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Adaptive fractional PID control of biped robots with time-delayed feedback

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Abstract. This paper presents the application of Fractional Order Time-Delay adaptive neural networks to the trajectory tracking for chaos synchronization between Fractional Order delayed plant, reference and Fractional Order Time-Delay adaptive neural networks. The proposed new control scheme is applied via simulations to control of a 4-DOF Biped Robot [1]. The main methodologies, on which the approach is based, are Fractional Order PID the Fractional Order Lyapunov-Krasovskii functions methodology. The structure of the biped robot is designed with two degrees of freedom per leg, corresponding to the knee and hip joints. Since torso and ankle are not considered, it is obtained a 4-DOF system, and each leg, we try to force this biped robot to track a reference signal given by undamped Duffing equation. The tracking error is globally asymptotically stabilized by two control laws derived based on a Lyapunov-Krasovski functional.

1 Introduction

Fractional calculus is a generalization of differential and integral calculus which involves generalized functions. The first to work this new branch of mathematics was Leibniz. Due to the growing interest in the applications of fractional calculation, in this work we obtain conditions that guarantee the tracking of trajectories of nonlinear systems generated by differential equations of fractional order which we will call plants (This term is widely used in engineering), which in our case will be a mechanical arm, a helicopter, a plane or limbs of a humanoid, all of fractional order.

The problem of tracking control of trajectories is very important, since the control function allows the non-linear system to carry out a previously assigned task, work or trajectory, for example, a mechanical arm and its objective is to cut a piece with a previously generated form, or the coupling of two aircraft in space. We include mathematical models with time delay, since the processing and transmission of information is important in this type of systems, which depending on the delay, these systems can generate undesirable oscillatory or chaotic dynamics, and cause instability in the mathematical model that describes the trajectory tracking error.

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The chapter is organized as follows: first, the general mathematical model of non-linear systems is proposed, as a second part, the Neural Network is proposed that will adapt to the non-linear system and the reference signal that both must follow, as a third part obtains the dynamics of the tracking error between the non-linear system and the reference, after obtaining conditions in the laws of adaptation of weights in the Neural Network and obtaining the control law that guarantees that the tracking error converges to zero , so that the non-linear system will follow the indicated reference signal, which is what was wanted to be demonstrated. Finally simulations are presented, which illustrate the theoretical results previously demonstrated.

There are several ways to define the fractional calculation, in this research we will use the well-known derivative of Caputo, which has equation (1):

$$aD_t^\alpha f(t) = \frac{1}{\Gamma(\alpha-n)}\int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau ,(n-1<\alpha<n) \quad (1)$$

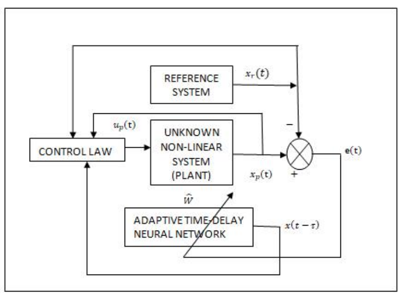

Fig. 1. Adaptive recurrent control diagram.

We use the scheme of figure 1 to indicate the procedure used in the obtaining of the laws of adaptation of weights and the laws of control that guarantee that the tracking error between the non-linear system, the neural network and the reference signal converges to zero.

2 Time-delay adaptive neural network and the reference

The nonlinear system, equation (5), which is forced to follow a reference signal:

$$\boldsymbol{aD_t^\alpha x_p = f_p(t, x_p(t) + x_p(t-\tau)),\ t \in [0,T],\ 0<\alpha\leq 1,} \quad (2)$$

$$x_p(\mathrm{t}) = \mathrm{g(t)}, \text{ where } x_p, f_p \in \Re^n,\ u \in \Re^m,\ g_p \in \Re^{nxn}.$$

The differential equation will be modeled by the neural network [2]: $aD_t^\alpha x_p = \mathrm{A}(x) + \mathrm{W}^*\Gamma_\mathrm{z}(x(t-\tau) + \Omega_\mathrm{u}$

The tracking error between these two systems:

$$w_{per} = x - x_p \text{ and } aD_t^\alpha w_{per} =- kw_{per} \quad (3)$$

We use the next hypotheses. The nonlinear system is [3]:

$$aD_t^\alpha x_p = aD_t^\alpha x + w_{per} = \mathrm{A}(x) + \mathrm{W}^*\Gamma_\mathrm{z}[x(t-\tau)] + w_{per} + \Omega_\mathrm{u} \quad (4)$$

where the W* is the matrix weights.

3 Tracking error problem

In this part, we will analyze the trajectory tracking problem generated by

$$aD_t^\alpha x_r = f_r(x_r,u_r),\ w_r, x_r \in \Re^n \quad (5)$$

The time derivative of the error is:

$$aD_t^\alpha e_n = aD_t^\alpha x - aD_t^\alpha x_r = \mathrm{A}(x) + \mathrm{W}^*\Gamma_\mathrm{z}[x(t-\tau)] + w_{per} + \Omega_\mathrm{u} - f_r(x_r,u_r) \quad (6)$$

The equation (6), can be rewritten as follows, and the unknown plant will follow the fractional order reference signal, if:

$$aD_t^{\alpha}e = \mathrm{A}e + \mathrm{W}^*\Gamma_z(x(t-\tau)) - \widehat{W}\Gamma_z(x_r(t-\tau)) - \mathrm{A}e + (A+I)(x-x_r) + \Omega(u-\alpha_r(\mathrm{t},\widehat{W})) \quad (7)$$

Now, $\widehat{W}$ is part of the approach, given by W^*. The Eq. (7), can be expressed as Eq. (8)

$$aD_t^{\alpha}e = \mathrm{A}e + (\mathrm{W}^* - \widehat{W})\Gamma_z(x(t-\tau)) + \widehat{W}\Gamma(z(x(t-\tau)) - z(x_r(t-\tau))) + (A+I)(x-x_r) - \mathrm{A}e + \Omega(u-\alpha_r(\mathrm{t},\widehat{W})) \quad (8)$$

$$\widetilde{W} = \mathrm{W}^* - \widehat{W} \text{ and } \widetilde{u} = u - \alpha_r(\mathrm{t},\widehat{W}) \quad (9)$$

And by replacing Eq. (9) in Eq. (8), we have:

$$aD_t^{\alpha}e = (A+I)e + \widetilde{W}\sigma(x(t-\tau)) + \widehat{W}\emptyset_{\sigma}(t-\tau) + \Omega u_2 \quad (10)$$

The control law, we will obtain using the fractional order Lyapunov-Krasovskii methodology.

4 Study of trajectory tracking error

Our mathematical model of the dynamics in the tracking error is described in (10). In this equation we can see that an equilibrium state of this system is $(e,\widehat{W}) = 0$.

Without loss of generality we can assume that the matrix A is given $A =- \lambda I, \lambda > 0$, where I is the identity matrix of order nxn.

For the study of the stability of the tracking error we propose the following PID control law [4], widely used in science and engineering.

We will determine conditions in the parameters that guarantee that the tracking error converges to zero, and we will also use the following control law [5].

$$\Omega u_2 = K_p e + K_i aD_t^{-\alpha}e + K_v aD_t^{\alpha}e - \gamma\left(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_{\phi}^2\right)e \quad (11)$$

We also include the following control law, $PI^{\lambda}D^{\alpha}$ [6]: $u(t) = K_p e(t) + K_i aD_t^{-\lambda}e(t) + K_d aD_t^{\alpha}e(t)$,

Substituting Eq. (11) in Eq. (10):

$$aD_t^{\alpha}e = \frac{-1}{a}(\lambda - 1 + K_p)e + \frac{1}{a}\widetilde{W}\sigma(x(t-\tau)) + \frac{1}{a}\widehat{W}\emptyset_{\sigma}(t-\tau) + \frac{1}{a}K_i aD_t^{-\alpha}e - \frac{\gamma}{a}\left(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_{\phi}^2\right)e \quad (12)$$

And if $w = \frac{1}{a}K_i aD_t^{-\alpha}e$, then $aD_t^{\alpha}w = \frac{1}{a}K_i e(t)$, [7], then Eq. (12) we rewrite as:

$$aD_t^{\alpha}e_n = \frac{-1}{a}(\lambda - 1 + K_p)e + \frac{1}{a}\widetilde{W}\sigma(x(t-\tau)) + \frac{1}{a}\widehat{W}\emptyset_{\sigma}(t-\tau) + w - \frac{\gamma}{a}\left(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_{\phi}^2\right)e \quad (13)$$

Let V be, the next candidate Lyapunov function as [8] and [9] **:**

$$V = \frac{1}{2}(e_n{}^T, w^T)(e_n, w)^T + \frac{1}{2a}tr\{\widetilde{W}^T\widetilde{W}\} + \frac{1}{a}\int_{t-\tau}^{t}[\emptyset_{\sigma}^T(s)\,\widehat{W}^T\widehat{W}\emptyset_{\sigma}(s)]ds \quad (14)$$

The fractional order time derivative of (14) along the trajectories of Eq. (13), and we select the next learning law from the neural network weights as in [10] and [11]:

$$tr\{aD_t^{\alpha}\widetilde{W}^T\widetilde{W}\} =- e^T\widetilde{W}\sigma(x(t-\tau)) \quad (15)$$

Then Eq. (13) is reduced to

$$aD_t^{\alpha}V = \frac{-1}{a}(\lambda - 1 + K_p)e^T e + \frac{e^T}{a}\widehat{W}\emptyset_{\sigma}(t-\tau) + \left(1 + \frac{K_i}{a}\right)e^T w - \frac{\gamma}{a}\left(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_{\phi}^2\right)e^T e + \frac{1}{a}[\emptyset_{\sigma}^T(t)\widehat{W}^T\widehat{W}\emptyset_{\sigma}(t) - \emptyset_{\sigma}^T(t-\tau)\widehat{W}^T\widehat{W}\emptyset_{\sigma}(t-\tau)] \quad (16)$$

Next, let's consider the following inequality proved in [12]

$$X^T Y + Y^T X \leq X^T \Lambda X + Y^T \Lambda^{-1} \mathrm{Y} \quad (17)$$

Which holds for all matrices $X,Y \in \Re^{nxk}$ and $\Lambda \in \Re^{nxn}$ with $\Lambda = \Lambda^T > 0$. Applying (22) with $\Lambda = I$ to the term $\frac{e^T}{a}\widehat{W}\emptyset_\sigma(t-\tau)$ from Eq. (16), where, we get

Here, we select $\left(1+\frac{K_i}{a}\right) = 0$ and $K_v = K_i + 1$, with $K_v \geq 0$ then $K_i \geq -1$, with this selection of the parameters from Eq. (16) is reduced to:

$$aD_t^\alpha V \leq \frac{-1}{a}\left(\lambda - 1 + K_p\right)e^T e - \frac{(\gamma-1)}{a}\left(\frac{1}{2}+\frac{1}{2}\left\|\widehat{W}\right\|^2 L_\phi^2\right)e^T \tag{18}$$

If $(\lambda - 1 + K_p) > 0$, $a > 0$, $(\gamma - 1) > 0$, so that: $aD_t^\alpha V \leq 0, \forall\, e, w, \widehat{W} \neq 0$, $e \neq 0$, is wanted to be demonstrate.

The control law is given by Eq. (19)

$$u_n = \Omega^\dagger[-\widehat{W}\Gamma\big(z(x_n(t-\tau)) - z\big(x_p(t-\tau)\big)\big) - (A+I)\big(x - x_p\big) + K_p e + K_i aD_t^{-\alpha} e + K_v aD_t^\alpha e - \Gamma\left(\frac{1}{2}+\frac{1}{2}\left\|\widehat{W}\right\|^2 L_\phi^2\right)e_n + f_r(x_r,u_r) - Ax_r - \widehat{W}\Gamma_z(x_r(t-\tau)) - x_r + x_p] \tag{19}$$

Theorem: The control law Eq. (19) and the neuronal adaptation law given by Eq. (15) guarantee that the fractional tracking error converges to zero, by which the tracking of trajectories of the non-linear system is guaranteed Eq. (4).

5 Identification of the unknown non-linear system by the neural network

Following the procedure above, we obtain the following control law. Let V be, the next candidate Lyapunov function as

$$V = \frac{1}{2}\left(e_p{}^T, w^T\right)\left(e_p, w\right)^T + \frac{1}{2a}tr\{\widetilde{W}^T\widetilde{W}\} + \frac{1}{a}\int_{t-\tau}^{t}\left[\emptyset_\sigma^T(s)\,\widehat{W}^T\widehat{W}\emptyset_\sigma(s)\right]ds \tag{20}$$

Then, (21) is reduced to

$$aD_t^\alpha V \leq \frac{-1}{a}\left(\lambda - 1 + K_p\right)(e_p{}^T)(e_p) - \frac{(\gamma-1)}{a}\left(\frac{1}{2}+\frac{1}{2}\left\|\widehat{W}\right\|^2 L_\phi^2\right)(e_p{}^T)(e_p) < 0 \tag{21}$$

The previous inequality guarantees that the identification of the non-linear system is satisfied, that is, the approach error converges to zero asymptotically

$$u_p = \Omega^\dagger[\widehat{W}\Gamma z(x_r(t-\tau)) - \widehat{W}\Gamma\big(z(x_n(t-\tau)) - z\big(x_p(t-\tau)\big)\big) - (A+I)\big(x - x_p\big) + K_p e + K_i aD_t^{-\alpha}e + K_v aD_t^\alpha e - \Gamma\left(\frac{1}{2}+\frac{1}{2}\left\|\widehat{W}\right\|^2 L_\phi^2\right)e_n - \Gamma\left(\frac{1}{2}+\frac{1}{2}\left\|\widehat{W}\right\|^2 L_\phi^2\right)e_p + f_r(x_r,u_r) - f_p\big(x_p\big) + Ax_p - Ax_r + \widehat{W}\Gamma_z\big(x_p\big) - x_r + x_p] \tag{22}$$

6 Simulation

The mathematical model, which describes the movement dynamics of the bipedal robot (figure 2), is obtained using the Euler-Lagrange equations [1], [13].

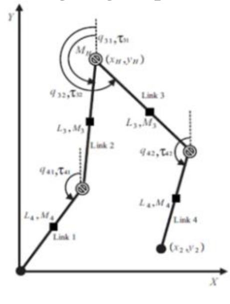

Fig. 2. Dynamic model of Biped Robot.

$$D(q(t)\ddot{q}(t) + C(q(t),\dot{q}(t))\dot{q}(t) + G(q(t)) = B\tau(t)$$

where $q(t) = [q_{31}(t)q_{32}(t)q_{41}(t)q_{42}(t)]^T$, is the generalized coordinates vector. As usual, $D(q(t))$ is the inertia matrix, bounded and positive definite, and $C(q(t),\dot{q}(t))$ is the matrix of Coriolis and centripetal forces. $G(q(t))$ represents a matrix of gravitational effects and B defines the input matrix. The vector $\tau(t) = [\tau_{31}(t)\tau_{32}(t)\tau_{41}(t)\tau_{42}(t)]^T$, defines the applied joint torques of the robot.

To illustrate the theoretical results obtained, we propose an example, which, as can be seen in the simulations, trajectory tracking is guaranteed and the neural network is described by:

$aD_t^{\alpha}x_p = \mathrm{A}(x) + \mathrm{W}^*\Gamma_z(x(t-\tau)) + \Omega_u$, with τ=25 sec, $A =- 20I, I \in \Re^{4x4}$, and, W^* is estimated using the learning law given in (15).

$\Gamma_z(x(t-\tau)) = (\tanh(x_1(t-\tau)), \tanh(x_2(t-\tau)),..., \tanh(x_n(t-\tau)))^{\mathrm{T}}$, $\Omega = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 00 & & 1 \end{pmatrix}^{\mathrm{T}}$ and the u is obtained using (22), and the reference signal that they have to follow, both the non-linear system and the neural network is given by the Duffing equation [14].

$$\ddot{x} - x + x^3 = 0.114\cos(1.1t) : x(0) = 1, \dot{x}(0) = 0.114$$

$$\frac{x(t)}{dt} = y(t)$$

$$\frac{y(t)}{dt} = x(t) - x^3(t) - \alpha y(t) + \delta\cos(\omega t)$$

Here, the conventional derivatives are replaced by the fractional derivatives as follows:

$$aD_t^{\alpha}x(t) = y(t)$$

$$aD_t^{\alpha}x(t) = x(t) - x^3(t) - \alpha y(t) + \delta\cos(\omega t)$$

where α, ω, δ, are the parameters of the Duffing differential equation, which we will use as a reference trajectory, that the non-linear system and the neural network have to follow.

As can be seen in figures 4 and 5, the tracking of trajectories in the states of the system are performed with satisfaction, while figure 3 shows the plane phase of the same fractional order differential equation. Figures 6, 7 show the torques applied to the ends of the bipedal robot. Parameter values of the fractional order, alpha (0.0001) and beta (0.0001) are included.

$$\alpha = 0.001,\ \beta = 0.001$$

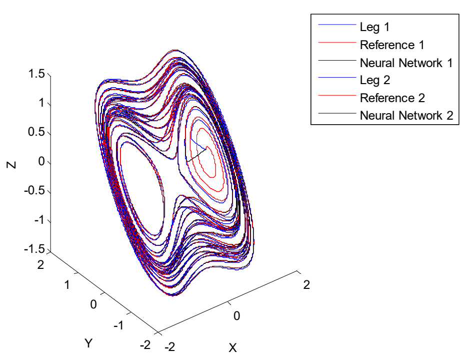

Fig. 3. A phase space trajectory of Duffing equation.

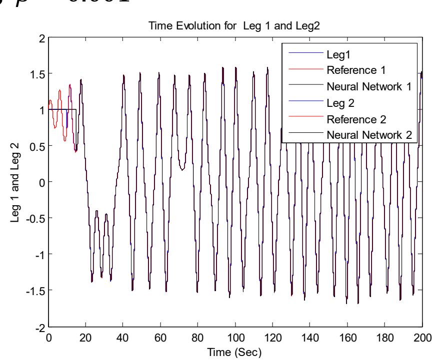

Fig. 4. Time evolution for the angular Position Leg 1 and Leg 2 (rad) of Link 1.

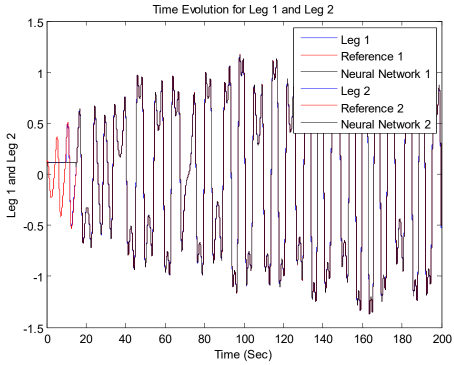

Fig. 5. Time evolution for the angular Position Leg 1 and and Leg 2(rad) of Link 2.

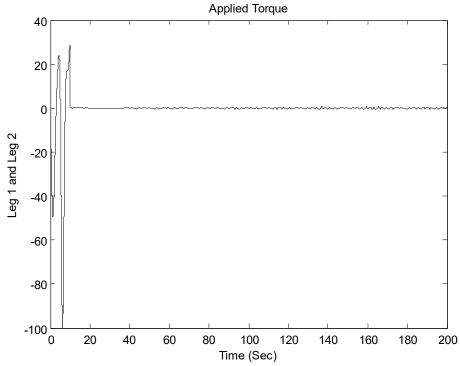

Fig. 6. Torque (Nm) applied to Leg 1 and Leg 2 of Link 1.

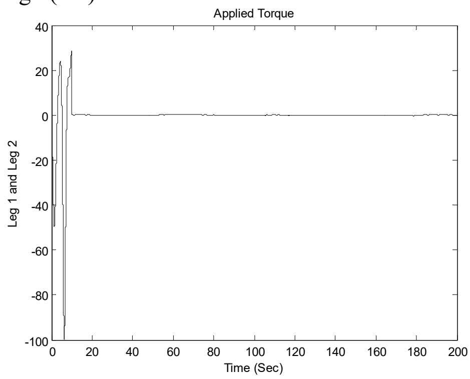

Fig. 7. Torque applied to Leg 1 of Link 2.

7 Conclusions

In this research work, conditions have been obtained in the parameters of the adaptive recurrent neural network, as well as laws of control and laws of neuronal adaptation, which, together, guarantee that the tracking error of trajectories between the non-linear system and the reference signal converges asymptotically to zero, so that trajectory tracking is develops with satisfaction.

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