Underwater multi-sensor Bayesian distributed detection and data fusion

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Abstract. The relationship of decision rule of sensor for each other is relevant to data fusion, so different topological network of sensors usually results in different performance. This paper considers the parallel and sequential topological data fusion in some detail and applies it to detect underwater signal with three sensors which respectively detects the energy, impulse width and frequency. In this paper, the signal detection model is specified for binary hypotheses testing problem. This paper compares the probabilities of error and Bayesian risk under both topologies corresponding to different value of priori probabilities of two hypotheses. Usually, the parallel architecture of detection and fusion with three sensors as specified in this paper needs to solve eleven nonlinear equations to determine the thresholds of three sensors and fusion rules, as to sequential architecture, five nonlinear equations need to be solved. So, this paper attempts to search numerical solutions for the parallel and sequential architecture of distributed detection and data fusion. Finally, this signal detection problem is simulated.

1 Introduction

Current most distributed detection and data fusion methods adopt probabilistic description for the observations of sensors and apply Bayesian rule to synthesize information. And multi-sensor Bayesian distributed detection and data fusion is an emerging technology, which is applied on a wide range of areas such as automated target recognition, field surveillance and so-on. In contrast to the multi-sensor Bayesian centralized detection which needs to collect the detail data from all sensors to make decision, multi-sensor Bayesian distributed detection refers to that each sensor could make local decision to be referred to the fusion center to make final decision. So multi-sensor data fusion refers to synergistic combination of information inferred by sensors for a better decision usually corresponding with a minimized Bayesian risk or less probabilities of error. Meanwhile, thanks to partial detection work locally finished on sensors, the computation complexity of data fusion center is reduced relative to Bayesian centralized distributed detection and data fusion, as well as transmission quantity between sensors and data fusion center is, which is in favor of transmitting messages for underwater low-rate circumstance[1]. However, there is a challenge for designing likelihood threshold of each sensor because of non-convex optimization when the number of sensors increases over five for parallel architecture of distributed detection and data fusion.

2 Distributed detection

In a binary hypothesis testing problem, each hypothesis represents either absence of a target or presence of the target, and it can also represent symbol zero or symbol one. Usually, the two hypotheses are respectively denoted by H_0 and H_1 . For Bayesian detection method, the priori probabilities of the two hypotheses are known and denoted by P_0 and P_1 . And what will be declared yields to a certain conditional probability under each hypothesis. This paper denotes the average cost or Bayesian risk function by R . And R is given by following equation.

$$R = \sum_{i=0}^1 \sum_{j=0}^1 C_{ij} P_i P(\text{declaring } H_j | H_i \text{ is present}) \quad (1)$$

where C_{ij} represents the Bayesian cost factor corresponding situation.

When it comes to three sensors, the conditional probability is related to the three declarations $\mathbf{u} = (u_1, u_2, u_3)$ respectively from three sensors. u_1 is 1 if sensor 1 declares H_1 or else is 0, likewise for u_2 and u_3 . So, the conditional probability above can be expressed by the following equation.

$$P(\text{declaring } H_j | H_i \text{ is present}) = \sum_{u_1=0}^1 \dots \sum_{u_3=0}^1 P(H_j | u_1, u_2, u_3) P(u_1, u_2, u_3 | H_i) \quad (2)$$

And the conditional probability $P(u_1, u_2, u_3 | H_i)$ is determined by observations of sensors, y_1 , y_2 and y_3 . For the case of observation of analog quantity, y is stochastically sampled in continuous space and yields to

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a certain conditional probability distribution. And hence,

$$P(u_3, u_2, u_1 | H_i) = \int_{z_1(u_1)} \dots \int_{z_3(u_3)} p(u_1, u_2, u_3, y_1, y_2, y_3 | H_i) dy_3 \dots dy_1 \quad (3)$$

where $z_k(u_k)$ expresses the region corresponding to declaration u_k for observation y_k .

In above equation, the factor $P(u_3, u_2, u_1 | H_i)$ usually means distributed detection process and the factor $P(H_j | u_3, u_2, u_1)$ does data fusion process. However, not in all distributed detection and data fusion architecture, the two processes can be clearly separated. As to sequential network, the two processes of distributed detection and data fusion are combined.

3 Parallel distributed detection

The architecture of parallel distributed detection is showed in Fig.1[2]. And three sensors receive the signal from source simultaneously and measure the quantity y_1, y_2 and y_3 , and then make each declaration at local sensor. At last they transmit their decisions u_1, u_2 and u_3 to fusion center to make the final decision u_0 .

Fig. 1. Parallel detection and data fusion architecture.

Denote that $\mathbf{u} = (u_1, u_2, u_3)$, $u_k = r_k(y_k)$, $k = 1, 2, 3$, and r_i represents the local decision rule at k^{th} sensor. And the fusion rule is $u_0 = r_0(u_1, u_2, u_3)$. First, we start with decision rule.

We rewrite equation (1) and (2) as follows,

$$R = \sum_{i=0}^1 \sum_{j=0}^1 C_{ij} P_i P(\text{declaring } H_j | H_i \text{ is present}) \quad (4)$$

$$P(\text{declaring } H_j | H_i \text{ is present}) = \sum_{u_1=0}^1 \dots \sum_{u_3=0}^1 P(H_j | u_1, u_2, u_3) P(u_1, u_2, u_3 | H_i) \quad (5)$$

then expand the factor $P(H_j | u_1, u_2, u_3)$ over two different u_0 , risk R is given by

$$R = C +$$

$$\sum_{\mathbf{u}^k} \{ P(u_0 = 1 | \mathbf{u}^{k1}) [C_F P(\mathbf{u}^{k1} | H_0) - C_D P(\mathbf{u}^{k1} | H_1)] + P(u_0 = 1 | \mathbf{u}^{k0}) [C_F P(\mathbf{u}^{k0} | H_0) - C_D P(\mathbf{u}^{k0} | H_1)] \} = C_k + \sum_{\mathbf{u}^k} A(\mathbf{u}^k) [C_F P(\mathbf{u}^{k1} | H_0) - C_D P(\mathbf{u}^{k1} | H_1)] \quad (6)$$

$$\text{where } C = C_{01}(1 - P_0) + C_{00}P_0, C_F = P_0(C_{10} - C_{00}), C_D = (1 - P_0)(C_{01} - C_{11}),$$

$$\mathbf{u}^k = (u_1, \dots, u_{k-1}, u_{k+1}, \dots, u_3), k = 1, 2, 3,$$

$$\mathbf{u}^{kl} = (u_1, \dots, u_{k-1}, u_k = l, u_{k+1}, \dots, u_N), N = 3, l = 0, 1$$

and $A(\mathbf{u}^k)$, C_k are given as follows,

$$A(\mathbf{u}^k) = P(u_0 = 1 | \mathbf{u}^{k1}) - P(u_0 = 1 | \mathbf{u}^{k0}) \quad (7)$$

$$C_k = C +$$

$$\sum_{\mathbf{u}^k} P(u_0 = 1 | \mathbf{u}^{k0}) [C_F P(\mathbf{u}^k | H_0) - C_D P(\mathbf{u}^k | H_1)] \quad (8)$$

Denote that $\mathbf{y}^k = (y_1, \dots, y_{k-1}, y_{k+1}, \dots, y_N)$ and three observations are conditionally independent of each other. So, the conditional probability $p(\mathbf{u} | H_j)$ and $P(\mathbf{u}^{kl} | H_j)$ is given as follows,

$$P(\mathbf{u}^{kl} | H_j) = \int_{\mathbf{y}} P(\mathbf{u}^{kl} | \mathbf{y}) p(\mathbf{y} | H_j) d\mathbf{y} = \int_{\mathbf{y}} P(u^k = l | y^k) P(\mathbf{u}^k | \mathbf{y}^k) p(\mathbf{y} | H_j) d\mathbf{y} \quad (9)$$

$$p(\mathbf{u} | H_j) = \int_{\mathbf{y}} P(\mathbf{u} | \mathbf{y}) p(\mathbf{y} | H_j) d\mathbf{y} \quad (10)$$

And suppose that each decision at local sensor is also conditionally independent of other sensors, that is

to say, $P(\mathbf{u} | \mathbf{y}) = \prod_{i=1}^3 P(u_i | y_i)$, as well as,

$$P(\mathbf{u}^k | \mathbf{y}^k) = \prod_{\substack{i=1 \\ i \neq k}}^3 P(u_i | y_i)$$

by substituting (9) and (10) into (6), the Risk R can be given by

$$R = C_k +$$

$$\sum_{\mathbf{u}^k} A(\mathbf{u}^k) \left[C_F \int_{\mathbf{y}} P(u^k = i | y^k) P(\mathbf{u}^k | \mathbf{y}^k) p(\mathbf{y} | H_0) d\mathbf{y} - C_D \int_{\mathbf{y}} P(u^k = i | y^k) P(\mathbf{u}^k | \mathbf{y}^k) p(\mathbf{y} | H_1) d\mathbf{y} \right] = C_k + \int_{\mathbf{y}^k} P(u^k = i | y^k) \left\{ \sum_{\mathbf{u}^k} \int_{\mathbf{y}^k} A(\mathbf{u}^k) P(\mathbf{u}^k | \mathbf{y}^k) [C_F p(\mathbf{y} | H_0) - C_D p(\mathbf{y} | H_1)] d\mathbf{y}^k \right\} d\mathbf{y}^k \quad (11)$$

Denote the factor by D_k

$$\sum_{\mathbf{u}^k} \int_{\mathbf{y}^k} A(\mathbf{u}^k) P(\mathbf{u}^k | \mathbf{y}^k) [C_F p(\mathbf{y} | H_0) - C_D p(\mathbf{y} | H_1)] d\mathbf{y}^k$$

and the decision rule can be given as follows,

$$P(u_k = 1|y^k) = \begin{cases} 1 & D_k \leq 0 \\ 0 & D_k > 0 \end{cases} \quad (12)$$

And consider that $p(\mathbf{y}|H_j) = p(\mathbf{y}^k|y_k, H_j)p(y_k|H_j)$, then we can get the likelihood ratio at local sensor denoted by t_k as follows,

$$\frac{P(y_k|H_1)}{P(y_k|H_0)} = \frac{\sum_{\mathbf{u}^k} C_F A(\mathbf{u}^k) \prod_{i=1, i \neq k}^3 P(u_i|H_0)}{\sum_{\mathbf{u}^k} C_D A(\mathbf{u}^k) \prod_{i=1, i \neq k}^3 P(u_i|H_1)} = t_k \quad (13)$$

When it comes to fusion rules for the case of three sensor, there are $2^3 = 8$ kinds of declarations $\mathbf{u}$. And we should divide them into two groups of declaring final decision H_0 or H_1 . So, there are $2^3 = 256$ kinds of divisions, or to say, fusion rules.

For any declaration $\mathbf{u}$, denoted by $\mathbf{u}^*$, the risk R is given by

$$R = K(\mathbf{u}^*) + P(u_0 = 1|\mathbf{u}^*) [C_F P(\mathbf{u}^*|H_0) - C_D P(\mathbf{u}^*|H_1)] \quad (14)$$

where $K(\mathbf{u}^*)$ represents the sum of risk caused by the rest kinds of $\mathbf{u}$, or to say, except current $\mathbf{u}^*$.

$$K(\mathbf{u}^*) = C + \sum_{\mathbf{u} \neq \mathbf{u}^*} P(u_0 = 1|\mathbf{u}) [C_F P(\mathbf{u}|H_0) - C_D P(\mathbf{u}|H_1)]$$

To reduce the Risk R , $P(u_0 = 1|\mathbf{u}^*)$ is given by

$$P(u_0 = 1|\mathbf{u}^*) = \begin{cases} 1 & [C_F P(\mathbf{u}|H_0) - C_D P(\mathbf{u}|H_1)] \leq 0 \\ 0 & [C_F P(\mathbf{u}|H_0) - C_D P(\mathbf{u}|H_1)] > 0 \end{cases} \quad (15)$$

and then we can get the fusion rules as follows,

$$\begin{aligned} u_0 &= 1 \\ \frac{P(\mathbf{u}^*|H_1)}{P(\mathbf{u}^*|H_0)} &> \frac{C_F}{C_D} \\ u_0 &= 0 \end{aligned} \quad (16)$$

which, by considering conditional independence, can be simplified as follows

$$\begin{aligned} u_0 &= 1 \\ \prod_{i=1}^3 \frac{P(u_i|H_1)}{P(u_i|H_0)} &> \frac{C_F}{C_D} \\ u_0 &= 0 \end{aligned} \quad (17)$$

There are eleven nonlinear equations, among which eight equations are like (17), and three are like (13).

4 Sequential distributed detection

The architecture of sequential distributed detection and fusion is showed in Fig.2[2]. Each sensor observes the source with the measurement result y_i . And the first sensor makes a decision u_1 and transmits it to the second. While the second makes a decision u_2 according to the former decision u_1 as well as its own

measurement y_2 , and so on for the last sensor. As to this kind architecture of detection, the last sensor makes the final decision which has fused all declarations from others, so there is no more extra fusion center, that is to say, $u_0 = u_N$

Fig. 2. Sequential detection and data fusion architecture.

Because the architecture is recursive, the detection rule can be derived from the situation of two sensors. The Bayesian risk is given by

$$R = \sum_{i,j,k} \iint_{y_1, y_2} C_{jk} P_k p(u_2 = j|u_1 = i, y_1, y_2, H_k) p(u_1 = i, y_1, y_2 | H_k) dy_1 dy_2 \quad (18)$$

And considering the conditional independence, risk R can be altered as follow,

$$R = \sum_{i,j,k} \iint_{y_1, y_2} C_{jk} P_k p(u_2 = j|u_1 = i, y_2) p(u_1 = i, y_1 | H_k) p(y_2 | H_k) dy_1 dy_2 \quad (19)$$

Considering $p(u_2 = 1|u_1, y_2) + p(u_2 = 0|u_1, y_2) = 1$, further derive it into

$$R = \sum_{i,k} \iint_{y_1, y_2} C_{jk} P_k p(u_1 = i, y_1 | H_k) p(y_2 | H_k) dy_1 dy_2 + \sum_i \int_{y_2} p(u_2 = 0|u_1 = i, y_2) \int_{y_1} (C_{0k} - C_{1k}) P_k p(u_1 = i, y_1 | H_k) p(y_2 | H_k) dy_1 dy_2 \quad (20)$$

When $\sum_k (C_{0k} - C_{1k}) P_k p(u_1 | H_k) p(y_2 | H_k) > 0$,

we let $p(u_2 = 0|u_1, y_2) = 0$, that is to say, u_2 or $u_0 = 1$. So, the decision rule of sensor 2 is given by

$$\begin{aligned} u_0 &= 1 \\ \frac{P(y_2|H_1)}{P(y_2|H_0)} &> \frac{C_F}{C_D} \cdot \frac{p(u_1|H_0)}{p(u_1|H_1)} \\ u_0 &= 0 \end{aligned} \quad (21)$$

And we can learn from (21), the likelihood threshold ratio of sensor 2 depends on the former decision u_1 . Denote the two different thresholds by t_2^1 and t_2^0 as follows,

$$t_2^1 = \frac{C_F}{C_D} \cdot \frac{p(u_1 = 1|H_0)}{p(u_1 = 1|H_1)} \quad (22)$$

$$t_2^0 = \frac{C_F}{C_D} \cdot \frac{p(u_1 = 0|H_0)}{p(u_1 = 0|H_1)} \quad (23)$$

Then we reckon back (19) to determine the decision rule of the first sensor. Considering that

$p(u_1=1|y_1) + p(u_1=0|y_1) = 1$, the risk, expanded over u_1 , is given by

$$R = \sum_{j,k} \iint_{y_1, y_2} C_{jk} P_k p(u_1, y_1 | H_k) p(y_2 | H_k) dy_1 dy_2 + \sum_j \int_{y_2} p(u_2=0|u_1, y_2) \sum_k \int_{y_1} (C_{0k} - C_{1k}) P_k p(u_1, y_1 | H_k) p(y_2 | H_k) dy_1 dy_2 \quad (24)$$

We make a substitution with D as follows,

$$D = \sum_{j,k} C_{jk} P_k p(u_1 | H_k) [p(u_2|u_1=0, H_k) - p(u_2|u_1=1, H_k)] \quad (25)$$

and expanding D over j and k before further reduction, we can get the decision rule of the first sensor as follows,

$$\begin{aligned} u_1 &= 1 \\ \frac{P(y_1|H_1)}{P(y_1|H_0)} &> \frac{C_F}{C_D} \cdot \frac{P_{F2}(t_2^1) - P_{F2}(t_2^0)}{P_{D2}(t_2^1) - P_{D2}(t_2^0)} \\ u_1 &= 0 \end{aligned} \quad (26)$$

where $P_{F2}(t_2^i)$ and $P_{D2}(t_2^i)$ represent $p(u_2=1|u_1=i, H_0)$ and $p(u_2=1|u_1=i, H_1)$ respectively.

As to sequential distributed detection with three sensors, the former two sensors are regarded as one equivalent sensor, and the problem of sequential distributed detection of multi-sensor is transformed into that of two, the latest one and the equivalent former one. And when likelihood ratio thresholds of the equivalent one is determined, it is regard as two sensors, and so on.

The simplified form of likelihood ratio thresholds of three sensors are expressed as follow,

$$\begin{aligned} t_1 &= \frac{C_F}{C_D} \cdot \frac{P_{F3}(u_1=1) - P_{F3}(u_1=0)}{P_{D3}(u_1=1) - P_{D3}(u_1=0)} \\ t_2^i &= \frac{C_F}{C_D} \cdot \frac{P_{F3}(u_1=1) - P_{F3}(u_1=0)}{P_{D3}(u_1=1) - P_{D3}(u_1=0)} \cdot \frac{P(u_2=i|H_0)}{P(u_2=i|H_1)} \end{aligned} \quad (27) \quad (28)$$

and

$$t_3^i = \frac{C_F}{C_D} \cdot \frac{P(u_2=i|H_0)}{P(u_2=i|H_1)} \quad (29)$$

In contrast with parallel distributed detection, the declarations of sensors are conditionally dependent, and there are two thresholds of likelihood ratio for the latter as functions of two decisions of the former, that is to say, the decision of latter sensor for the same measurement may be altered according to different decision of the former. And there are five nonlinear coupled equations which need to be solved simultaneously for sequential architecture.

5 Simulations

5.1 Parameters of simulation

Table 1 shows the Bayesian cost factors for later simulations. The cost factor for right declaration is 0, and the wrong declarations are both -1.

Fig. 3. Sensor 1, energy detection. Suppose that amplitude of noise obeys a Rayleigh distribution and amplitude of signal with noise obeys a Rice distribution.

Fig. 4. Sensor 2, pulse width detection. Suppose that width of noise obeys an Exponential distribution and width of signal with noise obeys a Rayleigh distribution.

Fig. 5. Sensor 3, frequency detection. Suppose that and that frequency of noise obeys an Exponential distribution and width of signal with noise obeys a Uniform distribution.

Table 1. Bayesian cost factors.

Cost Factors	Declaring H_0	Declaring H_1
Presenting H_0	0	-1
Presenting H_1	-1	0

Table 2. Likelihood functions at three sensors.

	Sensor 1	Sensor 2	Sensor 3
signal	Rice distribution; Amplitude 3 and sigma 1	Rayleigh distribution; Sigma 2.5	Uniform distribution; random variable in range from 3 to 6
noise	Rayleigh distribution; Sigma 1	Exponential distribution; Lambda 1, random variable in range from 0 to 8	Exponential distribution; Lambda 1, random variable in range from 3 to 6

Table 2 shows the likelihood functions of signal and noise at each sensor which are respectively depicted in Fig.3, Fig.4, and Fig.5.

5.2 Performance of each sensor

As showed in Fig.6, the ROC curves demonstrate that sensor 1 performs best, and the sensor 2 performs worse than sensor 1, and the sensor 3 performs worse than sensor 2.

Fig. 6. The detection performance of 3 sensors in simulation. The sensor 1 is the best and the second is secondary and the last is the lowest.

Fig.7 shows Bayesian risks of three sensors with respect to a range of probability of hypothesis H_0 . Because the cost factors are non-positive, the Bayesian Risk is negative. The higher the risk is, the better the sensor is. And additionally, the C_{00} and C_{11} are both equal to 0, and C_{01} and C_{10} are -1, So, the Bayesian risk can represent the minus Probability of Error.

Fig. 7. Bayesian risk versus probability of hypothesis H_0 .

And it also turns out that the performance of sensor 1 is the best, of sensor 2 secondary, and of sensor 3 the lowest. And when probability of hypothesis H_0 is 0.5, the Bayesian risks of sensor 1, 2 and 3 are respectively -0.123, -0.196 and -0.330.

5.3 Performance of parallel detection and data fusion

The complexity of computation for parallel distributed detection is exponential in number of sensors.[2,3]. The performance of parallel fusion, compared with that of sensor 1, is showed in Fig.8 which illustrates that the performance of parallel fusion is better than that of the best one of three sensors. And the likelihood ratio

thresholds of each sensor in parallel distributed detection are listed at Table 3.

Fig. 8. Comparison of Bayesian risk versus probability of hypothesis H_0 between sensor 1 and the parallel fusion.

Table 3 Likelihood ratio thresholds of the parallel.

$P(H_0)$	Likelihood Ratio threshold (t_1, t_2, t_3)			$P(H_0)$	Likelihood Ratio threshold (t_1, t_2, t_3)		
0	0.022	0.030	0.317	0.6	2.254	2.896	0.852
0.1	0.314	0.731	0.379	0.7	0.983	0.373	4.307
0.2	0.707	1.716	0.379	0.8	1.615	0.319	0.379
0.3	0.833	1.560	0.613	0.9	2.664	0.497	0.379
0.4	1.159	2.063	0.671	1.0	0.022	0.030	6.362
0.5	1.615	2.455	0.756				

Table 4 shows that the fusion rules for final declaration hypothesis H_1 represented by the decimal numbers of binary form as which declarations $\mathbf{u} = (u_1, u_2, u_3)$ is regarded, and u_1 corresponds to the highest bit. For the case of three sensors, there are 8 kinds of declaration. And the rules that are not listed at table 4 are the fusion rules for hypothesis H_0 .

Table 4. Fusion rules for final declaration $u_0=1$.

$P(H_0)$	Fusion rules for $u_0=1$	$P(H_0)$	Fusion rules for $u_0=1$
0	1,3,5,7	0.6	3,4,5,6,7
0.1	2,3,4,5,6,7	0.7	3,5,6,7
0.2	2,3,4,5,6,7	0.8	6,7
0.3	3,4,5,6,7	0.9	6,7
0.4	3,4,5,6,7	1.0	1,3,5,7
0.5	3,4,5,6,7		

5.4 Performance of Sequential detection and data fusion

First, for the simulation of the sequential detection and fusion, three sensors are placed in sequence in which the decision of the best sensor 1 is transmitted to sensor 2 with secondary performance and its decision is transmitted to the lowest sensor 3. And the performance of this sequence, compared with that of the best sensor 1 alone, is showed in Fig.9.

From Fig.9, we can learn that the performance of sequential detection and fusion is better than that of a single sensor 1. And the likelihood ratio thresholds of each sensor in sequential detection architecture are listed at Table 5. What is worth mentioning is that the

solutions of nonlinear equations are numerical by certain iterations, and its convergence rate may be not uniform, so its accuracy depends on the times and step length of iteration.

Fig. 9. Bayesian risk of the sequential detection and fusion versus probability of hypothesis H_0 .

Table 5. Likelihood ratio thresholds of the sequential.

$P(H_0)$	Sen. 1(t_1)	Sen. 2(t_2^0, t_2^1)		Sen. 3(t_3^0, t_3^1)	
0	0.049	0.153	0.030	0.403	0.317
0.1	0.267	1.7163	0.030	2.665	0.317
0.2	0.433	2.253	0.049	4.307	0.317
0.3	0.600	3.3883	0.070	5.642	0.317
0.4	0.707	4.533	0.070	6.362	0.317
0.5	0.833	5.896	0.122	6.362	0.317
0.6	0.982	8.3356	0.153	6.362	0.317
0.7	1.368	11.674	0.188	6.362	0.317
0.8	1.615	17.827	0.271	6.362	0.317
0.9	0.077	24.396	1.884	6.362	0.990
1.0	0.049	0.189	0.189	6.362	6.362

And then we switch the orders of the three sensors into another two sequence. In first one, the secondary sensor declares first and transmits its decision to the best sensor and then the decision of the best is transmitted to the lowest one. In second one, the lowest sensor declares first and transmits its decision to the secondary sensor and then the best sensor makes the final decision. The simulation result of the two orders, compared with the performance of the primary order, are shown in Fig.10.

Fig. 10. Simulation for sequence of the three sensors. The performance of sequence of sensors in which the better sensor is placed latter is better.

From Fig.10, we can conclude that the sequence of multiple sensors is relevant and that when the better sensor is placed latter, the final performance of the sequential distributed detection is better. Additionally, the numerical solutions of nonlinear equations are obtained by more iteration times and the result of

sequential detection is better than that in Fig.9. Generally, this sequence of the best sensor making the final decision is a good ideal but not always optimal. The counterexamples are showed in [4] where for the case of exponential distribution, however, it proves out that the best one makes the final decision [2,4].

As to this example by simulation, the performance of sequential distributed detection is better than that of parallel architecture. However, the both architecture of distributed detection can improve the performance of the best one detecting solo.

6 Conclusions

This paper discusses the detection and fusion rules of two architectures, the parallel and sequential distributed detection and fusion. And for the case of parallel distributed detection, the observations at different sensors are conditionally independent, and the decision of each sensor is only based on its own observations, so, in that way, the decisions of different sensors are also conditionally independent of each other. However, for the sequential distributed detection, although the observations are also conditionally independent, the decisions are not, because the thresholds of the latter sensor are based on the declaration of the former sensor. So, for sequential distributed detection, the sequencing is relevant. Generally, the latter the better sensor is placed, the better is the fusion performance.

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