

Pile bearing analysis based upon ultimate values of toe and skin resistance as well as their mobilization with settlement

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Abstract. The main task of pile foundation is the reduction of settlement and ensure the safety of the building. These two factors are in strongly relationship. In practice pile load capacity is calculated based on ultimate resistances including factor of safety. The load-settlement relationship is non-linear, so it would can happen that despite the applied load (several times smaller than ultimate pile load capacity), the settlement exceeds the allowable value, and in consequently it might cause damage of the structure. In the paper the method of determining ultimate resistances and mobilization of these resistances with settlement was presented based on the static pile load test results. The proposed method can be applied in pile load capacity verification.

1 Introduction

Pile is a part of deep foundation which allow to transfer load on the subsoil to deeper and stiffer layers in case of insufficient shear strength of upper layer. The purpose of using piles is not only transfer load on the deeper and stiffer layers but also use of strength of soft soils by skin resistance which occur on the shaft of the pile. Therefore, load transfer from the head of the pile to the surrounding soil is the sum of two components: toe resistance and skin resistance. In common used method pile load capacity is calculated based on ultimate values of toe resistance and skin friction. Design value of pile load capacity is determined using safety factor which includes imperfections in design methods, inappropriate design assumptions and imprecisions in piles technology. Very often piles are used in strongly difficult geotechnical conditions and in case of very responsible investment. Accurate design methods and verification are indispensable. Mistakes in the building foundation project can lead to a catastrophe or very serious failures. To avoid this, it is advisable to do verification. The most known and reliable verification method is static pile load test. In the design of piles, it is very important both the ultimate pile load capacity as the settlement of the pile that corresponds to the design load capacity. In the case of designing only for safety factors, it may turn out that although the safety factor was very large, the high settlement of the pile can lead to structural damage. It is shown in the Figure 1.

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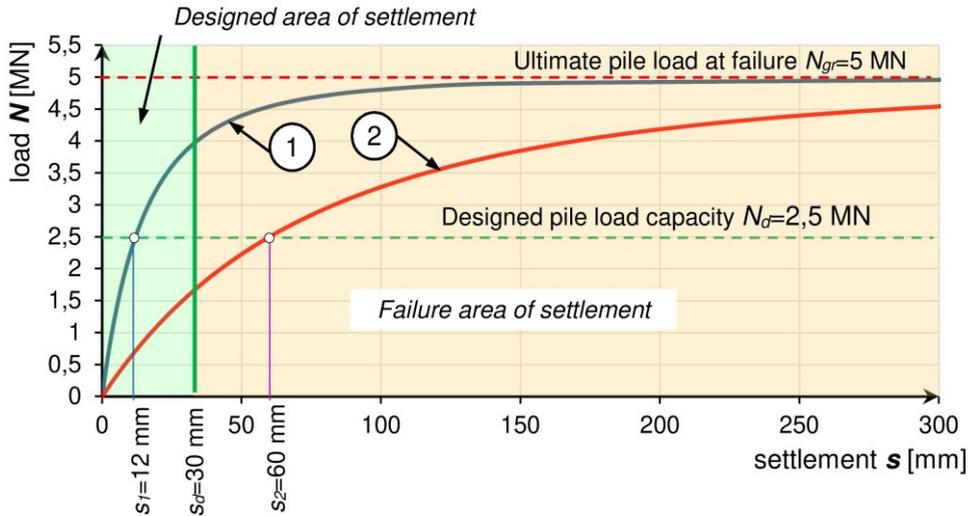


Fig. 1. Load-settlement curves of two different piles. s_1 , s_2 - settlement of the pile according to the designed load.

In the Figure 1 can be pointed two areas: designed area with settlement smaller than 30 mm and failure area with settlement greater than 30 mm. The allowable settlement of the pile may be differing from 30 mm, and it depends of building requirements, but mostly 30 mm meets these requirements. Both the piles have the same value of ultimate load equal to 5 MN, but at the designed load can be observed different settlement. It proves that while determining pile load capacity have to be taken into account not only bearing capacity but also allowable settlement of the pile.

2 Pile bearing capacity

2.1 Ultimate pile load at failure

Ultimate load does not mean the value of load that can be applied on the pile in real condition. The ultimate load is the value of load when very high and uncontrolled settlement is observed. In mathematical description the ultimate load is achieved at infinitive settlement of the pile. It is shown in the Figure 1. The ultimate load is the sum of ultimate toe resistance and skin friction (ultimate skin resistance). In practical approach the ultimate resistances are calculated independently because of differ mechanism of failure. Research indicated that the resistances depends on each other but it is still too difficult to unambiguous define these interdependences [1–3].

2.2 Ultimate toe resistance of the pile

The ultimate toe resistance can be calculated based on soil parameters or cone penetrations test CPT results. The ultimate toe resistance depends from proper mechanism of soil shear. Lots of theories proves that ultimate toe resistance increase with deep, but the certain deep exist below which toe resistance remains constant. This phenomenon may be influenced by dilative and contractive behaviour of soil. Behaviour of the same soil in different state of stress is strongly different. Long pile toe resistance is influenced by contractive soil phenomena generally.

Ultimate toe resistance for piles embedded in sands can be calculated from Eq (1).

$$q_b = N_q \sigma_v' \tag{1}$$

where:

q_b – unit ultimate to resistance [kPa],

N_q – coefficient of bearing capacity,

σ_v' – effective vertical stress in soil at the toe of the pile [kPa].

The N_q coefficient theory was developed by Terzaghi, Meyerhof, Vesic, Janbu [4–6]. The range of this coefficient r is very large (1 to over 500). The main parameter need to calculate N_q is angle of internal friction in soil below the toe of the pile φ' .

In cohesive soils ultimate toe resistance can be calculated based on undrained shear strength of soil c_u from Eq (2).

$$q_b = N_c c_u \tag{2}$$

where:

N_c – bearing capacity coefficient usually assumed to 9 [7].

Very useful kind of soil investigation in calculating unit toe resistance is cone penetration test. In this method mechanism of toe interaction with soil is likened to cone penetration. In failure the mechanisms are very similar because both ultimate toe resistance and resistance under the cone refer to very large settlement. The main difference is diameter of cone and pile. For larger diameters the unit toe resistance is smaller.

$$q_b = C_p q_c \tag{3}$$

where:

C_p – toe adjustment factor according to table 1.

q_c – resistance under the cone of CPT near to the toe of the pile.

Table 1. Toe adjustment factor for toe resistance calculation [8].

Pile type	Type of soil	C_p range	C_s range
Displacement	sand, gravel	0,2 ÷ 0,57	0,004 ÷ 0,009
Replacement		0,02 ÷ 0,26	0,0034 ÷ 0,0066
Displacement	silt, clay	0,3 ÷ 1,0	0,028 ÷ 0,086
Replacement		0,34 ÷ 0,83	0,0069 ÷ 0,012

The calculated values of unit toe resistance rarely exceed 12 MPa for sands and 6 MPa in cohesive soils. It should be also stated that not all proposed equations refer to ultimate load that has been previously explained in the introduction of the article.

2.3 Ultimate skin friction of the pile

Skin resistance at failure is a skin friction. In contrast to ultimate toe resistance the failure of skin friction can be achieved at small settlement. The skin friction is the unit skin friction multiplied by shaft surface area. The shaft of the pile very often goes through strongly inhomogeneous layers. It is complicating calculation procedure. Lots of method do not

include it in calculation, because there is no clear and certain method which allow to properly take it into account. Skin friction of the pile can be calculated similarly to toe resistance, based on state of stress, angle of internal friction of soil, undrained shear strength of soil or resistance obtained from CPT.

Skin friction of piles embedded in sands can be calculated from Eq (4).

$$q_s = \beta \sigma'_v = K \sigma'_v \tan \delta_k \tag{4}$$

where:

q_s – unit skin friction of the pile [kPa],

$\beta = K \tan \delta_k$ – skin friction coefficient,

K – earth pressure coefficient after pile installation which can be described as mean value of passive, active and rest earth pressure coefficient $K = \frac{K_a + K_0 + K_p}{3}$ [9],

$\tan \delta_k$ – coefficient of friction in the pile-soil interface,

σ'_v – effective vertical stress in soil [kPa].

The presented method called also β -method based on friction law theory. Skin friction is calculated based on radial stress on the shaft of the pile. It indicated that rough piles at large depth rich higher values than smooth piles at smaller depths. In Eq. (4) it can be seen that the horizontal component of stress in soil play main role. The spread of horizontal stress values can be very wide. Before pile installation usually K is smaller than 1. Pile installation process can disrupt it, and in displacement piles can increase K to generate passive earth pressure, and sometimes in replacement piles can decrease K to generate active earth pressure. Choosing the proper K coefficient in designing piles is very difficult. Skin friction calculated on Eq. (4) can achieve 300 kPa but it should not exceed 150 kPa.

In cohesive soils skin friction is determined based on undrained shear strength of soil according to Eq. (5).

$$q_s = \alpha c_u \tag{5}$$

where:

α – adhesion factor between 0,3 to 1,0. according to Figure 2.

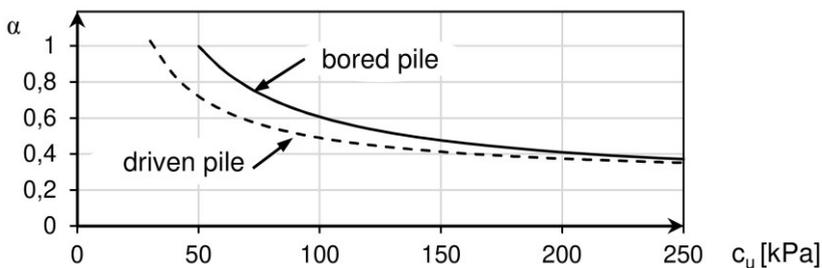


Fig. 2. Adhesion factor α versus undrained shear strength for bored and driven piles (adapted from Coduto 1994) [10].

Skin friction calculated based on Eq (5) is between 30 to 90 kPa.

Another method which allow to determine skin friction is method based on CPT results [11, 12].

$$q_s = C_s q_c \quad (6)$$

where:

C_s –skin adjustment factor according to table 1,

q_c – resistance under the cone of CPT along the shaft of the pile.

Skin friction calculated based on the CPT method can achieve almost 250 kPa. Unit skin friction is usually much smaller (even 200 times) than unit toe resistance Eq. (7).

$$\frac{f_s}{q_c} = R_f \in (0,5\%; 8\%) \quad (7)$$

where:

f_s –unit skin friction in the CPT cone shaft,

q_c – resistance under the cone of CPT.

It has to be stated that skin friction is the unit average skin friction multiplied by shaft surface area, and ultimate toe resistance is the unit toe resistance multiplied by toe surface area. Ratio of skin friction and toe resistance versus H/D ratio is presented on Fig. 3.

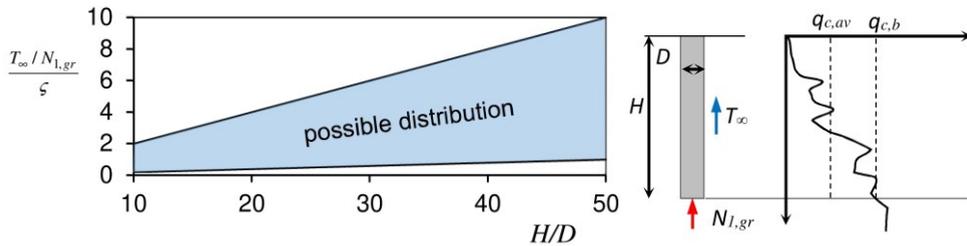


Fig. 3. Ratio of skin friction versus toe resistance. $\zeta = \frac{q_{c,av}}{q_{c,b}}$ - coefficient of soil layer variation in range from 0.1 to 1 (usually 0.3-0.5).

3 Mobilization of toe and skin resistance with settlement of the pile

Mobilization of toe and skin resistance is the increment of these resistances with settlement of the head of the pile. This is very important issue in pile design, because one of the most important is settlement reduction in comparison with shallow foundations. The example of differ skin and toe resistance mobilization is presented in Figure 3.

The settlement curve $N_2(s)$ can be divided on skin friction curve $T(s)$, and toe resistance curve $N_1(s)$. In the Figure 4. skin friction compared to toe resistance is mobilized much faster. Furthermore, a little settlement is needed to generate maximum of skin friction. After that skin friction remains constant or decreasing to ultimate value of skin friction. The peak of skin resistance, which can be observed in figure 4 can be caused by dilative behaviour of soil typical for dense sand in small stress or by preconsolidated soil.

Static pile load test is the kind of field investigation which allow to determine relationship between load and settlement of the pile. One of the method of static pile load test analyses is Meyer-Kowalow approximation curve called also M-K curve [13]. Approximated M-K curve (8) was described by three parameters C_2 , $N_{gr,2}$ and κ_2 .

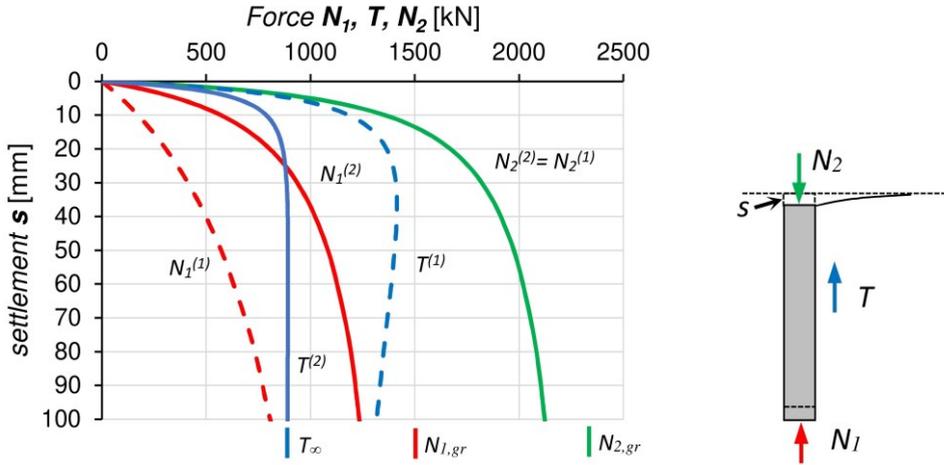


Fig. 4. Different distribution of load due to different way of skin and toe resistance mobilization. Superscript ⁽¹⁾ – distribution way No. 1. Superscript ⁽²⁾ – distribution way No. 2. Ultimate values of resistances are the same.

$$s(N_2) = \frac{C_2 N_{gr,2}}{\kappa_2} \left[\left(1 - \frac{N_2}{N_{gr,2}} \right)^{-\kappa_2} - 1 \right] \quad (8)$$

Which can be also described as (9).

$$N_2 = N_{gr,2} \left[1 - \left(1 + \frac{\kappa_2 s}{C_2 N_{gr,2}} \right)^{-1/\kappa_2} \right] \quad (9)$$

where:

C_2 – inverse of Winkler coefficient concerning to the head of the pile [mm/kN],
 $N_{gr,2}$ – ultimate pile load capacity when uncontrolled settlement is observed [kN],
 κ_2 – dimensionless functional parameter of settlement curve.

First derivative of Eq (9) was equal to (10).

$$N_2' = \frac{1}{C_2} \left(1 + \frac{\kappa_2 s}{C_2 N_{gr,2}} \right)^{-1 - \frac{1}{\kappa_2}} \quad (10)$$

$$N_2'(s = 0) = \frac{1}{C_2} \quad (11)$$

Second derivative of Eq (9) was equal to (13).

$$N_2'' = -\frac{1}{C_2 N_{gr,2}} (\kappa_2 + 1) \left(1 + \frac{N_2}{N_{gr,2}} \right)^{\kappa_2} \quad (13)$$

$$N_2''(N_2 = 0) = -\frac{1}{C_2 N_{gr,2}} (\kappa_2 + 1) \quad (14)$$

Tangent equation of the derivative N_2' can be described by Eq (15).

$$N_2'(N_2) = N_2''(0)N_2 + N_2'(0) = -\frac{1}{C_2 N_{gr,2}} (\kappa_2 + 1)N_2 + \frac{1}{C_2} \quad (15)$$

In the Polish code [14] the ultimate pile load capacity is the value of load when $N_2'(N_2) = 0$ but the equations (14, 15) are usually calculated based on force range from the end of settlement curve obtained from static pile load test. Author of the article proposed that the design value of pile load capacity could be calculated based on the formulae (15). To determine design load it should equate the Eq. (15) to zero using $(\kappa_2 + 1,4)$ instead of $(\kappa_2 + 1)$ because for $\kappa_2 = 0$ it then will be similar to shallow foundation when $FS = 1,4$ usually is used.

$$N_2'(N_2) = 0 \rightarrow N_{2,d} = \frac{N_{gr,2}}{\kappa_2 + 1,4} \quad (16)$$

where: $N_{2,d}$ – design value of pile load capacity [kN].

Using the proposed method the settlement according to the design load is independent of optimized κ_2 parameters. It can also be assumed that the factor of safety FS can be described by (13).

$$FS = \frac{N_{gr,2}}{N_{2,p}} = \kappa_2 + 1,4 \quad (17)$$

It can be observed that in high values of κ_2 the increasing of settlement in each subsequent step of load is significant, and although the FS equal to 2, the settlement exceeds allowable value. The Eq. (17) allow to find compromise between allowable settlement and design value of load which take into account also factor of safety.

4 Practical approach of the analysis

In practical approach one displacement Tubex piles was chosen. The piles were used as a deep foundation of bridge on the Odra river in Szczecin in north-west Poland. The pile was made in cohesionless soil like sands, sand with silt and gravels. The soil properties were presented in fig. 5 and in Table 2.

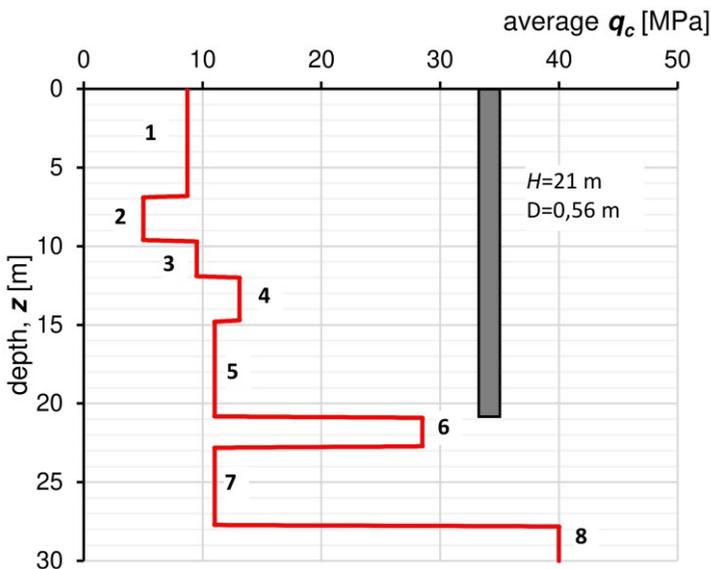


Fig. 5. Results of CPT near to the analysed pile.

Table 2. Soil geotechnical parameters.

No	Soil	γ' [kN/m ³]	ϕ [°]	average q_c [MPa]
1	Gr	8	36	8.7
2	Or	3	12	5.0
3	FSa	7.5	33.2	9.5
4	FSa/siSa	8.5	37	13.1
5	FSa/siSa	8.5	35.2	11.0
6	MSa/FSa	9	38.5	28.5
7	MSa/FSa	8.5	35.2	11.0
8	MSa/FSa	8.5	37.5	40.0

The pile was finally tested using static pile load test method. The axial load has been applied and settlement has been measured. Firstly the load-settlement relationship was approximated using Meyer-Kowalow method widely described in [15–17]. During approximating the following relationship was obtained (18), (19).

$$C_2 = 6 \cdot 10^{-6} \kappa_2^2 - 3,0 \cdot 10^{-5} \kappa_2 + 0,002278 \quad (18)$$

$$N_{gr,2} = 2562,3 \kappa_2 + 3393,1 \quad (19)$$

The equations allow to approximate only one parameter κ_2 which was determined using the (20) condition.

$$\delta^2 = \sum (s_{i,meas} - s_{i,calc})^2 = \min \quad (20)$$

where: $s_{i,meas}$ –settlement measured at i-step [mm], $s_{i,calc}$ – settlement calculated at i-step [mm].

Using formulas which were determined based on laboratory and field static pile load tests [18] it is possible to determine toe resistance based on M-K parameters according to static pile load test. The results were shown in Table 3.

Table 3. M-K curve parameters and other parameters of the toe resistance and skin resistance curves.

$N_{gr,2}$ [kN]	C_2 [mm/kN]	κ_2 [-]	$N_{gr,1}$ [kN]	C_1 [mm/kN]	κ_1 [-]	C_t [mm/kN]	T_∞ [kN]
7053	0.00225	1.428	4127	0.00592	1.190	0.00362	2926

For the chosen pile using Eq. (8,16,17) the following results were obtained: $FS = 2,828$, $N_{2,d} = 2494 \text{ kN}$, $s(N_{2,p}) = 9,61 \text{ mm}$. The method of $N_{2,d}$ force determination is presented in Figure 6.

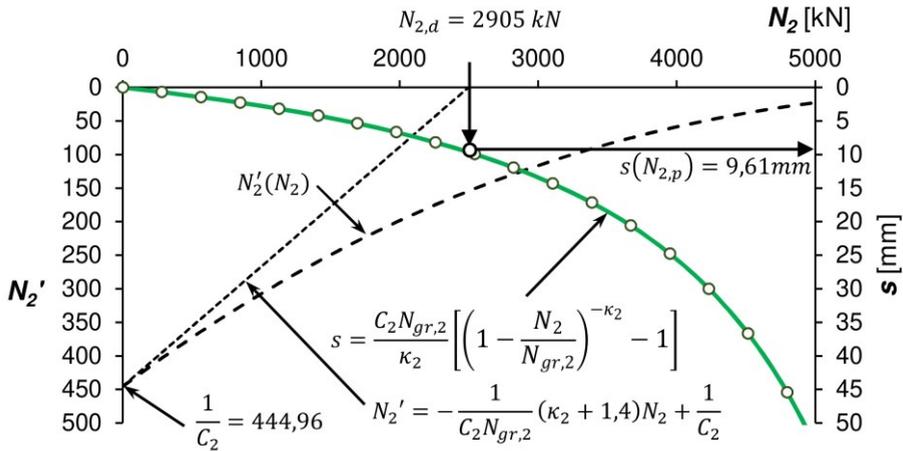


Fig. 6. Design method of pile load capacity determination and according settlement of the pile.

The parameters which were presented in Table 3 allow to determine the toe and skin resistance participation in load transfer. It was presented in Figure 7.

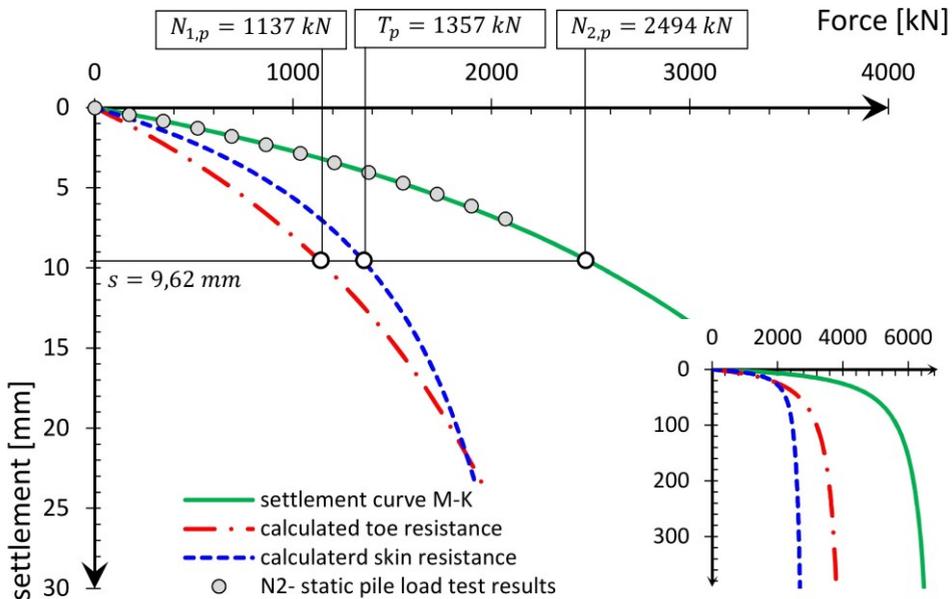


Fig. 7. Mobilization of toe resistance and skin resistance due to the settlement of the example pile.

Based on the transfer mechanism which was showed in Fig. 7 it can be assumed that skin resistance at design load equals 1357 kN, when toe resistance equals 1137 kN. At failure skin friction equals 2927 kN, and ultimate toe resistance equals 4126 kN. It indicates that the degree of effort for the skin and the toe of the pile are equal 46% and 28%, respectively. The ultimate resistances calculated based on Eq. (1,3,4,6) are similar to the values obtained from static pile load test. It can be a method of soil parameters verification or design assumptions. The comparison of the resistances was presented in Table 4.

Table 4. Skin friction, toe bearing capacity and ultimate pile load capacity calculated based on different methods.

Method	Necessary parameters	T_{∞} [kN]	$N_{1,gr}$ [kN]	$N_{2,gr}$ [kN]
1 <i>CPT- method</i>	q_c from CPT $C_s = 0.0069$ $C_p = 0.6$	2482	4212	6694
2 <i>β- method</i>	φ, σ'_v from soil investigation $K = \frac{\kappa_a + \kappa_o + \kappa_p}{3} = K(\varphi)$ $\delta_k = \varphi$ $N_q = \left(\tan \varphi + \sqrt{1 + \tan^2 \varphi} \right)^2 e^{2\eta \tan \varphi}$ $\eta = 0.58\pi$	2728	3027	5755
3 <i>M-K- method</i>	Meyer-Kowalow approximation parameters: $C_2, N_{gr,2}, \kappa_2$ [13] and Żarkiewicz formulas for $C_1, N_{gr,1}, \kappa_1$ [18]	2927	4126	7053

Coefficient of soil layer variation used in Fig. 3 is equal to 0.34. For the ratio of $H/D = 37,5$ and the ultimate resistance for method 3 (Table 4), the relationship $\frac{T_{\infty}}{s^{N_{1,gr}}}$ is equal to 2.09 and this is within the range presented in Fig. 3.

5 Conclusions

In the paper the method of pile load capacity analyses was proposed. The maximum load which can be applied to the head of the pile is the sum of ultimate toe resistance and skin friction. Mobilizing of these values require very high settlement of the pile. In practice expected settlement in many solutions should not exceed 50 mm. At this settlement it could be happen that skin resistance achieve the ultimate value, and sometimes decrease after that settlement. This case in not a failure, because the toe resistance is still increasing. Despite this can also be expected sudden increase in settlement of the pile which is sometimes very dangerous because it may cause damages and consequently failure of the construction.

In the proposed case the main parameter influenced on pile-soil behaviour is parameter κ_2 . Small values of κ_2 indicate that the behaviour is approximately linear so can be used less factor of safety. The greater κ_2 indicate that the load-settlement relationship is strongly non-linear and it this case the determinant of pile capacity should be allowable settlement of the pile.

The analysed example showed that not only ultimate resistances are important but also the way of their mobilization with settlement. The toe resistance at failure was 40 % bigger than skin friction but in allowable settlement the participation was different: toe resistance was 20% smaller than skin resistance. The proposed method can be used in diagnostics of pile load capacity.

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