

# Numerical calculation and measurement for the focus field of concave spherical acoustic lens transducer

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**Abstract.** How to accurately calculate the sound field formed by acoustic lenses is an important basis for the design of acoustic lens transducers. The radiation sound field distribution of the physical model of acoustics lens is simulated by numerical methods, including the ray propagation method and the wave propagation method. The ray propagation method can only get the focal length without considering the wave characteristics property, while the wave propagation method takes into account the amplitude and phase factors of the wave, and by which the distribution of the whole sound field can be got. The relationship between the property of refractive wave and incident angle of incident wave is analyzed, and theoretical results of the distribution of the focal field are obtained. The actual sound field of the real transducer is measured by acoustic field scanning system, and the measured results of focal length and focal area are obtained. The comparison and analysis of the numerical data and measured data show that the wave propagation method can be used to predict the focus field of concave spherical acoustic lens transducer accurately and effectively.

## 1 Introduction

An acoustic lens transducer combined with a planar vibrating element and an acoustic lens can produce an effective focusing sound field. Acoustic lens focusing transducer has been widely used in many fields, including ultrasonic microscopy, ultrasonic non-destructive testing, imaging sonar and so on. Acoustic lenses are usually arranged in a coaxial plane and a curved surface. Since the sound velocity of the acoustic lens material is often greater than that of the propagation medium, the concave lens will form a focusing effect. In order to get designed focusing sound field of acoustic lens transducer, it is necessary to simulate the distribution of sound field in advance to judge the accuracy of transducer design. How to accurately calculate the sound field formed by acoustic lenses is an important basis for the design of acoustic lens transducers.

There are many simulating methods including ray method, integral method, and finite element method [1], boundary element method and so on. Each method has its advantages. The simple calculation is always carried out by the ray method, by which the approximate focal length of acoustic lens can be obtained. However, it should be noted that the ray method is not appropriate to calculate the size of the focal area, the sidelobes of the focal plane, and the sound field distribution outside of the focal area. The integral method containing mixing method and wave method refers to the use of Huygens principle, where the concave sphere surface of the acoustic lens is used as a radiation source area. The sound field is calculated by Rayleigh integral equations.

Under the mix method condition, the wave pressure at any place of the concave sphere surface is the same, and there is no difference in amplitude and phase. In fact, this assumption is the same as the self-focusing concave spherical piezoelectric ceramics, and it is quite different from the actual situation of concave spherical acoustic lens focusing. Wave fluctuation is considered carefully in the whole process from vibrating element to the focus by wave method, that is, the refracted acoustic waves from the different places of the concave sphere surface of acoustic lens have different amplitude and phase. This is close to the actual situation and easy to realize. Most researchers consider the larger phase effect and omit the smaller amplitude influence [2, 3].

Based on ray propagation theory and wave propagation theory, this paper deduces the numerical calculation equations of the focusing acoustic field radiated by acoustic lens. The pressure of acoustic field is simulated and compared with the measured results. The purpose of this study is to verify the accuracy of the numerical method for calculating the focusing acoustic field of acoustic lens by the wave propagation theory, and to provide a predictive method for the design of acoustic lens transducer.

## 2 The numerical calculation method of the model

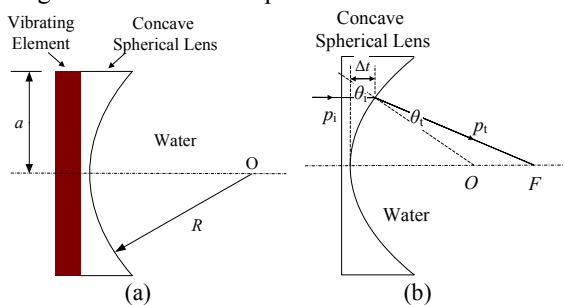
### 2.1 The model of concave spherical acoustic lens transducer

The following assumptions are made to simplify the

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simulating model. Firstly, the influence of Lamb wave and shear wave in the solid acoustic lens are neglected. Secondly, the absorption and attenuation of sound energy in the material along the sound wave propagation path are neglected. Thirdly, the interference effect caused by refraction between different material interfaces is neglected.

As shown in Fig.1, the acoustic lens transducer is composed of a vibrating element and an acoustic lens, working in water. The surface of vibrating element is a circular plane. The acoustic lens is a plane-concave structure, one side surface of which is a circular plane; the other side surface is a concave sphere. The vibrating element is attached to the plane of the acoustic lens. The sign  $a$  is the radius of the vibrating element. The sign  $R$  is the curvature radius of the concave spherical surface. The sign  $O$  is the concave spherical center.



**Fig. 1.** Concave spherical acoustic lens and wave propagation path.

**2.2 The ray method for describing the sound wave propagation process**

According to the theory of optical ray imaging, it is assumed that sound wave propagates in the form of rays, and the propagation process generated is shown in Fig.1(b). The sound wave generated by the vibrating element enters into the acoustic lens from the plane side along the normal direction. Then the wave refracts into water on the concave sphere surface of the acoustic lens and intersects the center axis of the transducer at the point  $F$ , which is the focal point. The distance between  $F$  point and the center point of the concave sphere is the focal length.

According to Snell's law, the refractive index of lens medium entering water can be obtained as

$$n = \frac{\sin \theta_i}{\sin \theta_t} = \frac{c_1}{c_2} \tag{1}$$

where  $\theta_i$  is the incidence angle,  $\theta_t$  is the refraction angle,  $c_1$  the longitudinal wave velocity in lens,  $c_2$  is the sound velocity in water. Formula (1) shows that the sound wave refraction angle value at the interface between two medias is related to the velocity values of sound in the two medias. Since the refraction angle increases with the increase of the sound velocity in the refractive medium, the incident angles of acoustic wave are changing with the incident positions on the concave sphere surface of the acoustic lens. The expression for the intersection point length of the refraction wave and the central axis of the acoustic lens transducer can be deduced according to

the formula (1) and the geometric relationship shown in Fig.2(b) as

$$F = R \left( 1 + \frac{1}{\sqrt{n^2 - \sin^2 \theta_i} - \sqrt{1 - \sin^2 \theta_i}} \right) \tag{2}$$

when  $\theta_i$  or  $a$  is small, the formula (1) has a simple expression as

$$F \approx R \left( 1 + \frac{1}{n-1} \right) \tag{3}$$

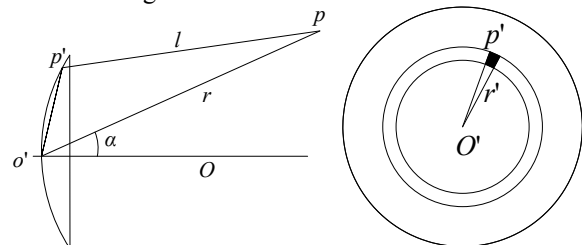
**2.3 The wave method for describing the sound wave propagation process**

When the sound wave propagation process is describing by the wave method, the wave generated by the vibrating element entering into the acoustic lens vertically is approximately plane wave. The distances between the central axis to the points at the concave sphere surface of lens are different. The phase of incident waves varies with the position of the incident point, so does the refractive waves. The acoustic wave at any other point of the concave surface of the acoustic lens has a phase delay of about  $\Delta t$  as compared with the center of the concave sphere. The incident sound wave on the interface point with the same length from the central axis has the same phase, and so does the corresponding refraction wave. Most researchers take this into account and consider the effect of phase difference when calculating the sound field.

On the interface of the two media, the refractive coefficient varies with the incident angle of the acoustic wave. Then It is evident that the amplitudes of the refraction wave at the points of different lengths from the center axis on the concave sphere surface are also different. Researchers basically ignored this situation and did not take into account the impact of amplitude differences when simulating.

**2.2.1 The formula for calculating the self focusing sound field of concave spherical surface**

The amplitude and phase of the radiation surface of the concave spherical self focusing transducer are the same, as shown in Fig.2.



**Fig. 2.** Concave Spherical Element.

Using the Helmholtz-Huygens integral formula, the sound pressure of point  $P$  in the radiation field of a concave spherical self-focusing transducer is calculated as

$$p = \frac{p_0}{2\pi} \int_0^{2\pi} \int_0^{r'_m} j \frac{e^{-jkl}}{l} r' dr' d\beta'$$

$$p_0 = k \rho c u_0 e^{j\omega t} \quad (4)$$

$$l = \sqrt{r^2 - 2rr' \sqrt{1 - \frac{r'^2}{4R^2}} \sin \alpha \cos \beta' + r'^2}$$

where the sign P is the sound pressure at a point in water. The sign  $\rho$  is the density of water. The sign  $c$  is the sound speed in water. The sign  $k$  is the wave number in water. The sign  $u_0$  is the amplitude of the radiation surface velocity. The sign  $r'$  is the distance from a micro-cell P' on the concave sphere to the center O'. The sign  $r'_m$  is the maximum value of  $r'$ . The sign  $r$  is the distance from the center O' of the concave sphere to the point P. The sign  $l$  is the distance from the micro-cell P' on the concave sphere to the point P. The sign  $\alpha$  is the angle between the two lines  $r$  and  $l$ . The sign  $\beta'$  is the projection of the central angle of the circle for the micro-cell P' on the concave sphere.

### 2.2.2 The acoustic field calculating formula only relating to the phase differences of refraction wave

When only the refraction wave phase on the concave sphere of the acoustic lens is considered, the initial phases of the micro-cells on the concave sphere are different. Adding the phase factor into the integral term the formula for calculating the radiated sound pressure produced by the transducer at a certain point in water would be as

$$p(r, \alpha) = \frac{p_0}{2\pi} \int_0^{2\pi} \int_0^{r'_m} j \frac{e^{-jk \left( l + \frac{r'^2}{2nR} \right)}}{l} r' dr' d\beta' \quad (5)$$

### 2.2.3 The sound field calculating formula relating to the differences of both amplitude and phase

The refraction coefficient of sound wave which is obliquely incident is

$$D = \frac{p_t}{p_i} = \frac{2}{1 + \frac{1}{m} \sqrt{\frac{n^2 - 1}{\cos^2 \theta_i} + 1}} \quad (6)$$

where, the signs  $\rho_1$  and  $\rho_2$  are respectively the densities of lens and water. The sign  $m$  is the density ratio of water to lens,  $m = \rho_1/\rho_2$ . The signs  $p_i$  and  $p_t$  are respectively the pressure values of incident wave and refractive wave. From the formulas (5) and (6), adding the factors of both phase and amplitude into the integral term, the formula for calculating the radiated sound pressure produced by the transducer at a certain point in water would be as

$$p(r, \alpha) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{r'_m} j p' \frac{e^{-jk \left( l + \frac{r'^2}{2nR} \right)}}{l} r' dr' d\beta' \quad (7)$$

$$p' = \frac{2p_0}{1 + \frac{1}{m} \sqrt{\frac{n^2 - 1}{\left( 1 - \frac{r'^2}{2R^2} \right)} + 1}}$$

## 3 Simulation and measurement results

### 3.1 The design of real acoustic lens transducer

The acoustic lens transducer has been designed and fabricated in the structure shown in Fig.1 and Fig.3(b). The working frequency  $f$  is 250 kHz. The vibration element, as shown in Fig.3(a), is composed of piezoelectric ceramic block particles, which has pure thickness vibration mode and can effectively avoid the transverse coupling effect. Its radius is 50mm. The lens material is hard aluminum, and the element is bonded to the plane side of the aluminum lens. The opening radius on the concave sphere side is 50mm, and the curvature radius is 51mm. The longitudinal wave velocity of sound in the aluminum lens is 6320 m/s and the density is 2790 kg/m<sup>3</sup>. The velocity of sound in water is 1500 m/s and the density is 1000 kg/m<sup>3</sup>.

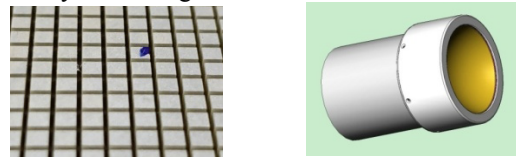


Fig. 3. The vibrating element and Acoustics Lens Transducer.

### 3.2 Simulating results using ray method

By the simple approximate formula (3) of ray method, the focal distance of the concave spherical acoustic lens is 66.9 mm. However, the intersection distance of the refracted sound and the central axis of the acoustic lens changes in the range of 64.1 mm to 66.9 mm with the incident angle, as shown in Figure 4. The acting of all refraction waves on the concave sphere surface of the acoustic lens are uniform in that range, and the final focus is located at 65.5mm, which is the middle of the range. But, in practice, as the refraction near the edge of the concave sphere of the acoustic lens gradually weakens, the interference effect on the focal point also decreases, and the final focal length should be slightly greater than 65.5mm.

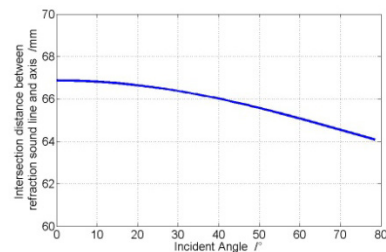
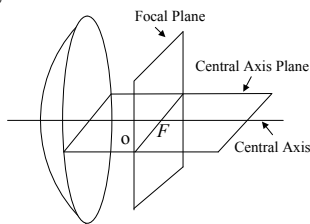


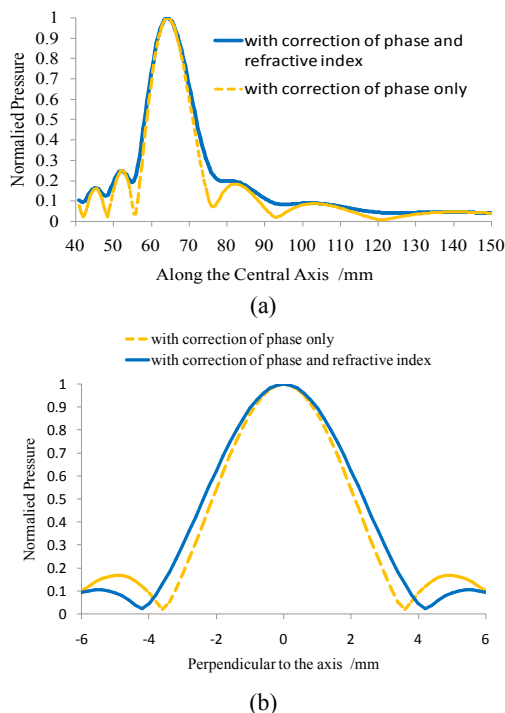
Fig. 4. The relationship between intersection distance and incident angle.

### 3.3 Simulation results using wave method

By MATLAB software, using the formulas (5) and (7), the acoustic field distributions have been calculated under the two different conditions, one is that only the phase difference is referred, and another is that both the phase and amplitude differences are referred, as mentioned above. Fig.5 shows the diagram of the central axis plane and the focal plane of the concave spherical acoustic lens. The central axis plane is the plane where the central axis lies on. The focal plane is a plane perpendicular to the central axis and passing through the focus point. The sound pressure distributions along the central axis, the central axis plane and the intersection line, on which the central axis plane and the focal plane intersect, are calculated respectively. The results are shown in Fig.6 and 7.



**Fig. 5.** The focal plane and central axis plane of the transducer.

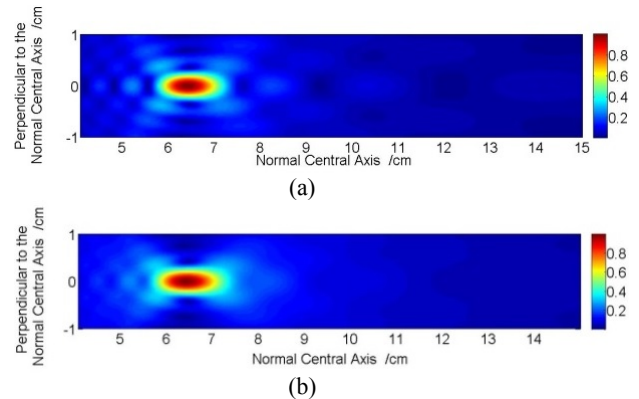


**Fig. 6.** The simulating result of sound wave pressure under two different correction conditions.

Fig.6(a) shows the normalized sound pressure distribution along the central axis of the acoustic lens transducer. The point with maximum sound pressure is the focal point. From the data, the focal length is 65mm, and the -3dB length of the focal area is 8.5mm relating to only the phase correction. The -3dB length is 9.5mm relating to both the phase and amplitude corrections. The difference of the two -3dB focal length is about 10%.

Fig.6(b) shows the normalized sound pressure distribution along the direction vertical to the axis. From the data, the -3dB width of the focal area is 3.2 mm

relating to only the phase correction. The -3dB width is 3.6mm relating to both the phase and amplitude corrections. And the difference between the two -3dB width is about 11%.



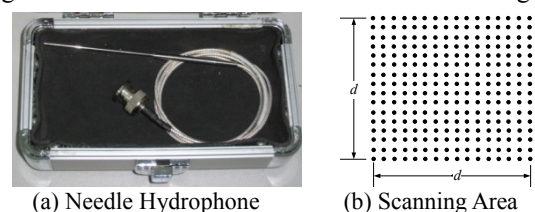
**Fig. 7.** The simulating result of sound wave pressure on the central axis plane under two different correction conditions.

Fig.7 is the numerical simulation of the sound field distribution in the plane of the center axis of the concave lens. Fig.7(a) shows the normalized sound pressure distribution on the centric axis plane relating to only the phase correction. Fig.7(b) shows the normalized sound pressure distribution on the centric axis plane relating to both the phase and amplitude corrections. The interference effect of the latter is obviously weaker than that of the former, and the pressure fluctuation of the latter is less severe than that of the former.

It can be seen that the focal area size is enlarged a little when the influence of refractive index is taken into account. This is because the amplitude distribution of the concave sphere surface of the acoustic lens is equivalently weighted. It is also found that the closer to the edge of the lens, the smaller the refractive index value of the wave is. In addition, it is clear that the more obvious the amplitude suppression effects on the refractive wave is, the weaker the interference effect and the focusing effect in the acoustic field are. Besides, we also noted that the more dispersed the sound beam is, the more gently the sound pressure fluctuate.

### 3.4 Comparison of simulation and measured results for the focal plane of acoustic lens transducer

By using the formula (7), the acoustic field distributions have been calculated under the condition that both the phase and amplitude differences are referred, as mentioned above. The sound field distribution of focal plane has been measured by using probe hydrophone and with acoustic field scanning method. The schematic diagram of the measurement scheme is shown in Fig.8.

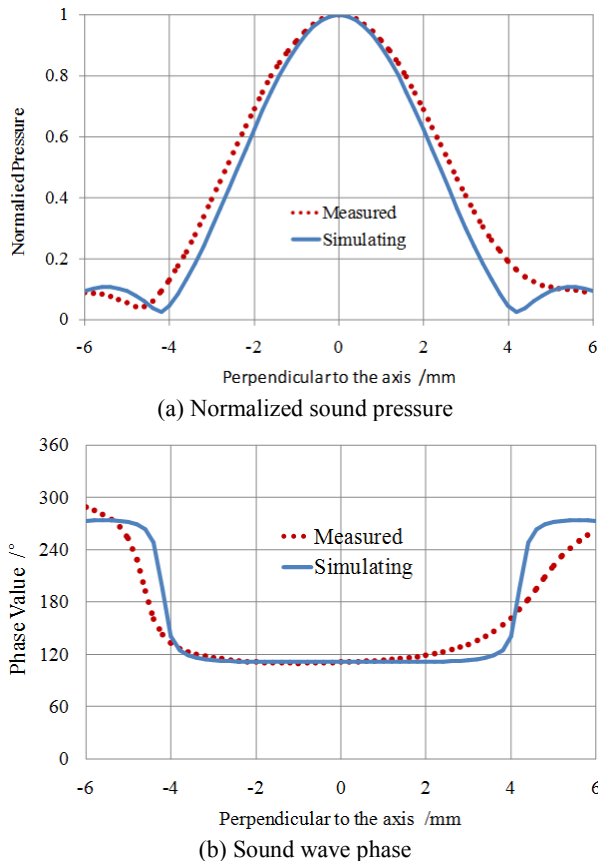


**Fig. 8.** The scanning measurement area with needle hydrophone.

Fig.8(a) is a probe hydrophone with a diameter of about 1.5mm, which has little effect on the radiated sound field. Fig.8(b) shows the scanning area of the focal plane and the distribution of the measuring points. The distance between the measuring points is 0.2mm with uniform interval.

The measurement results are shown in Fig.9 and Fig.10 respectively. Fig.9 shows the sound field distribution along the intersection line of the central axis plane and the focal plane. Fig.10 shows the normalized distribution of sound pressure on the focal plane.

Fig.9(a) is the normalized sound pressure distribution on the intersection line, and the focal length is 65.1mm, which is about 2% different from the results calculated by the ray method and the wave method. The simulating -3dB width of focal area is 3.6mm, the measured data is 3.9mm, the difference is about 8%. Fig.9(b) is the phase pressure distribution on the intersection line. It can be seen that the phase value keeps basically the same in the main lobe. From the measured results, it can be seen that the measuring plane is not completely perpendicular to the central axis, which is slightly biased.



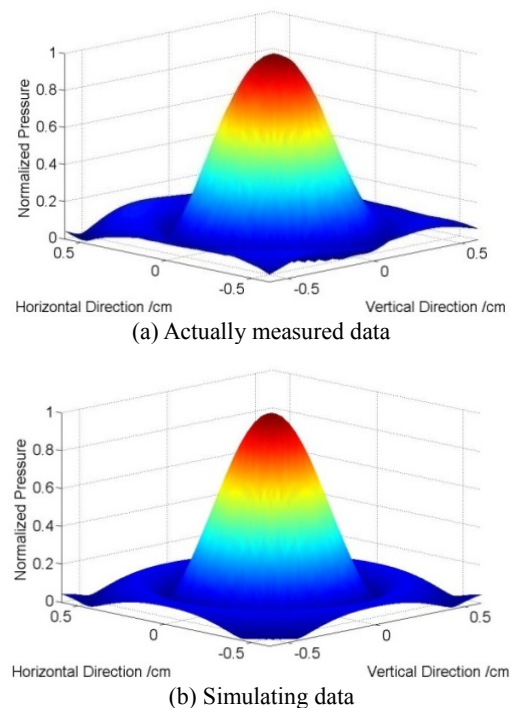
**Fig. 9.** The actually measured result and simulating result with correction of phase and refractive index on the direction perpendicular to the central axis.

Fig.10(a) and (b), whose shapes are the same, are respectively the measured and simulated normalized sound pressure distributions on the focal plane. Fig.10 shows that the acoustic lens transducer has good central symmetry.

## 4 Conclusions

In this paper, we demonstrated that the focal length of a concave spherical acoustic lens transducer can be preliminarily predicted by using the ray method. The distribution of the acoustic field can be accurately calculated by using the wave propagation method combined with the Helmholtz-Huygens integral formula, and the results calculated under the conditions of both phase and amplitude correction are more in line with the actual situation than those with phase correction only. The total results show that the radiated sound field of the concave spherical acoustic lens transducer calculated by wave theory is very close to the actual situation, which is useful for the design and research of the concave spherical acoustic lens focusing transducer.

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**Fig. 10.** The actually measured result and simulating result with correction of phase and refractive index on the focal plane perpendicular to the central axis.

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