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# The non-stationary dynamic of a beam under the elastic-plastic impact of a conical indenter

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**Abstract.** The problem of impact interaction of a conical indenter with the surface of a steel beam under elastic-plastic deformation is considered. The analysis of the dynamic reaction of the beam to the impact is considered analytically and numerically. The analytical solution is based on the phenomenological model of conical indentation and the solution of wave equations of beam dynamics at impact. The numerical study of the indentation process was carried out using the ANSYS software. The dynamic response of the beam is analyzed for different geometric parameters and different values of strength characteristics. Such tasks are widely used in construction practice in the implementation of non-destructive methods of assessing the mechanical characteristics of structures.

## 1 Introduction

Assessment of mechanical properties of steel structures elements is one of the most important tasks of building diagnostics at any stage of their operation. When designing it is important to know the most significant mechanical properties, such as yield strength, tensile strength, elongation, and hardness [1,2]. Recent years the non-destructive testing methods are widely spread in the civil construction field especially for existing structures [3,4]. There are many types of research show the possibility of an assessment of the parameters of creep [5] of materials, fatigue [6] and welding residual properties [7].

A hybrid, numerical–analytical model is presented in [8] to investigate the transient response of a simply supported elastic–plastic beam subjected to the impact of a sphere. The solution is obtained with the assumption that the contact region is small and elastic-plastic behavior is described by Stronge's model [9,10] for spherical indenter.

An axisymmetric contact problem on the indentation of a rigid conical indenter into an elastic transversely isotropic half-space with a functionally graded transversely isotropic coating is considered in [11]. Elastic moduli of the coating vary in depth according to arbitrary continuous positive functions, independent of each other.

Dynamic response of a sandwich beam using a yielding criterion by considering the time inhomogeneity of foam core deformation is established in [12]. The collapse of a foam core under high-velocity impact leads to the change of the plastic neutral surface and beam cross-sectional area.

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An analytical approximation and numerical solution of the transverse impact interaction of a compact body with an infinitely long Euler-Bernoulli beam is studied in [13]. The beam is initially stationary, linear and dissipation-free, a Hertzian spring mediates body-beam contact, and the body is otherwise rigid. Impact interaction obeys two nonlinear differential equations with a fractional order derivative.

For some reasons, we use conical indenter as more informative for rough surfaces of structures. The choice of the indenter form is an important task, and it has determined the response of the structure element. The investigations considering indentation of dual [14] and multiple indenter tips [15] show large interest in using indenters of complex shape, consisting of two or more connected profiles without coupling. To develop a model of beam dynamic response prediction during conical impact indentation, we need to solve a task of the interaction of indenter with elastoplastic beam and to evaluate their use.

## 2 Materials and methods

We consider the steel beam with the yield strength $\sigma_y$, elastic modulus $E$ and Poisson ratio $\nu$. Plastic deformation in a beam obeys the Tresca conditions, with the plastic constant k=0.5$\sigma_y$.The conical indenter is considered as an elastic body with modulus $E_k$ and Poisson coefficient $\nu_k$. The beam of length $l$ is freely supported at the ends, on which the indenter falls in the middle between the supports. When oscillating, the elements of the beam perform the only translational motion.

The General equation of transverse vibrations of the rod has the form

$$\frac{d^2y}{dt^2}+a^2\ \frac{d^4y}{dx^4}=0 \quad , \quad a^2=\frac{EJg}{F\gamma} \tag{1}$$

where $J$ is a moment of inertia, $F$ - a cross-sectional area of a beam, $\gamma$ - weight per unit volume of beam material.

The General expression of free transverse oscillations has the form

$$y=\sum_{i=1}^{\infty}\sin\frac{i\pi x}{l}(A_i\cos p_it+B_i\sin p_it) \tag{2}$$

$p_1=ak_i^2=\dfrac{a\pi^2}{l^2}, p_2=\dfrac{4a\pi^2}{l^2}, p_3=\dfrac{9a\pi^2}{l^2},...$ are the frequency of sequential kinds of oscillation.

We consider the expression in brackets as generalized coordinates $q_i$. We formulate the Lagrange equations for the given system and obtain an expression for the deflection of the beam upon impact in the form

$$y=\sum_{i=1,3,5}^{\infty}\frac{l^2 2g}{i^2\pi^2aF\gamma l}\int_0^t P(t)\sin\frac{i^2\pi^2a(t-t_1)}{l^2}dt_1. \tag{3}$$

where $P(t)$ - contact force to be determined.

Full displacement of the indenter upon impact

$$h=\alpha+y \tag{4}$$

where $\alpha$ - indentation depth, $y$ – beam deflection.

Then the integral equation for calculating the impact parameters

$$v_0 t - \int_0^t \frac{1}{m} dt_1 \int_0^{t_1} P dt_2 = \alpha(P) + \sum_{i=1,3,5}^{\infty} \frac{2gl^2}{i^2 \pi^2 a F \gamma l} \int_0^t P(t) \sin \frac{i^2 \pi^2 a (t - t_1)}{l^2} dt_1 \tag{5}$$

where $v_0$ – initial velocity of the indenter.

In [1,2] approximate solution of elastic-plastic problems is considered by solving two problems: the elastic and rigid-plastic with the following assumptions:

1. The displacement $\alpha$ of any point of interacting solids is the sum of the elastic $\alpha_e$ and plastic $\alpha_p$ components (Figure 1)

$$\alpha = \alpha_e + \alpha_p \tag{6}$$

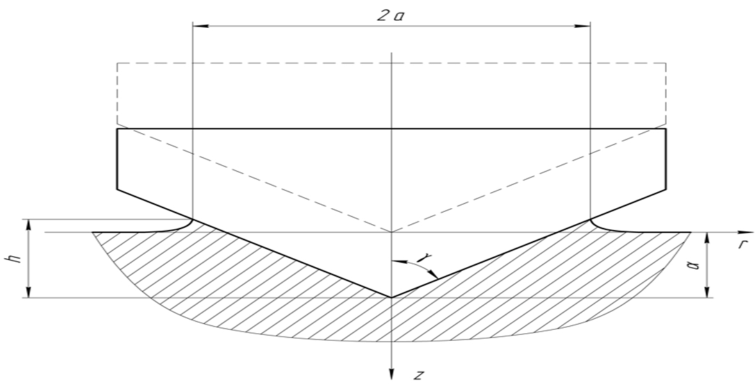



**Fig.1.** Diagram of indentation of a cone.

2. In the contact zone, the distribution of contact pressures is determined by the elastic solution for cone

$$q(F, r) = -q_0 \ln\left(a/r - \sqrt{(a/r)^2 - 1}\right), \quad q_0 = P/\left(\pi a^2\right) \tag{7}$$

3. The average pressure in the contact zone does not exceed Brinell stresses.

$$q_0 \le \lambda k, \quad \lambda \approx 5.7 \tag{8}$$

4. The growth of the plastic deformation occurs when the conditions (8) are performed

$$q_0 = \lambda k, \qquad dP/dt > 0 \tag{4}$$

So, the dependence of indentation force $F(\alpha)$ is [1]

$$\alpha = C F^{1/2} \quad , \tag{6}$$

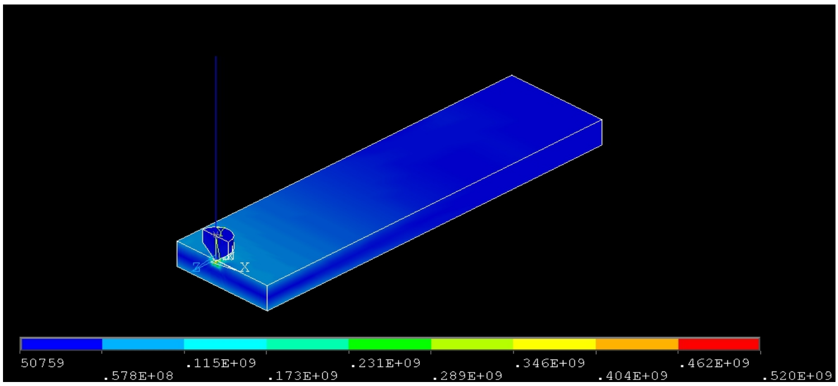
$$C = ctg(\gamma)(1 - \delta^*)(\lambda k \pi)^{-1/2} + (1 + (2\delta^* - 2)/\pi)(\lambda k \pi)^{1/2} E^{-1}.$$

where δ* - coefficient determined at the experiment and expressing the pile-up of material during indentation.

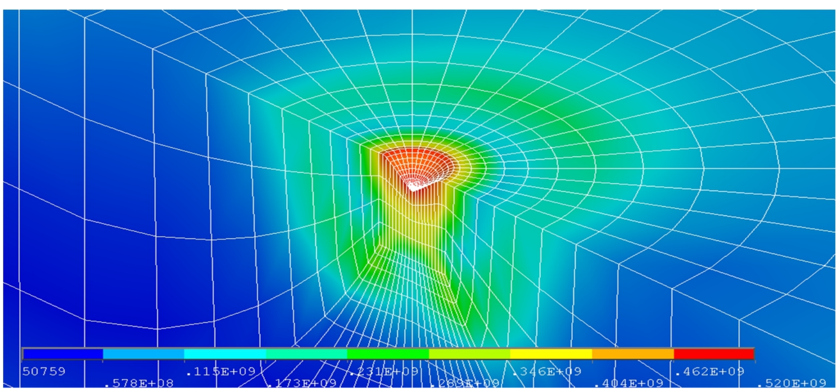
The solution of integral equation (5) is carried out numerically considering the dependence (6) for $P(t)$ and on the assumption that within the integration step the contact force varies linearly.

## 3 Finite element analysis and experimental data

The stress-strain state of the steel beam by impact cone indentation was investigated with the finite element method. We considered a beam $l$=60, 100 and 160 mm, thickness $h_b$=5 mm and width $b$=22 mm, the indenter hit on the beam at the distance 60 mm, 100 mm, and 160 mm from one side. The scheme of the model is shown in Figure 2.




a)




b)

**Fig.2.** FEM modeling: a) 1/2 of the beam with indenter; b) FE web of the model.

Material of the beam is steel with 0,09% C, 2% Mn and <1% Si and $\sigma_y$=290 MPa.

For modeling the elastic-plastic properties of the material, the 3D higher order 20-node elements that exhibit quadratic displacement interpolation were used. For a more accurate solution to contact area mesh refinement was performed (Figure 2). As we can see (Figure 2a) the impact was inflicted at the beam center and with the 5 mm offset of the edge. For the simulation of the beam, the model of a bilinear isotropic hardening has been used in the plastic deformation zone that depends on the stress-strain state.

## 4 Results and discussion

The results of an experimental study with the numerical analysis during the impact of conical indenter 90° was compared (Figure 3).

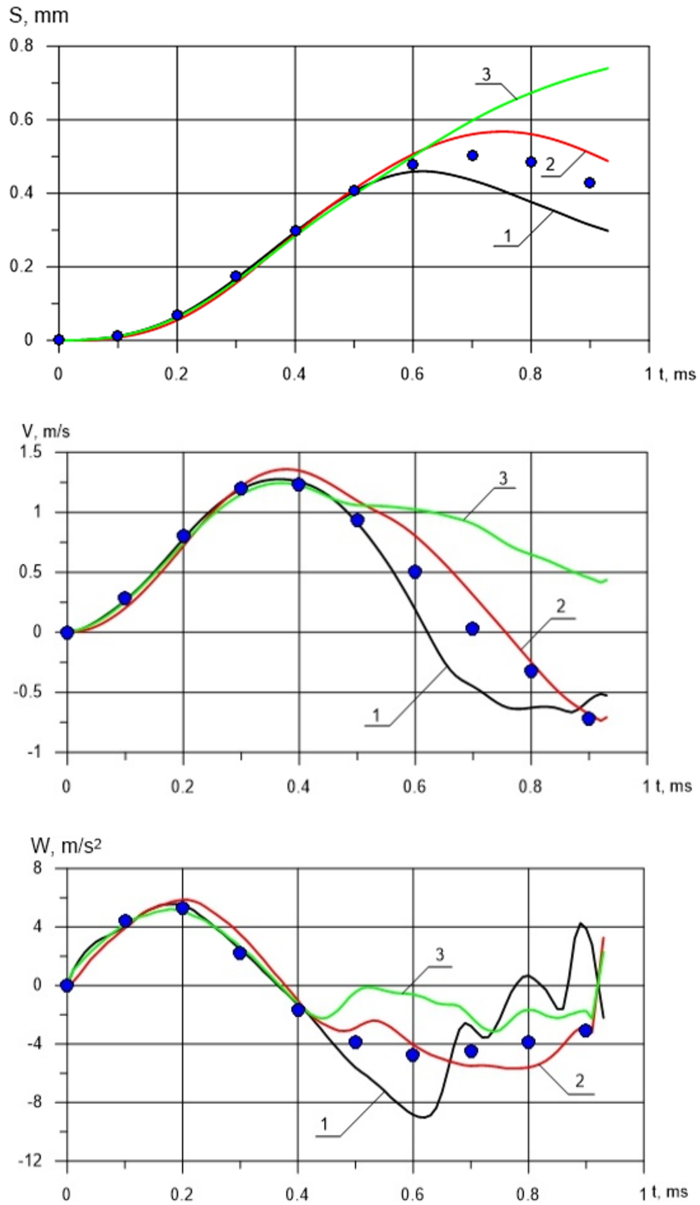



**Fig. 3.** The dependence of the displacement *S(t)*, velocity *V(t)*, and acceleration *W(t)* vs. time for a conical indenter: 1 – beam *l*=60 mm, 2 – *l*=100 mm, 3 – *l*=160 mm, ● - experimental data *l*=100 mm

Comparison of the results of theoretical and experimental studies have shown good convergence. Fig. 3 presents the dependence of the displacement *S(t)*, velocity *V(t)* and the acceleration *W(t)*. The discrepancy between the results of theoretical and experimental studies in the determination of kinematic parameters was in the range: 12% for displacement, 13% for speed and 13% for acceleration. It is seen, that in the active phase of indentation up to approximately 0.4 ms, the curves almost coincide. This indicates that the mechanical characteristics of the beam with constant impact parameters play a key role. The discrepancy begins to rise in the next phase when the geometrical parameters of the beam begin to play a leading role.

## 5 Conclusions

The problem of non-stationary beam dynamics at the impact of a conical indenter taking into account the elastic-plastic deformation in the contact zone is considered. The analytical solution is obtained, the numerical analysis by the finite element method is carried out. The data of numerical analysis were compared with the experimental data obtained at a special shock installation. The obtained models allow us to calculate the results of a collision with beam structures using non-destructive testing methods.

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