was prepared, with 101x101 points, to test the capacity of the 2D method with an shock wave [9].

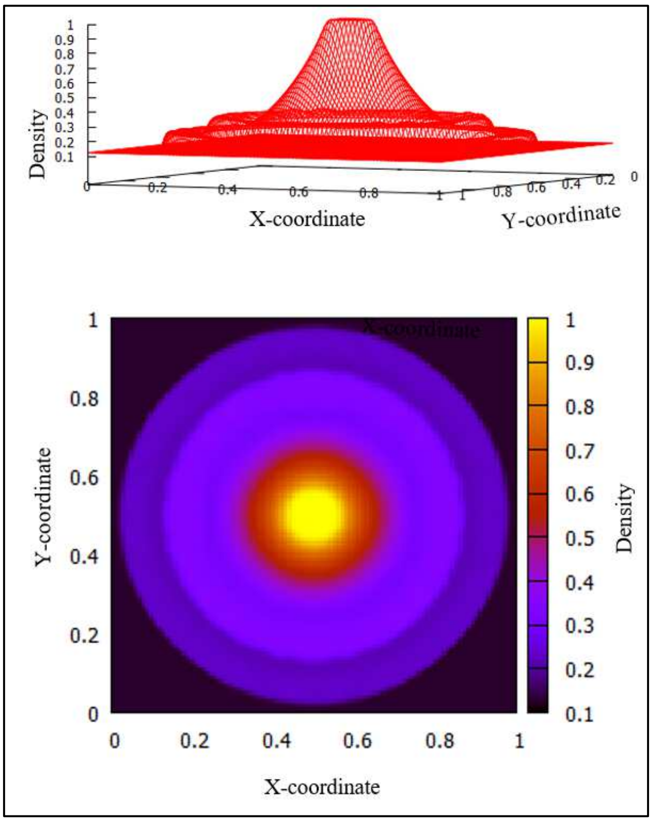

Figure 4 - Evolution of the density for a 2D Sod shock at t=0.144.

Figures 4, 5 and 6 show that despite the use of a square mesh in Cartesian coordinates, the model's behaviour remains correct towards the shocks.

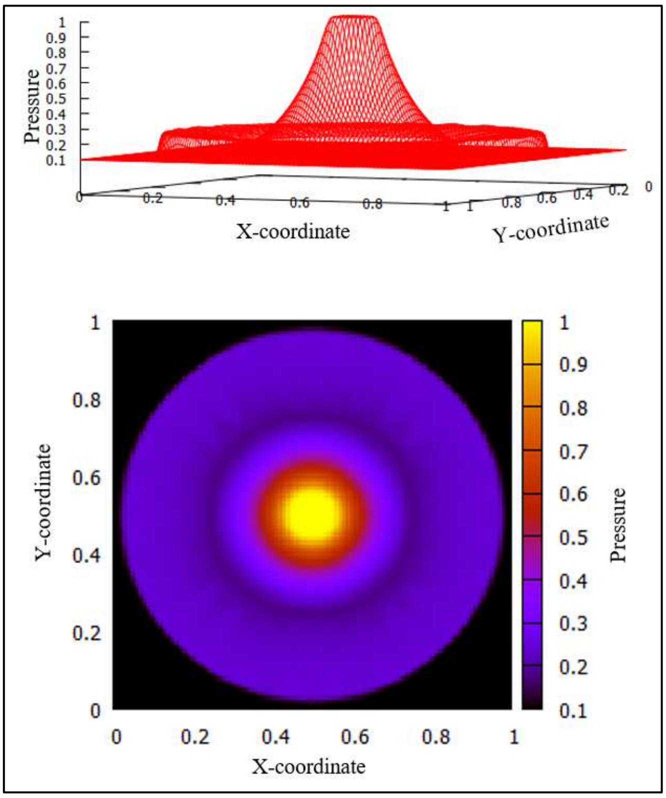

Figure 5 - Pressure evolution for a 2D Sod shock at t=0.144.

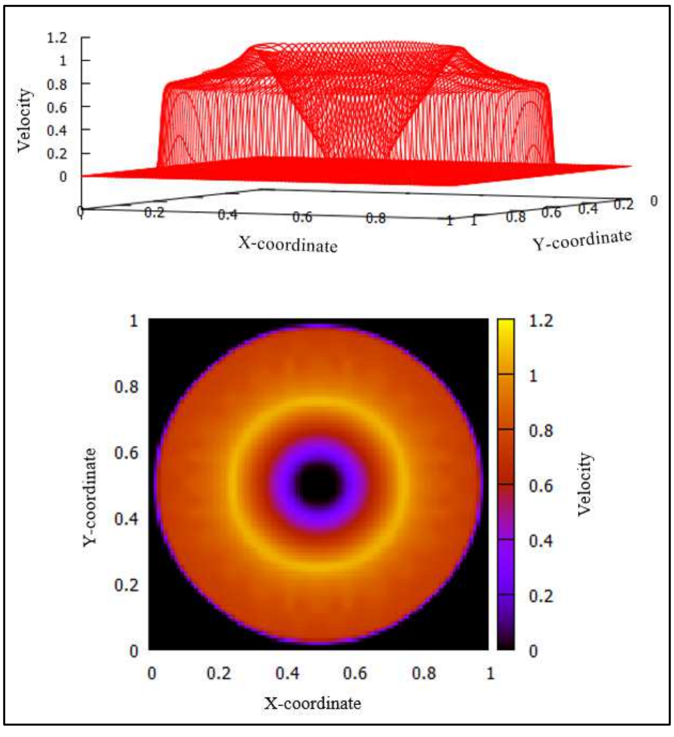

Figure 6 - Velocity evolution for a 2D Sod shock at t=0.144.

Concluding remarks

Based on the results obtained, we can conclude that the mathematical formulation used for the resolution of two-dimensional single-specie flows in the presence of discontinuity is robust and able to capture shocks with acuity.

In order to improve this method, it is interesting to develop the program for multi-species flows in three-dimensional cases.

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