Strong coherent interference suppression based on second-order cone null steering beamforming

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Abstract. For active sonar systems, coherent interference, such as direct-path wave and strong multi-path, will shield the weak target echo signal, which is one of the main factors affecting the target detection performance. Null steering beamforming is an important signal processing method that suppresses strong interference. In this paper, the second-order cone programming (SOCP) method is used to perform null steering beamforming on the uniform linear array. Beam null is caused in the direction of interference to suppress strong coherent interference in the active sonar system during detection. However, there are two limitations that can cause decline of the algorithm performance. A norm-control method is proposed to limit the response near the end-fire direction of linear array, and an array manifold compensation method is introduced to solve the array manifold mismatch. Simulated and experimental data was used to evaluate the performance and verify the feasibility of the methods.

1 Introduction

In plenty of modern underwater work, the first task is to detect the target. However, in the process of actual detection, the marine environment is often complicated, and the data received by the array is mixed with other interferences besides the target signal. Especially when the intensity of the interference source is large, the resolution of the target will be affected, and even the target will be completely covered. Therefore, how to detect weak targets under strong interference is directly related to the success of subsequent analysis work.

For active sonar systems, the interference includes both coherent and non-coherent interference. Coherent interference mainly includes direct-path interference, multi-path interference, etc. Non-coherent interference mainly comes from other non-coherent sources and underwater targets. Due to the long distance of detection, the signal attenuation is severe, and the received signal is often weak and is masked by strong interference in a complex underwater acoustic environment. The interference suppression technique extracts the desired signal by suppressing the interference signal[1].

The second-order cone scheme[2] is a subset of convex optimization[3-5]. With the development of convex optimization algorithms, in 1997, S. Boyd and H. Lebert introduced convex optimization techniques into beam optimization algorithms[6]. Yan Shefeng, Ma

Yuanliang, Sun Chao et al., applied the second-order cone programming (SOCP) technique to the optimal array processing technology, and solved the weight problem in beam optimization[7-11]. Huang Cong proposed a direct path interference suppression method based on zero constraint condition with phase correction, which applied the method of array manifold compensation to interference suppression in 2013[12-13].

The basic idea of designing a beam by SOCP is to set the ideal beam shape, and then approximate the designed beam to the ideal beam by SOCP. This method can adjust multiple indicators of the array and is suitable for arrays of any shape. However, there is a limitation when designing beams of linear array with classic SOCP: the response near the end-fire direction will be too large, even exceed the main lobe response, which will affect the performance of the algorithm and reduce the range of detection angle. The reason is the same conditions are used to constrain beams in classic SOCP, since the beam parameters are inconsistent at different directions of linear array. Therefore, a control method is required to limit the response near the end-fire direction of linear array.

Another limitation of SOCP is that the beam design method relies entirely on theoretical values and is independent of the received data, so its ability to respond to the environment is insufficient. In practical applications, the array manifold mismatch is often caused by various errors, resulting in a decrease in beamforming

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performance, and it is necessary to compensate the array manifold.

The rest of the paper is organized as follows. Section 2 proposes an improved norm-control method of designing a null beam by SOCP. Section 3 analyzes the performance of SOCP null steering beamforming. Section 4 illustrates a method of array manifold compensation. Section 5 gives the experimental results, and Section 6 concludes this work.

2 Improved SOCP beam design with norm-control method

It is necessary to consider the main lobe width, side lobe level, beam null depth, and robustness of the beam when designing the weight. The general expression for beam design by SOCP is:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \xi_i \\ \text{s.t.} \quad & \mathbf{a}^H(\theta_0)\mathbf{w} = 1 \\ & \|\mathbf{a}^H(\theta_{ML})\mathbf{w} - \mathbf{p}_d(\theta_{ML})\| \leq \xi_1 \\ & |\mathbf{a}^H(\theta_{SL})\mathbf{w}| \leq \xi_2 \\ & |\mathbf{a}^H(\theta_{NL})\mathbf{w}| \leq \xi_3 \\ & \|\mathbf{w}\|^2 \leq \xi_4 \end{aligned} \quad (1)$$

where $\mathbf{a}(\theta)$ represents the array manifold vector of angle θ ; θ_0 represents the direction of the desired signal; $\mathbf{p}_d$ represents the result of conventional beamforming (CBF); θ_{ML} represents the main lobe interval; θ_{SL} represents the side lobe interval; θ_{NL} represents the beam null interval; $\mathbf{w}$ represents the designed array weight vector, 2-norm (Euclidean norm) of the weight vector represents the robustness of the beam.

When the direction of strong interference is relatively fixed, θ_{NL} is taken to contain the angle of interference. The null can be seen as a side lobe with a lower response. The expression for the null beam design by SOCP is:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \xi_4 \\ \text{s.t.} \quad & \mathbf{a}^H(\theta_0)\mathbf{w} = 1 \\ & |\mathbf{a}^H(\theta_{SL})\mathbf{w}| \leq \xi_2 \\ & |\mathbf{a}^H(\theta_{NL})\mathbf{w}| \leq \xi_3 \\ & \|\mathbf{w}\|^2 \leq \xi_4 \end{aligned} \quad (2)$$

Through the formula (2), a weight vector $\mathbf{w}$ with the minimum 2-norm can be designed, while the side lobe level is under ξ_2 , the null depth is under ξ_3 , and the signal in the main lobe direction does not decay.

Simulation was conducted to illustrate the process of interference suppression. Consider a 48-element uniform linear array (ULA) with inter-sensor spacing of 0.25m. The echo direction of the target was -10° . There was a strong coherent interference from 50° in space. The interference-to-signal ratio (ISR) was 40dB, and the center frequency of the signals was 1.5 kHz. When the signal is processed by CBF, the weak target is masked by the side lobes of strong interference.

Design a null beam by SOCP to make the side lobe level under -20dB ($\xi_2 = -20\text{dB}$), and the null depth

under -60dB ($\xi_3 = -60\text{dB}$). The beam null interval was between 35° and 65° . Fig.1 shows the designed null beam.

Fig. 1. Null beam designed by SOCP.

Fig.2 shows the results of null steering beamforming and CBF. It can be seen that the beam designed by SOCP can suppress strong interference and make the target visible. The target is 40dB-lower than the interference.

Fig. 2. Results of null steering beamforming and CBF.

However, there is a limitation when designing beams by SOCP for ULAs. For CBF, beams pointing in different directions have different main lobe widths. The closer to the end, the wider the main lobe; the closer to the normal, the narrower the main lobe. According to formula (2), if the same conditions are used to constrain beams at different directions, the 2-norm of the beam weight near the end-fire direction will be too large. The 2-norm of the weight represents the response of the beam to white noise, so the beamforming result will increase near the end-fire direction.

Fig. 3. Results without norm-control method.

Simulation was conducted to illustrate this problem. The conditions for the array and signal were the same as before. The incidence direction of the weak target echo and the strong coherent interference were -60° and 50° , respectively. The result is shown in Fig.3.

It is obvious that the strong coherent interference is suppressed and the weak targets can be detected. However, the side lobe near the end-fire direction is higher than the main lobe, because of the large 2-norm. This phenomenon affects the performance of the algorithm and reduces the range of detection angle.

Aiming at this problem, it is necessary to increase the width of the main lobe near the end-fire direction to limit the 2-norm of the weight.

The algorithm is improved based on the general expression instead of formula (2) to design the null beam. The improved expression is:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \xi_4 \\ \text{s.t.} \quad & \mathbf{a}^H(\theta_0)\mathbf{w} = 1 \\ & \|\mathbf{a}^H(\theta_{ml}^0)\mathbf{w} - \mathbf{p}_d(\theta_{ml}^0)\| \leq \xi_1 \\ & |\mathbf{a}^H(\theta_{SL})\mathbf{w}| \leq \xi_2 \\ & |\mathbf{a}^H(\theta_{NL})\mathbf{w}| \leq \xi_3 \\ & \|\mathbf{w}\|^2 \leq \xi_4 \end{aligned} \quad (3)$$

Fig. 4. Results with and without the norm-control method

Equation (3) is the improved expression with the norm-control method, where θ_{ml}^0 represents the main lobe interval when the beam pointing at θ_0 . The range of θ_{ml}^0 changes with θ_0 . The closer θ_0 is to the end-fire direction, the larger the range of θ_{ml}^0 . The 2-norm of $\mathbf{w}$ can be limited by adjusting the main lobe width adaptively.

Simulation was conducted to prove the effect of the norm-control method. The simulation conditions for the array and signal were the same as before. The azimuth spectrum of beamforming with the norm-control method is shown in Fig.4.

Fig.4 shows that the improved SOCP beam design with Norm-control method can effectively solve the problem of excessive response near the end-fire direction.

3 Performance analysis

Simulations were conducted to analyze the performance of null steering beamforming by SOCP. In this paper, the mainlobe-to-sidelobe ratio (MSR), the ratio of the main lobe to the highest side lobe, is used to describe the performance of beamforming.

According to Section 2, when designing a null beam by SOCP, it does not depend on the actual received data at all. Design can be done only through theoretical array manifold vectors. However, the actual array manifold tends to deviate from the theoretical array manifold. The main source of deviations is: (1) the complexity of the underwater acoustic channel, which causes the signal received by the elements, has different amplitude and phase; (2) the amplitude and phase consistency deviation of each channel; (3) the deviations of the placement position and the fixing device.

The beam designed by SOCP is sensitive to these deviations. Therefore, the influence of the array manifold mismatch to the algorithm needs to be discussed.

The theoretical array manifold vector of a ULA is expressed as:

$$\mathbf{a}_{th}^{ULA}(\theta) = [1, e^{-j\frac{2\pi}{\lambda}d \sin \theta}, \dots, e^{-j\frac{2\pi}{\lambda}(N-1)d \sin \theta}] \quad (4)$$

λ represents the wavelength, d represents the inter-sensor spacing, and N represents the number of elements. According to equation (4), the phase difference between neighbouring elements is stable, and the amplitude of each element equals to 1. The actual array manifold vector with amplitude and phase error is expressed as:

$$\begin{aligned} \mathbf{a}_a^{ULA}(\theta) = [A_1 e^{-j\varphi_1}, A_2 e^{-j(\frac{2\pi}{\lambda}d \sin(\theta) + \varphi_2)}, \dots \\ \dots, A_N e^{-j(\frac{2\pi}{\lambda}(N-1)d \sin(\theta) + \varphi_N)}] \end{aligned} \quad (5)$$

The vector $\mathbf{A} = [A_1, A_2, \dots, A_N]$ represents the amplitude error, and the vector $\boldsymbol{\Phi} = [\varphi_1, \varphi_2, \dots, \varphi_N]$ represents the phase error. $\mathbf{A}$ and $\boldsymbol{\Phi}$ both obey Normal distribution.

$$\mathbf{A} \sim \mathcal{N}(0, \sigma_A^2) \quad (6)$$

$$\boldsymbol{\Phi} \sim \mathcal{N}(0, \sigma_\varphi^2) \quad (7)$$

Simulations were conducted to analyze the influence of amplitude error and phase error respectively. Consider a 48-element ULA with inter-sensor spacing of 0.25m. The incidence direction of the weak target echo and the strong coherent interference were 0° and 50° , respectively. The center frequency of the signals was 1.5 kHz.

3.1 Influence of amplitude error

When σ_A , the standard deviation in formula (6), gradually increases from 0 to 0.2, MSRs of the output under different amplitude errors are shown in Fig.5.

Fig.5 shows that MSR decreases as the amplitude error increases. When the amplitude error increases to 0.08, MSR drops to 5dB. If MSR is less than 5dB, it is considered that the target is difficult to be distinguished from the azimuth spectrum.

Fig. 5. MSRs under different amplitude errors.

3.2 Influence of phase error

When σ_ϕ , the standard deviation in formula (7), gradually increases from 0° to 8° , MSRs of the output under different phase errors are shown in Fig. 6.

Fig. 6. MSRs under different phase errors.

Fig. 6 shows that MSR decreases as the phase error increases. When the phase error increases to 4.5° , MSR drops to 5dB.

4 Array manifold compensation method

According to Section 3, the azimuth spectrum is sensitive to the array manifold mismatch. Therefore, before using SOCP beamforming, the array manifold error needs to be compensated.

The compensation method is: (1) select a data segment with only strong interference and noise, and perform DFT on the received signals of each channel. (2) Select the complex number, corresponding to the frequency f_0 in the DFT result, to form a vector $\mathbf{R}_a$. $\mathbf{R}_a$ represents the actual array manifold at f_0 . (3) Calculate the theoretical array manifold vector of the array at f_0 , denoted as $\mathbf{R}_{th}$. (4) Calculate the deviation of the array manifold, $\mathbf{R}_d$, based on $\mathbf{R}_a$ and $\mathbf{R}_{th}$: $\mathbf{R}_d = \mathbf{R}_a / \mathbf{R}_{th}$. (5) When processing the data containing the target, multiply the actual array manifold $\mathbf{R}_a^t$ by the deviation $\mathbf{R}_d$ firstly, to obtain the theoretical array manifold $\mathbf{R}_{th}^t$ of the data segment, and then perform beamforming.

Simulation was conducted to illustrate the effect of array manifold compensation. The simulation conditions for the array and signal were the same as in Section 2. Suppose that the array had an amplitude error with a standard deviation of 0.1 and a phase error with a standard

deviation of 5° . The array manifold was compensated using the above method, and the output results before and after the compensation are shown in Fig. 7.

Fig. 7. Results before and after the compensation.

The results show that, when the array manifold mismatch, the azimuth spectrum side lobes are high and the target cannot be accurately distinguished; after the compensation, the performance of beamforming is improved and the target can be clearly distinguished.

It is worth noting that this method is not applicable under all conditions. The compensated phase will affect the DOA (Direction of Arrival) results. The DOA errors under different phase errors and ISRs are shown in Fig. 8.

Fig. 8. DOA errors under different phase errors and ISRs.

5 Experimental results

After several simulations, the real environmental data collected in the South China Sea were used for testing. The data were recorded by a horizontal ULA with 48-element hydrophones with inter-sensor spacing 0.25m. There is a target and a single-frequency sound source with a frequency of 1.5 kHz in the sound field. The direct-path wave of the sound source and the echo of the target are incident on the hydrophone array from different directions, and overlap in the time domain, as shown in Fig. 9.

Fig. 9. Time domain schematic diagram of the signal.

Since the coherent direct-path interference is stronger than the target echo (ISR is about 30 dB), the target echo is masked by strong interference in the CBF result, as shown in Fig.10.

Fig. 10. CBF results of experimental data.

Fig.10 shows the incident direction of the direct-path wave was -53° . Design a beam with null between -65° and -45° by SOCP. The result of null steering beamforming is shown in Fig.11

Fig.11. Null steering beamforming results of experimental data.

The target is merged in Fig.11, because of the performance degradation caused by errors in the amplitude response and phase consistency of each array element. Perform null steering beamforming after compensating the array manifold. The result is shown in Fig.12.

Fig. 12. Null steering beamforming results after array manifold compensation.

From this experiment, the results demonstrate that the null steering beamforming by SOCP can effectively suppress strong coherent interference. When there is an error in the actual array manifold, the method of array manifold compensation can improve the performance of the algorithm.

6 Conclusions

In this paper, an improved method to design a null steering beam by SOCP is used to suppress strong coherent interference in active detection. Aiming at the high response near the end-fire direction, a norm-control method is proposed to control the norm by adaptively adjusting the main lobe width. The method can control the norm of the weight while ensuring the angular resolution.

Simulations were conducted to analyse the influence of array manifold mismatch, and a method of array manifold compensation is illustrated.

The feasibility of the improved SOCP beam design method and the array manifold compensation method was verified through experimental data.

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