

Modeling and simulation of an underactuated system

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Abstract :

One of the most active research areas in mechatronic systems is the control of mechanical systems controlled by electronic systems using computer programs. These programs execute algorithms called control laws.

Our study focuses on the control of underactuated mechanical systems: Case of a reversed two-wheeled pendulum. It consists of developing a control law to stabilize this system. This class of system is rich in practical as well as theoretical applications (SEGWAY, Acrobot robots ...) and this is why the control synthesis for underactuated mechanical systems constitutes a very active research axis and still constitutes an open domain for technological research.

Keywords: Mechatronics, Underactuated, Nonlinear, Modeling, Simulation

1- Introduction

Nowadays, the development of mechatronic systems creates a real revolution in the industrial world. Indeed, in our daily life, the majority of the systems we operate are the mechatronic ones. A mechatronic system is an heterogeneous system combining subsystems and different phenomena from various engineering sectors: thermal, automatic, electrical, etc.

The mechanical systems can be grouped in three major classes according to their degree of actuation. Thus, a mechanical system can be fully actuated, over-actuated (or redundant), or underactuated:

- The fully actuated mechanical systems have as many actuators as degrees of freedom; each degree of freedom can be individually controlled, which probably makes this category of mechanical systems the easiest to control.
- The over-actuated or redundant mechanical systems have more actuators than degrees of freedom.
- The underactuated mechanical systems have fewer actuators than degrees of freedom. This class includes most land, marine, submarine and air vehicles, but also walking robots (non-rigid bodies). They have several advantages: weight reduction, production costs and energy expenditure ... On the other hand, they have dynamic constraints that are generally non-linear and non-integrable.

This paper aims to study the modeling and simulation of a SEGWAY which can be a very good example of a mechatronic system.

2 – The Modeling Underactuated Systems

It should be emphasized that none of the techniques proposed in the literature can universally apply to any underactuated mechanical system. The main underactuated mechanical systems used in the literature are: **i)** The pendulum on a cart (PC), **ii)** The wheeled inverted pendulum (WIP), **iii)** The planar manipulator (PM).

As for the main theses that have dealt with the evaluation of the reliability of mechatronic systems; these are the following:

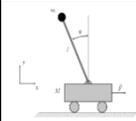
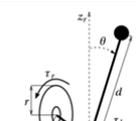
			
Partial linearization	[OS99]	[HS08]	[DLMO00]
Backstepping	[OS99]		
Passivity / energy	[BLM99]		
Sliding mode		[HGM*10]	[BLX95]
Approximation / linearization		[KKK05]	[DLMO00]

Table n° 1 : Different underactuated mechanical systems and their control methods

3- Modeling of a SEGWAY

A Segway can be modeled by a wheeled inverted pendulum. This underactuated mechanical system has been particularly studied because, despite its simplicity, it indicates a practical interest for locomotion, as evidenced by the commercialization of the SEGWAY, a compact personal vehicle based on the model of the wheeled inverted pendulum. The wheeled inverted pendulum is schematically shown in the following figure:

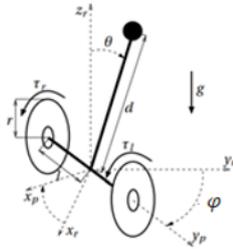


Figure 1 : wheeled inverted pendulum.

The Parameters of our system are the following :

Parameters	Definition	Unity
θ	Pendulum angle to the vertical	rad
φ	Pendulum orientation in a horizontal plane (x _r ,y _r),	rad
x	Position along the x _p axis	m
d	Distance between the rotating wheels axis centre and the pendulum gravity centre	m
m_s, m_c	Respective masses of the pendulum and of each one of the wheels	kg
l	Half distance between the wheels	m
R, r	Wheel radius	m
I_{2p}, I_{3p} I_2, I_3	Pendulum inertia respectively around the z _p axis aligned with the pendulum, and the y _p axis aligned with the wheels axis.	kg m ²
t_l, t_r α_3, β_3	Couples	N m

Table n° 2 : Variables used in the modeling

-The Dynamic Model :

The dynamic model of the wheeled inverted pendulum is given by [KKK05]:

$$3(m_c+m_s)\ddot{x} - m_s d \cos \theta \ddot{\theta} + m_s d \sin \theta (\dot{\theta}^2 + \dot{\varphi}^2) = -\frac{t_l+t_r}{r}$$

$$[(3l^2 + \frac{r^2}{2})m_c + m_s d^2 \sin^2 \theta + I_{2p}] \ddot{\varphi} + m_s d^2 \sin \theta \cos \theta \dot{\varphi} \dot{\theta} = \frac{l(t_l-t_r)}{r}$$

$$m_s d \cos \theta \ddot{x} + (-m_s d^2 - I_{3p}) \ddot{\theta} + m_s d^2 \sin \theta \cos \theta \dot{\theta}^2 + m_s g d \sin \theta = t_l + t_r$$

(1)

The dynamic model gives nonlinear equations of the system's motion. For the resolution of these equations, we linearize them around the equilibrium position. In this position, the system is in its quasi-equilibrium state. So, we could develop the linearized model under the assumption that the variation of the inclination angle is small enough to be neglected. We then have the three linearized equations of motion on the state of equilibrium as follows:

$$\ddot{x} = \frac{m_s^2 d^2 g}{3m_c I_3 + 3m_c m_s d^2 + m_s I_3} \varphi - \frac{(m_s d^2 + I_3)R + m_s d}{3m_c I_3 + 3m_c m_s d^2 + m_s I_3} (\alpha_3 + \beta_3)$$

(2)

$$\ddot{\theta} = \frac{2L/R}{6m_c L^2 + m_c R^2 + 2I_2} (\alpha_3 - \beta_3)$$

(3)

$$\ddot{\varphi} = \frac{m_s d g (3m_c + m_s)}{3m_c I_3 + 3m_c m_s d^2 + m_s I_3} \varphi - \frac{(m_s d)/R + 3m_s m_c}{3m_c I_3 + 3m_c m_s d^2 + m_s I_3} (\alpha_3 + \beta_3)$$

(4)

After linearization, we reorganized it in the form of the following a state space:

$$\dot{x} = AX + Bu$$

(5)

$$y = CX + Du$$

(6)

Where the vectors X and u are given by:

$$X = [x \ \dot{x} \ \theta \ \dot{\theta} \ \varphi \ \dot{\varphi}]^t, \quad u = [\alpha_3 \ \beta_3]^t$$

(7)

The matrixes A and B can be identified as in the equations below:

(8)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{m_s^2 d^2 g}{3m_c I_3 + 3m_c m_s d^2 + m_s I_3} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{m_s d g (3m_c + m_s)}{3m_c I_3 + 3m_c m_s d^2 + m_s I_3} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ -\frac{(m_s d^2 + I_3)}{R} - m_s d & -\frac{(m_s d^2 + I_3)}{R} - m_s d \\ \frac{0}{3m_c I_3 + 3m_c m_s d^2 + m_s I_3} & \frac{0}{3m_c I_3 + 3m_c m_s d^2 + m_s I_3} \\ \frac{2L/R}{6m_c L^2 + m_c R^2 + 2I_2} & \frac{-2L/R}{6m_c L^2 + m_c R^2 + 2I_2} \\ \frac{0}{-m_s d/R - 3m_c m_s} & \frac{0}{-m_s d/R - 3m_c m_s} \\ \frac{0}{3m_c I_3 + 3m_c m_s d^2 + m_s I_3} & \frac{0}{3m_c I_3 + 3m_c m_s d^2 + m_s I_3} \end{bmatrix}$$

(9)

The matrix C is defined as an identity matrix (6, 6) and the matrix D as a null matrix, which gives y = X.

4-Wheeled inverted pendulum response

To study the stability of the system we will determine the impulse response of the open-loop system using MATLAB.

MATLAB is the reference tool for numerical simulation. It offers advanced possibilities in terms of identification or control. More generally, it allows solving a wide variety of simulation problems.

Simulation are carried out in an environment MATLAB/SIMULINK. The differential equation governing the dynamics of the system are integrated by using the method Runge-Kutta(function ode45 of MATLAB) .

Figure 2 present diagram SIMULINK

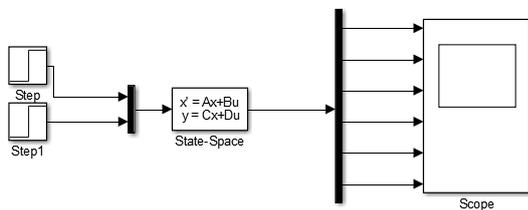


Figure 2 : diagram SIMULINK

Apply to the input an amplitude, we will get the following results

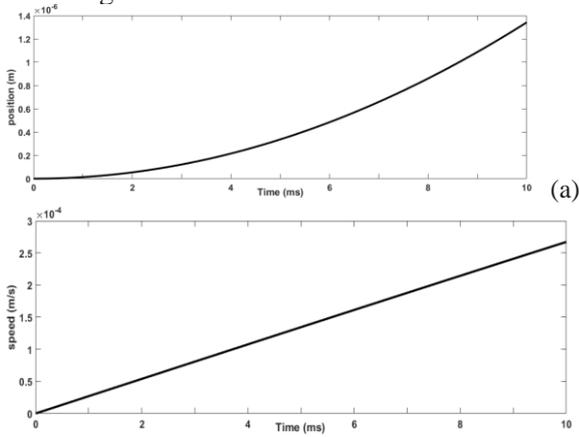


Figure 2-1 : impulse response of the system x (a) & \dot{x} (b)

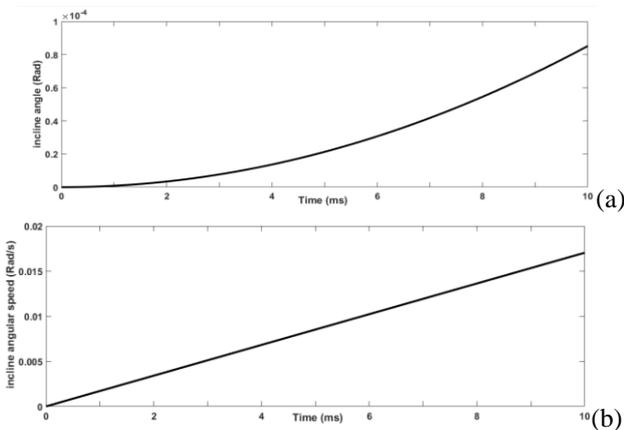


Figure 2-2 : impulse response of the system θ (a) & $\dot{\theta}$ (b)

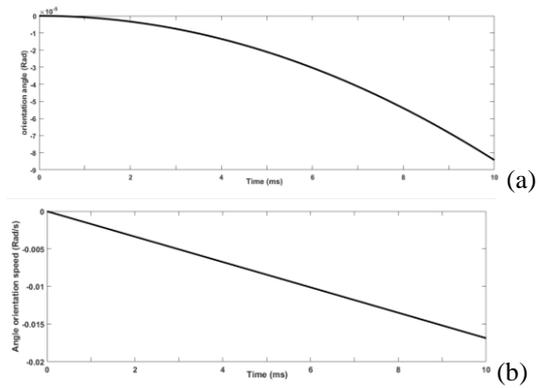


Figure 2-3 : impulse response of the system φ (a) et $\dot{\varphi}$ (b)

Figures 2 show that the kinematic parameters of the system increase over time, thus indicating the instability of the system.

Conclusion :

In this paper we proposed the modeling and simulation of a Segway using the wheeled inverted pendulum (WIP). We have shown that the open loop control is unstable, which then requires the determination of the proper closed loop control law in order to make it stable.

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