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Optimum threshold of group formation in multi-agents

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Abstract. This paper presents a simple multi-agent model of group formation, while the behaviour is complicated. The statistical characteristics of the formation are suddenly changed at some states. Their agents are carrying characteristic vectors, meet each other in non-dimensional free spaces without any restrictions randomly, and the agents make groups in the spaces if their characteristics are similar, that is, they have high similarities. Actually, for a given threshold on similarities, when the characteristic vectors between two agents are similar in the threshold, the agents join into one. They are not only for agents but also for groups, i.e. two groups can become into one if they are similar, and we repeat it. On the other hand, making groups decrease a satisfaction on groups. In this paper, we show, for a given threshold, there is not only an optimal threshold to maximize the satisfactions among groups which satisfy the threshold, but also the other one to minimize the satisfactions. The thresholds to maximize satisfactions are experimentally approximated by some function on the size of characteristic vectors rather than the number of agents.

1 Introduction

To discover a new model is essential that the model is to be simple. Our model is of multiple agents, and each agent is quite simple. And, also the interactions among agents are simple, but the behavior sometimes becomes complex. In this paper, we present a new model for multi-agents to form groups, their agents meet each other at random in free moving spaces autonomously and combine the groups into a new group if the characteristic vectors are similar or compatible (an extended similarity). We suppose each agent carrying a characteristic vector that are properties represented by numerical vectors, and assume that when two persons meet each other in the free spaces and also their similarities are higher than a certain threshold, they form a new group. That is, if an agent is close to any other one in the degrees of similarities, two agents form a new group. It applied to not only agents but also groups, i.e. two agents/groups are combined into new one. In this paper, we first define a similarity, that is a compatibility by inner product between two characteristic vectors of agents, and extend this definition to the similarity between two groups consisting groups, that is a set of agents. However, forming group might reduce the satisfactions of the groups by getting the size of groups larger, because each group consists of similar agents,

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but each agent is different more or less in general. For an example, suppose A is similar to B, B is similar to C, and they form an agent group. Then, A is always similar to C? When an agent group becomes larger, the satisfaction of the agent becomes lower. Proving a threshold is a lower limit of "ideal height", which allows to form groups. When the threshold value becomes lower, the ideal height also becomes lower. Conversely, the ideal height becomes higher when the threshold becomes higher. Forming groups is to become lower on the satisfaction of groups. Then, it is difficult to form a group if the ideal is too either high or low. This paper presents an interesting result for forming groups, and there is not only a threshold value to maximize the amount of the satisfactions of the groups, but also the other threshold to minimize the amount of satisfactions. By choosing two threshold values appropriately, we show that the amount of the fitness among the group can become the highest, and can also become lowest. It says that two kinds of optimal thresholds we have.

The nearest work to ours is [1, 2] for looking another kind of partner in the context of two kinds of agents as same as in our free moving spaces, and two agents become partners each other. The partners are keeping finite time before disassociating, where the compatibility of two agents is defined as well as symmetric, and the goal is to maximize the duration keeping the pairs. [3] is discussion on the real world, not so small, on the conference community of DBLP. It uses the techniques of decision trees, time series and diffusion, they are trying to find some incentive to join the conference community so that to present their papers to the community is not only to related the number of friends, but also how those friends connected to other friends within or out of the community. Even, they are close to their research topics of interest. However, the community as well as others is complicated for obtaining explicit results, how many friends are interrelated. The Schelling segregation model [4] is not only a simple model to produce a segregation among groups as well as our work, also the results are also simple. But, these behaviors are sometimes complicated as [5, 6]. Our discussions can be applied to our real world as collective dynamics [7] and scaling features [7, 8] as we have to try in the next. These are on the discussions in colored free spaces rather than complete free spaces.

This paper is organized as following. We define our model and present an example in the following section. In Section 3, we show our experiments and our discussions. We present our conclusions in the final section.

2 Proposed model of group formation

Consider N agents $A_1, A_2, ..., A_N$ of which each has a fixed feature S-dimensional vector $\mathbf{x}_i$, and they are moving at random in a moving free space. Then, the agents form groups consisting of some agents if their agents meet at random in the space, their feature vectors match or the similarity value is higher than a certain threshold, i.e. we consider a model in which agents with high compatibility between agents form groups.

Suppose two agents A_i and A_j ($i \neq j$), and two have the feature vector $\mathbf{x}_i$ and $\mathbf{x}_j$, respectively. For the simplicity, assume every elements of their vectors are either 1 or -1. Then we define a compatibility $C'_{i,j}$ between A_i and A_j as well as in [1, 2]

$$C'_{i,j} = (\mathbf{x}_i \cdot \mathbf{x}_j + S)/2,$$

where · denotes an inner product between two vectors, and S is the number of charcteristic vectors. Then, we have $C'_{i,j} = C'_{j,i}$ and $0 \leq C'_{i,j} \leq S$, i, j=1,..,N. Also, $C'_{i,j}$ follow a binomial distribution Bin(S,1/2). The higher similarity $\mathbf{x}_i$ and $\mathbf{x}_j$ becomes the higher compatibility $C'_{i,j}$, and vice versa.

We suppose all the agents belong to one of the groups $G_1, ..., G_N$, and they are exclusive. Further, we extend the degree of conformity between agents to the degree of conformity between groups.

$C_{i,j}=\frac{\sum_{k\in G_i, l\in G_j, k<l} C'_{k,l}}{|G_i|\,|G_j|}$

$C_{i,j}$ denote a compatibility between G_i and G_j, and it is the amount of the compatibilities of all the agents belonging to their groups.

Similar to $C'_{i,j}$, we have a symmetric $C_{i,j} = C_{j,i}$ and $0 \leq C_{i,j} \leq S$. Two groups G_i and G_j meet in free spaces, and when their compatibility $C_{i,j}$ are higher than a threshold τ ($0 \leq \tau \leq S$), two groups united into one group, and a new group is generated. We note that when $C_{i,j} = \tau$, two groups G_i and G_j into one with even probability 1/2.

A satisfaction S_i of a group G_i is defined by the sum of inner products as:

$S_i = \sum_{k,l\in G_i, k\neq l} x_k \cdot x_l$

We assume that the satisfaction of G_i is zero, if G_i are a singleton. S_i are the sum of inner products of two agents among groups, and the sum of the satisfactions for every groups by:

$S_{sum} = \sum_{i=1}^{M} S_i$.

By varying the threshold τ, we can find the threshold τ_{op} to maximize the S_{sum}. Also, we can find the other optimal threshold to minimize the satisfactions.

Example 1 (table 1):

Consider the five agents A_1, A_2, A_3, A_4 and A_5, whose vectors x_i have the same dimension $S = 4$, and we define the vectors as

$\mathbf{x}_1 = [1,-1,1,1]$,

$\mathbf{x}_2 = [-1, 1, 1,-1]$,

$\mathbf{x}_3 = [1, -1, 1,1]$,

$\mathbf{x}_4 = [1, 1, 1,-1]$, and

$\mathbf{x}_5 = [1, 1, -1,1]$.

Table 1. C'_{ij} in Example 1.

C'ij	1	2	3	4	5
1		1	4	2	2
2	1		1	3	1
3	4	1		2	2
4	2	3	2		2
5	2	1	2	2	

The compatibilities can be computed, for an example, $C'_{1,2}$ is as

$C'_{1,2} = \frac{x_1 \cdot x_2 + 4}{2} = 1$.

At the initial agent setting, each agent belongs to a group having only its own group as follows:

$G_1 = \{1\}$, $G_2 = \{2\}$, $G_3 = \{3\}$, $G_4 = \{4\}$, $G_5 = \{5\}$.

Set the threshold $\tau = 2.5$, and suppose G_1 and G_3 meets at random, and merge into one by $C_{1,3} = C'_{1,3} = 4 > \tau$ and the following groups are formed.

$G_1 = \{1, 3\}$, $G_2 = \{2\}$, $G_3 = \{4\}$, $G_4 = \{5\}$.

Also, if G_2 and G_3 encounter and $C_{2,3} = C'_{2,4} = 3 > \tau$, G_2. Then, the running groups are:

$G_1 = \{1, 3\}$, $G_2 = \{2,4\}$, $G_3 = \{5\}$,

The compatibilities between the groups are as:

$C_{1,2} = \frac{C'_{1,2}+C'_{1,4}+C'_{3,2}+C'_{3,4}}{2\times 2} = 1.5 < \tau$,

$C_{1,3} = \frac{C'_{1,5}+C'_{3,5}}{2\times 1} = 2.0 < \tau$, and

$C_{2,3} = \frac{C'_{2,5}+C'_{4,5}}{2\times 1} = 1.5 < \tau$.

Forming groups are inactive, anymore. The compatibilities of their groups are:
$S_1 = \mathbf{x}_1 \cdot \mathbf{x}_3 + \boldsymbol{x_3} \cdot \mathbf{x}_1 = 8.0$,
$S_2 = \mathbf{x}_2 \cdot \mathbf{x}_4 + \boldsymbol{x_4} \cdot \mathbf{x}_2 = 6.0$, and
$S_3 = 0$.

The amount S_{sum} of the compatibility of the agents is:
$S_{sum} = S_1 + S_2 + S_3 = 14.0$

The final group composition obtained may differ depending on the order to encounter each other in the free space. In this research, the group encounters randomly, and it does not depend on group size. But, we will see in the following section that the thresholds to maximize the satisfactions do not depend on the number of agents.

3 Experiments and the results

We examined the formation of groups by agents and computed the thresholds to maximize S_{sum}. We present the following algorithm:

(1) Provide thirty agents, i.e. N=30. Each element of all the feature vectors is either 1 or -1 with even probability.

(2) Two groups meet in the free space randomly and are unified into one if the compatibility of G_i and G_j is greater than the threshold $\boldsymbol{\tau}$. If the threshold is equal to $\boldsymbol{\tau}$, they are unified with even probability.

(3) Repeat (2), until group formation is impossible.

(4) Finally, we show the amount S_{sum} of satisfiability.

We examined 1,000 times for each threshold $\boldsymbol{\tau}$, and their average values are shown in figure 1 and 2. The figures show that there is the optimum threshold $\boldsymbol{\tau}_{op}$ so that the thresholds to maximize the compatibilities S_{sum} can be experimentally approximated by the following expression:

$$\boldsymbol{\tau_{op}} = \frac{\boldsymbol{S}}{\boldsymbol{2}} + \frac{\sqrt{\boldsymbol{S}}}{\boldsymbol{3}}.$$

We present a comparison of the experimental values and the approximated expression values shown in figure 3. Although it looks like a straight line, the values of the second term become relatively smaller as $\boldsymbol{\tau}$ is larger. On the other hand, the thresholds to minimize the S_{sum} can be find in the figures, and also the values are independent to the number of agents, and the values, the optimum thresholds, are low fifty.

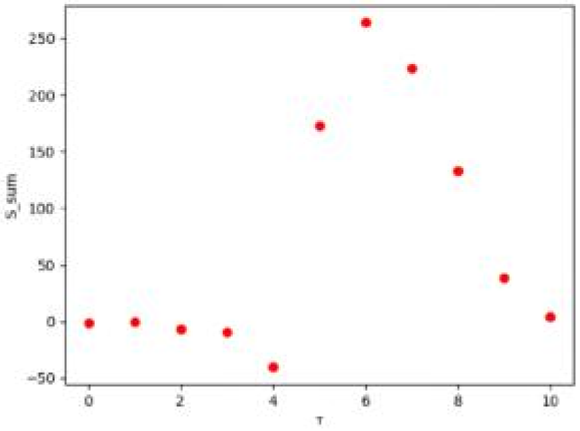

Fig. 1. $\boldsymbol{\tau}$ vs. S_{sum}. The experiment of the case N=30, S=10.

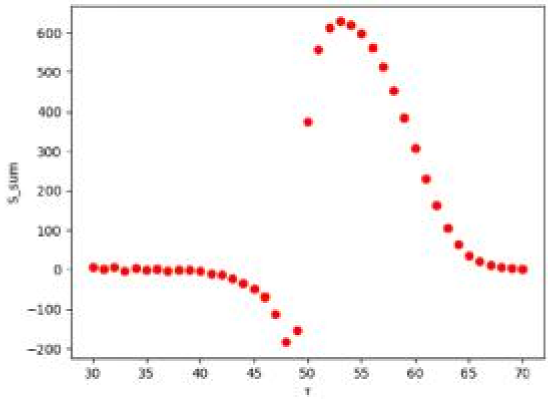

Fig. 2. τ vs. S_{sum}. The experiment of the case N=30, S=100.

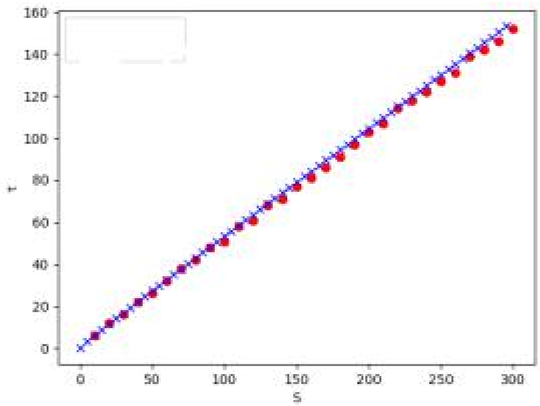

Fig. 3. S vs. τ(maximized). The experimental values (red dot) and approximated values (×).

We have presented the group formation of agents in free spaces. For every two agents encounter each other in the free space, and join into a new group if two are similar. The compatibilities $C'_{i,j}$ to represent the similarities follow binomial distribution Bin(S,1/2), so that the average is S/2, and the variance is S/4 along normal distributions. If two agent similarities is greater than a given threshold $\boldsymbol{\tau}$, they form new groups. So, our model simple and also no restrictions to apply on real world. To become the size of the groups larger decrease the satisfactions of groups lower. So, we expect to derive an optimum threshold and also to find the theoretical values S_{sum}. Our experiments show that our discussions are true. The similar approach can be applied to the threshold which the satisfactions make worst.

4 Conclusions

This paper experimentally showed that, when agents are carrying feature vectors and they meet at random in moving spaces forming groups, the optimum thresholds exist.

We have discussed the group formation of agents in free spaces. For every two agents meet each other in the free space, and join into a new group if two are similar. To become the size of groups larger decreases to become the satisfactions of groups lower. So, we expect to derive an optimum threshold and also to find the theoretical values S_{sum}. Our experiments show that our discussions are true. Similarly, the threshold to minimize the compatibilities can be found. Choosing the worst threshold will not only form a group, but it also become at the worst level of satisfaction.

For our future works, since our experiment was conducted on a simple network that the agent groups randomly meets in free spaces, we will perform on more complex networks as in [7, 8]. Furthermore, consider the model in which the probability of meeting depends on the size of the groups, and we will derive the theoretical expression of S_{sum}.

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