Passive manipulation of low resonance scattering frequency associated with subsonic Rayleigh waves on submerged polymer spheres

Yanling Zhou^{1,2}, Jun Fan^{1,2,*}, and Bin Wang^{1,2}

¹Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai Jiao Tong University, 200240 Shanghai, China

²State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, 200240 Shanghai, China

Abstract. Observation of the backscattering form function for a solid polymethylmeth-acrylate(PMMA) sphere reveals a significant enhancement at low frequencies, which is associated with the subsonic Rayleigh waves resonance. The presence of these resonances suggests that solid PMMA spheres may be used as passive sonar targets for certain applications. The slightly deformed PMMA spheres with regular corrugation are constructed for studying and manipulating the low-frequency resonances of subsonic Rayleigh waves. For the regular-corrugated PMMA spheres, the first-order approximate of Rayleigh normal mode solution is obtained with the perturbation method and the backscattering form function is analyzed in each order. It can be found that coupling among multiple resonance modes is introduced by the boundary deformation and leads to the shift of resonance frequency. An approximate formula for the frequency shift with respect to the parameters of corrugated boundary is deduced. Based on this formula, it is possible to evaluate resonance frequency of subsonic Rayleigh waves quickly and then manipulate acoustic scattering field of the slightly deformed PMMA spheres in water passively.

1 Introduction

Solid-polymer objects have not received much attention in the study of backscattering primarily because the large absorption tends to limit the response at high frequencies. In addition to intrinsic attenuation, PMMA objects have other properties which make them interesting from the standpoint of scattering theory: PMMA objects typically have shear and Rayleigh wave velocities which are less than the speed of sound in water, and the density of PMMA is very close to water[1]. Different from the mechanism of the mid-frequency scattering enhancement[2-4] caused by subsonic antisymmetric Lamb waves in the previous studies on metallic shells, Hefner and Marston[5] studied the backscattering form function of PMMA sphere at low frequencies and interpreted a significant enhancement due to the tunneling mechanism which is shown to be associated with the subsonic Rayleigh waves in detail. Previous studies on PMMA objects have largely dealt with the typical solid spheres and spherical shells[5,6], and the presence of backscattering enhancement at low frequencies suggests that solid PMMA spheres may make good candidates for using as passive sonar targets for certain applications.

The echo characteristics of rough surface can be applied to ultrasonic nondestructive testing, sea surface and bottom reverberations modeling, classification and detection of marine plankton and so on, so the research on acoustic scattering of rough surface is very

meaningful. The regular-corrugated surface is a special case of randomly rough surface, whose underwater acoustic scattering remains to be further explored. For example, J. A. Fawcett used a hybrid Kirchhoff/diffraction method to study the high-frequency pulse acoustic scattering characteristics of rigid spheres and cylinders with sine square corrugation in the time domain[7], and obtained the wide-band frequency acoustic scattering characteristics from spherical shells using numerical method in the frequency domain[8]. In recent years, it has been found that acoustic metamaterials based on artificial microstructures such as regular-corrugated surface can produce many new physical phenomena[9], and the grasp of the echo characteristics from regular-corrugated targets is helpful to manipulate acoustic scattering field with the acoustic superstructure surface in water passively.

Below the first-order approximate of Rayleigh normal mode solution is obtained with the perturbation method and an approximate formula for the resonance frequency shift is deduced in Sec.2. In Sec.3, the acoustic scattering field of the regular-corrugated PMMA spheres is calculated by analytical and numerical method. In Sec.4, we put forward some issues for further study and its potential applications.

2 The perturbation theory

* Corresponding author: fanjun@sjtu.edu.cn

For a regular-corrugated PMMA sphere, the first-order approximate of Rayleigh normal mode solution is obtained with the perturbation method[10-12].

Fig. 1. The regular-corrugated PMMA solid sphere.

Neglecting time factor $e^{-i\omega t}$, the incident pressure is written as:

$$p_{inc}(r, \varphi) = \sum_{n=0}^{\infty} i^n (2n+1) j_n(kr) P_n(\cos \varphi) \quad (1)$$

where $k=\omega/c$ is wavenumber, j_n is the n th order spherical Bessel function, and P_n is the n th order Legendre polynomial.

We consider this solid sphere with a slightly deformation boundary defined as

$$r(\varphi) = a + \varepsilon f(\varphi) \quad (2)$$

The main assumption is that the deformation function $|\varepsilon f(\varphi)| \ll a$, a is the radius of sphere, and ε is the corrugated height.

It is convenient to look for the acoustic scattering field in the following form

$$p_{scat}(r, \varphi) = \sum_{n=0}^{\infty} i^n (2n+1) \left[\begin{aligned} &b_n h_n^{(1)}(kr) P_n(\cos \varphi) \\ &+ \varepsilon \sum_{p \neq n} \alpha_{bp} h_p^{(1)}(kr) P_p(\cos \varphi) \end{aligned} \right] \quad (3)$$

In the case of spherically symmetric geometry, the acoustic scattering field of the solid sphere can be completely described by the two scalar potentials, Φ and Ψ , which are with respect to longitudinal wave and shear wave, respectively and can be expressed as follows

$$\begin{aligned} \Phi(r, \varphi) &= \sum_{n=0}^{\infty} i^n (2n+1) \left[\begin{aligned} &c_n j_n(k_d r) P_n(\cos \varphi) \\ &+ \varepsilon \sum_{p \neq n} \alpha_{cp} j_p(k_d r) P_p(\cos \varphi) \end{aligned} \right] \\ \Psi(r, \varphi) &= \sum_{n=0}^{\infty} i^n (2n+1) \left[\begin{aligned} &e_n j_n(k_s r) P_n(\cos \varphi) \\ &+ \varepsilon \sum_{p \neq n} \alpha_{ep} j_p(k_s r) P_p(\cos \varphi) \end{aligned} \right] \end{aligned} \quad (4)$$

where b_n, c_n, e_n are the scattering coefficients, and $\alpha_{bp}, \alpha_{cp}, \alpha_{ep}$ are unknown coupling coefficients, $h_n^{(1)}$ is the spherical Bessel function of the first kind. k_d and k_s represent longitudinal wavenumber and shear wavenumber respectively.

The following are continuity conditions on the interface between both media

$$\tau_{rr}(a + \varepsilon f(\varphi), \varphi) + p(a + \varepsilon f(\varphi), \varphi) = 0 \quad (5)$$

$$u_r^{(2)}(a + \varepsilon f(\varphi), \varphi) = u_r^{(1)}(a + \varepsilon f(\varphi), \varphi) \quad (6)$$

$$\tau_{r\varphi}(a + \varepsilon f(\varphi), \varphi) = 0 \quad (7)$$

By imposing the boundary continuity conditions (5) ~ (7) one gets

$$[d_{6 \times 6}] \begin{pmatrix} b_n \\ c_n \\ e_n \\ \alpha_{bp} \\ \alpha_{cp} \\ \alpha_{ep} \end{pmatrix} = \begin{pmatrix} A_1^* \\ A_2^* \\ A_3^* \\ A_4^* \\ A_5^* \\ A_6^* \end{pmatrix} \quad (8)$$

According to the Clem's law, we can obtain

$$b_n = \frac{B_n}{D_n} \quad \alpha_{bp} = \frac{A_{bpn}}{D_n} \quad (9)$$

The form function is defined as

$$f(r, \varphi) = \frac{2r}{a} e^{-ikr} \sum_{n=0}^{\infty} i^n (2n+1) \left[\begin{aligned} &b_n h_n^{(1)}(kr) P_n(\cos \varphi) \\ &+ \varepsilon \sum_{p \neq n} \alpha_{bp} h_p^{(1)}(kr) P_p(\cos \varphi) \end{aligned} \right] \quad (10)$$

The characteristic equation of the slightly deformed sphere with regular corrugation is written as

$$D_n(x) = 0 \quad (11)$$

Correspondingly, the quasistationary eigenvalue, $kr \equiv x$, can be represented as the following series

$$x = x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \dots$$

where x_0 is the complex solution of solid sphere characteristic equation.

The root of equation(11) is

$$x = x_0 (1 - \varepsilon A_{mn}) \quad (12)$$

$$\text{where } A_{mn} = \frac{(2n+1)}{2} \int_0^\pi \frac{f(\varphi)}{a} P_n(\cos \varphi) P_n(\cos \varphi) \sin \varphi d\varphi$$

So an approximate formula for the frequency shift of the regular-corrugated spheres is obtained as

$$\Delta x = x - x_0 = -\varepsilon x_0 A_{mn} \quad (13)$$

3 Theoretical calculation and numerical simulation

The backscattering form function of regular-corrugated PMMA spheres is calculated using COMSOL Multiphysics software, and compared with the approximate analytical Rayleigh normal mode solutions in Sec. 2, as shown in Fig.2. In formula (2), $a=25.4\text{mm}$, $f(\varphi)=\sin^2(m\varphi)$, m is the corrugated period, $\xi=\varepsilon/a$ stands for normalized deformation, and the material parameters used are shown in Table 1. Only the first-order approximation is considered in the perturbation method, and the couplings among some normal modes due to the slightly deformed surface are neglected.

It can be observed that these solutions are consistent very well except for $ka \approx 4.5$. The first-order approximate of Rayleigh normal mode solution is feasible to calculate the acoustic scattering of regular-corrugated spheres.

Table 1. Material parameters used in calculation.

Material	Density (kg/m ³)	Longitudinal velocity (m/s)	Shear velocity (m/s)
PMMA	1190	2690	1340
water	1000	1479	—

Fig. 2. The backscattering form function of regular-corrugated spheres is calculated using the finite element method(FEM)and the perturbation method(PM) (a) $\xi=2\%$, $m=2$, (b) $\xi=2\%$, $m=3$.

Fig. 3. The backscattering form function for a PMMA sphere and the spectrum for different orders (a) the summed form function, (b)~(j) spectrum for different orders $n=2\sim10$.

The backscattering form function of PMMA spheres and the spectrum for different orders $n=2\sim10$ are calculated by the Rayleigh normal mode solution, as shown in Fig.3. It can be seen that the first resonance is associated with the partial wave $n=2$, $n=3\sim10$ and so on. Mastering these characteristics is helpful in analyzing the

mechanism of the slightly deformed PMMA spheres with regular corrugation.

Fig. 4. For the slightly deformed PMMA spheres with regular corrugation, the backscattering resonance frequency shift is calculated with the PM and approximate formula. (a) $m=2$, $\xi=3\%$, (b) $m=2$, $\xi=5\%$.

The acoustic backscattering of regular-corrugated PMMA spheres is calculated with the perturbation method and the form function is analyzed in each order. It can be found that coupling among multiple resonance modes is introduced by the boundary deformation and leads to the shift of resonance frequency, which is compared with the frequency shift from the approximate formula in Sec. II, as shown in Fig.4. The triangles connected by solid line indicate the frequency shift in each order with the perturbation method, and the circles with solid line show the result using approximate formula given in Sec. 2. The radius of a PMMA sphere is now slightly deformed with $m=2$, $\xi=3\%$ in Fig.4(a), and $m=2$, $\xi=5\%$ in Fig.4(b). The comparison result shows that the resonance frequency shift for the slightly deformed PMMA spheres with regular corrugation can be estimated by formula(13). Based on this formula, it is possible to evaluate resonance frequency of subsonic Rayleigh waves quickly. The resonance frequency shift is associated with the corrugated height and period, and more sensitive to the corrugated period and height as the frequency increases. Therefore, the acoustic scattering field of the slightly deformed PMMA spheres in water can be manipulated passively by changing the parameters of corrugated boundary, such as corrugated height and period and so on.

4 Conclusions

The first-order approximate of Rayleigh normal mode solution is obtained with the perturbation method for the regular-corrugated PMMA spheres, and an approximate formula for the resonance frequency shift is deduced,

which is introduced by coupling among multiple resonance modes due to the slightly deformed boundary. However, it is worthwhile to have a further study on the coupling mechanism among multiple normal modes and the technology used to manipulate the acoustic scattering field of the slightly deformed PMMA spheres.

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