MODELISATION TRIDIMENSIONNELLE DU COMPORTEMENT STATIQUE DES PLAQUES MULTICOUCHES MAGNETO-ELECTRO-ELASTIQUE REPOSANTES SUR UN SUPPORT ELASTIQUE

THREE-DIMENSIONAL MODELING OF THE STATIC BEHAVIOR OF MAGNETO-ELECTRO-ELASTIC MULTILAYER PLATES BASED ON AN ELASTIC SUPPORT

M. HAMIDI[1], S. ZAKI[1,2], M. ABOUSSALEH[1]

1. Equipe M2I, Laboratoire des Sciences et Métiers de l'Ingénieur, Université Moulay Ismail, ENSAM Meknès, Maroc
2. Equipe M2AI, Laboratoire Mécanique Mécatronique et Commande, Université Moulay Ismail, ENSAM Meknès, Maroc

Abstract

In this communication, the state space method is used to analyze the static behavior of laminated magneto-electro-elastic rectangular plates with simply supported boundary conditions based on an elastic support. The mathematical formulation is elaborated in a general form and an arbitrary number of layers as well as the orthotropic behavior can be considered. The methodology is based on the mixed formulation, in which basic unknowns are formed by collecting displacements, stresses, electrical displacements, electrical potential, magnetic induction and magnetic potential. As special case, multilayered rectangular plate is analyzed under the surface loading with simply supported boundary conditions based on an elastic support. The procedure of calculation shows that the formulation presented here is simple and direct.

Résumé

Dans cette communication, la méthode d'espace d'état est utilisée pour analyser le comportement statique des plaques rectangulaires magnéto-électro-élastiques avec des conditions aux limites simplement appuyés basées sur un support élastique. La formulation mathématique est élaborée sous une forme générale avec un nombre arbitraire de couches et tenant en compte le comportement orthotrope des matériaux. La méthodologie est basée sur la formulation mixte, dans laquelle des inconnues de base sont formées en collectant les déplacements, les contraintes, les déplacements électriques, le potentiel électrique, l'induction magnétique et le potentiel magnétique. En cas particulier, la plaque rectangulaire multicouche est analysée sous une charge de surface avec des conditions aux limites simplement appuyées basées sur un support élastique. La procédure de calcul montre que la formulation présentée ici est simple et directe.

Mots clefs: *Plaque multicouche, magnéto-électro-élastique, support élastique, méthode d'espace d'état, statique.*

Keywords: *Multilayer plates, magneto-electro-elastic, elastic support, state space method, static.*

1. Introduction

The multilayered magneto-electro-elastic plates are in nowadays an important component in recent smart and intelligent structures. These materials exhibit magneto-electric-mechanical coupling effect in that they produce an electric field and a magnetic field when deformed and, conversely, undergo deformation when subjected to an electric field or a magnetic field.

Some exact results of three-dimensional (3D) static analyses of single-layer and multiple-layer rectangular plates are also available in the literature. S. Zaki [1] treated the static behavior of multilayered elastic plates, with different orthotropic angles of fibers, resting on the Winkler-Pasternak elastic foundation. J. Wang [2,3] have developed an exact 3D solution for the static behavior of multilayered magneto-electro-elastic plate subjected to mechanical and electrical loading [2]. They have also studied the free vibration of the same plate after applying the electric potential on the top and on the bottom surfaces of the plate [3].

In this communication, we derived an analytical 3D solution for the static behavior of multilayered magneto-electro-elastic plate with simply supported boundary conditions based on a Winkler-Pasternak elastic foundation.

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2. Mathematical modeling

2.1 Constitutive equations

Let us consider an *N*-layered magneto-electro-elastic rectangular plate under simply supported edge conditions. The dimensions of the plate are $Lx \times Ly \times H$ (Lx, Ly and H respectively being the length, the width and the depth). We assume that the layers of the plate are orthotropic. Layer j is bonded by the lower interface z_{j-1} and the upper interface z_j with thickness $h_j = z_j - z_{j-1}$. It is obvious that $z_0 = 0$ and $z_N = H = \sum_{j=1}^{N} h_j$.

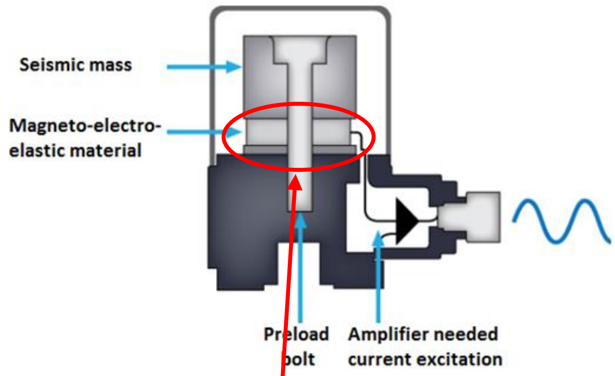

Figure 1: Schematic representation of vibration sensor

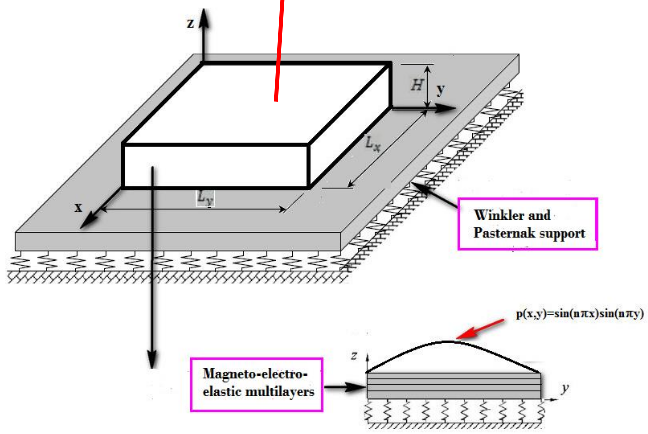

Figure 2: Schematic representation of magneto-electro-elastic multilayer based on elastic support

Out of the orthotropic axis of the layer *j*, the behavior laws of magneto-electro-elastic materials are [4]:

$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix} = [\bar{C}]\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix} - [\bar{e}]\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix} - [\bar{q}]\begin{pmatrix}H_x\\ H_y\\ H_z\end{pmatrix} \quad (1.a)$$

$$\begin{pmatrix}D_x\\ D_y\\ D_z\end{pmatrix} = [\bar{e}]^T\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix} + [\bar{\varepsilon}]\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix} + [\bar{d}]\begin{pmatrix}H_x\\ H_y\\ H_z\end{pmatrix} \quad (1.b)$$

$$\begin{pmatrix}B_x\\ B_y\\ B_z\end{pmatrix} = [\bar{q}]^T\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix} + [\bar{d}]\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix} + [\bar{\mu}]\begin{pmatrix}H_x\\ H_y\\ H_z\end{pmatrix} \quad (1.c)$$

$$[\bar{C}] = \begin{pmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16}\\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26}\\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36}\\ 0 & 0 & 0 & C_{44} & C_{45} & 0\\ 0 & 0 & 0 & C_{45} & C_{55} & 0\\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66}\end{pmatrix};\ [\bar{e}] = \begin{pmatrix}0 & 0 & e_{31}\\ 0 & 0 & e_{32}\\ 0 & 0 & e_{33}\\ e_{14} & e_{24} & 0\\ e_{15} & e_{25} & 0\\ 0 & 0 & e_{36}\end{pmatrix}$$

$$[\bar{q}] = \begin{pmatrix}0 & 0 & q_{31}\\ 0 & 0 & q_{32}\\ 0 & 0 & q_{33}\\ q_{14} & q_{24} & 0\\ q_{15} & q_{25} & 0\\ 0 & 0 & q_{36}\end{pmatrix};\ [\bar{\varepsilon}] = \begin{pmatrix}\varepsilon_{11} & \varepsilon_{12} & 0\\ \varepsilon_{12} & \varepsilon_{22} & 0\\ 0 & 0 & \varepsilon_{33}\end{pmatrix};$$

$$[\bar{d}] = \begin{pmatrix}d_{11} & d_{12} & 0\\ d_{12} & d_{22} & 0\\ 0 & 0 & d_{33}\end{pmatrix};\ [\bar{\mu}] = \begin{pmatrix}\mu_{11} & \mu_{12} & 0\\ \mu_{12} & \mu_{22} & 0\\ 0 & 0 & \mu_{33}\end{pmatrix}$$

In these relationships, D_x, D_y, D_z and B_x, B_y, B_z are the components of the electric and magnetic displacements respectively. The parameters E_x, E_y, E_z and H_x, H_y, H_z are the components of the electric and magnetic fields respectively, the terms [C],[ε] and [μ] being the elastic, dielectric, and magnetic permeability coefficients, respectively. The terms [e], [q] and [d] are the piezoelectric, piezomagnetic, and magnetoelectric coefficients, respectively.

2.2 Static solution of multilayered plates

The plate being simply supported on its borders, the boundary conditions are:

$$\begin{cases} u_x(x,0,z) = u_x(x, L_y, z) = u_y(0,y,z) = u_y(L_x,y,z) = 0\\ u_z(0, L_y,z) = u_z(L_x,0,z) = u_z(L_x,L_y,z) = u_z(0,0,z) = 0\\ \Phi(0, L_y,z) = \Phi(L_x,0,z) = \Phi(L_x,L_y,z) = \Phi(0,0,z) = 0\\ \Psi(0, L_y,z) = \Psi(L_x,0,z) = \Psi(L_x,L_y,z) = \Psi(0,0,z) = 0\end{cases}$$

where $u_x(x,y,z), u_y(x,y,z), u_z(x,y,z)$ are the elastic displacements, Φ (x,y,z) is the electric potential and $\psi(x,y,z,t)$ is magnetic potential at the point (x,y,z). Based on the Fourier series ([2], [4]), the solution components are assumed in the following form:

$$\begin{Bmatrix}u_x(x,y,z)\\ u_y(x,y,z)\\ u_z(x,y,z)\\ \phi(x,y,z)\\ \psi(x,y,z)\end{Bmatrix} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\begin{Bmatrix}U_{mn}(z)\cos(px)\sin(qy)\\ V_{mn}(z)\sin(px)\cos(qy)\\ W_{mn}(z)\sin(px)\sin(qy)\\ \phi_{mn}(z)\sin(px)\sin(qy)\\ \psi_{mn}(z)\sin(px)\sin(qy)\end{Bmatrix}$$

And

For: $p=\frac{n\pi}{L_x}$; $q=\frac{m\pi}{L_y}$,

$0 \le x \le L_x$; $0 \le y \le L_y$; $z_{j-1} \le z \le z_j$; m,n ∈ N*

$$\begin{Bmatrix} \sigma_{xx}(x,y,z) \\ \sigma_{yy}(x,y,z) \\ \sigma_{zz}(x,y,z) \\ \tau_{yz}(x,y,z) \\ \tau_{xz}(x,y,z) \\ \tau_{xy}(x,y,z) \\ D_x(x,y,z) \\ D_y(x,y,z) \\ D_z(x,y,z) \\ B_x(x,y,z) \\ B_y(x,y,z) \\ B_z(x,y,z) \end{Bmatrix} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \begin{Bmatrix} \sigma_{xxmn}(z)\sin(px)\sin(qy) \\ \sigma_{yymn}(z)\sin(px)\sin(qy) \\ \sigma_{zzmn}(z)\sin(px)\sin(qy) \\ \tau_{yzmn}(z)\sin(px)\cos(qy) \\ \tau_{xzmn}(z)\cos(px)\sin(qy) \\ \tau_{xymn}(z)\cos(px)\cos(qy) \\ D_{xmn}(z)\cos(px)\sin(qy) \\ D_{ymn}(z)\sin(px)\cos(qy) \\ D_{zmn}(z)\sin(px)\sin(qy) \\ B_{xmn}(z)\cos(px)\sin(qy) \\ B_{ymn}(z)\sin(px)\cos(qy) \\ B_{zmn}(z)\sin(px)\sin(qy) \end{Bmatrix}$$

The state vector approach is based on the mixed formulation of solid mechanics in which $U_{mn}, V_{mn}, D_{zmn}, B_{zmn}, \sigma_{zzmn}, \tau_{xzmn}, \tau_{yzm}, \phi_{mn}, \psi_{mn}$ and W_{mn} are taken as basic unknowns. The field equations can be recast in the following matrix form ([2][3],[4]):

$$\frac{\partial}{\partial z}\eta_{mn}^{1}(z) = A_{mn}^{(j)}\eta_{mn}^{1}(z) \quad \text{and} \quad \eta_{mn}^{2}(z) = B_{mn}^{(j)}\eta_{mn}^{1}(z)$$

Where:

$$\eta_{mn}^{1}(z) = (U_{mn}, V_{mn}, D_{zmn}, B_{zmn}, \sigma_{zzmn}, \tau_{xzmn}, \tau_{yzm}, \phi_{mn}, \psi_{mn}, W_{mn})^T$$

$$\eta_{mn}^{2}(z) = (\sigma_{xxmn}, \sigma_{yymn}, \tau_{xymn}, D_{xmn}, D_{ymn}, B_{xmn}, B_{ymn})^T$$

The functions $\eta_{mn}^{1}(z)$and $\eta_{mn}^{2}(z)$are respectively the primary and secondary state vectors, and:

$$A_{mn}^{(j)} = \begin{bmatrix} 0 & A_1^{(j)} \\ A_2^{(j)} & 0 \end{bmatrix} \text{ and } B_{mn}^{(j)} = \begin{bmatrix} B_1^{(j)} & 0 \\ 0 & B_2^{(j)} \end{bmatrix}$$

$A_{mn}^{(j)}$ being 10x10 matrix, $A_1^{(j)}$ and $A_2^{(j)}$ are 5x5 matrices, $B_{mn}^{(j)}$ is a 7x10 matrix $B_1^{(j)}$ and $B_2^{(j)}$ being 3x5 and 4x5 matrices respectively. The relationships of these matrices are detailed in ([2][3],[4]).

According to the theory of the solution of ordinary differential equations, the solutions of state vector equations can be expressed as follows.

$\eta_{mn}^{1}(z)$= $\exp(A_{mn}^{(j)}z)\,\eta_{mn}^{1}(0)$= P (h) $\eta_{mn}^{1}(z_0)$

Where : $h = z - z_0$;

For N-multilayered magneto-electro-elastic plates, there are: $\eta_{mn}^{1}(z_1)$ = P (h_1) $\eta_{mn}^{1}(z_0)$

$\eta_{mn}^{1}(z_2)$ = P (h_2) $\eta_{mn}^{1}(z_1)$

⋮

$\eta_{mn}^{1}(z_N)$= P (h_N) $\eta_{mn}^{1}(z_{N-1})$

Considering the conditions of interface continuity, we have: $\eta_{mn}^{1}(z_N)$= [R]$\eta_{mn}^{1}(z_0)$

Where: [R] = [P (h_N).... P (h_2) P (h_1)]

[R] being 10x10 matrix: $\eta_{mn}^{1}(z_N)$ =

$$\begin{pmatrix} R(1;1) & R(1;2) & \cdots & \cdots & \cdots & \cdots & R(1;10) \\ R(2;1) & R(2;2) & & & & & \vdots \\ \vdots & & \ddots & & & & \\ & & & & & & \vdots \\ \vdots & & & \ddots & & & \\ & & & & & & \vdots \\ \vdots & & & & \ddots & & \\ & & & & & & \vdots \\ \vdots & & & & & R(9;9) & R(9;10) \\ R(10;1) & & \cdots & \cdots & \cdots & R(10;9) & R(10;10) \end{pmatrix} \begin{Bmatrix} U_{mn}(z_0) \\ V_{mn}(z_0) \\ D_{zmn}(z_0) \\ B_{zmn}(z_0) \\ \sigma_{zzmn}(z_0) \\ \tau_{xzmn}(z_0) \\ \tau_{yzmn}(z_0) \\ \phi_{mn}(z_0) \\ \psi_{mn}(z_0) \\ W_{mn}(z_0) \end{Bmatrix}$$

For z_0 and z_N : $D_{zmn}(z), B_{zmn}(z), \sigma_{zzmn}(z), \tau_{xzmn}(z), \tau_{yzm}(z)$ are knowen, the other variables $U_{mn}(z), V_{mn}(z), \phi_{mn}(z), \psi_{mn}(z), W_{mn}(z)$ in z=z_0 are given by:

$$\begin{pmatrix} U_{mn}(z_0) \\ V_{mn}(z_0) \\ \phi_{mn}(z_0) \\ \psi_{mn}(z_0) \\ W_{mn}(z_0) \end{pmatrix} = \begin{pmatrix} R(3;1) & R(3;2) & R(3;8) & R(3;9) & R(3;10) \\ R(4;1) & R(4;2) & R(4;8) & R(4;9) & R(4;10) \\ R(5;1) & \vdots & \vdots & \vdots & R(5;10) \\ R(6;1) & \vdots & \vdots & \vdots & R(6;10) \\ R(7;1) & R(7;2) & R(7;8) & R(7;9) & R(7;10) \end{pmatrix}^{-1} \text{x}$$

$$\left(\begin{pmatrix} D_{zmn}(z_N) \\ B_{zmn}(z_N) \\ \sigma_{zmn}(z_N) \\ \tau_{zxmn}(z_N) \\ \tau_{zymn}(z_N) \end{pmatrix} - \begin{pmatrix} R(3;3) & R(3;4) & R(3;5) & R(3;6) & R(3;7) \\ R(4;3) & R(4;4) & R(4;5) & R(4;6) & R(4;7) \\ R(5;3) & \vdots & \vdots & \vdots & R(5;7) \\ R(6;3) & \vdots & \vdots & \vdots & R(6;7) \\ R(7;3) & R(7;4) & R(7;5) & R(7;6) & R(7;7) \end{pmatrix} \begin{pmatrix} D_{zmn}(z_0) \\ B_{zmn}(z_0) \\ \sigma_{zmn}(z_0) \\ \tau_{zxmn}(z_0) \\ \tau_{zymn}(z_0) \end{pmatrix} \right)$$

2.3 Static solution of multilayered plates based on an elastic support

We assume that multilayered rectangular plate are based on a Winkler-Pasternak elastic support, the constraint σ_z in $z = z_0$ is given by: $\sigma_z(x,y,z_0) = \beta W(z_0)$

$$\sigma_z(x,y,z) = K_w W(z_0) - a^{-2}K_{px}\frac{\partial^2 W(z_0)}{\partial x^2} - b^{-2}K_{py}\frac{\partial^2 W(z_0)}{\partial y^2}$$

Introducing this expression in a general static solution of multilayered plates, we found:

$$\begin{pmatrix} U_{mn}(z_0) \\ V_{mn}(z_0) \\ \phi_{mn}(z_0) \\ \psi_{mn}(z_0) \\ W_{mn}(z_0) \end{pmatrix} = \begin{pmatrix} R(3;1) & R(3;2) & R(3;8) & R(3;9) & R(3;10)+\beta R(3;5) \\ R(4;1) & R(4;2) & R(4;8) & R(4;9) & R(4;10)+\beta R(4;5) \\ R(5;1) & \vdots & \vdots & \vdots & R(5;10)+\beta R(5;5) \\ R(6;1) & \vdots & \vdots & \vdots & R(6;10)+\beta R(6;5) \\ R(7;1) & R(7;2) & R(7;8) & R(7;9) & R(7;10)+\beta R(7;5) \end{pmatrix}^{-1} \text{x}$$

$$\left(\begin{pmatrix} D_{zmn}(z_N) \\ B_{zmn}(z_N) \\ \sigma_{zmn}(z_N) \\ \tau_{zxmn}(z_N) \\ \tau_{zymn}(z_N) \end{pmatrix} - \begin{pmatrix} R(3;3) & R(3;4) & R(3;6) & R(3;7) \\ R(4;3) & R(4;4) & R(4;6) & R(4;7) \\ R(5;3) & \vdots & \vdots & R(5;7) \\ R(6;3) & \vdots & \vdots & R(6;7) \\ R(7;3) & R(7;4) & R(7;6) & R(7;7) \end{pmatrix} \begin{pmatrix} D_{zmn}(z_0) \\ B_{zmn}(z_0) \\ \tau_{zxmn}(z_0) \\ \tau_{zymn}(z_0) \end{pmatrix} \right)$$

Next step for this work is to establish a numerical study for our equations, to analyze the influence of the elastic support parameters on the static behavior of the magneto-electro-elastic multilayered plates.

Conclusion

In this communication, a methodological approach to analyze the static behavior of multilayered magneto-electro-elastic plates has been presented for structures with simply supported boundary conditions based on an elastic support. The model is formulated in general way allowing account for the orthotropic angle of fibers, and an arbitrary number of layers. The presented method can be used for multilayered elastic, piezoelectric and electromagnetic plates based on an elastic support.

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