

Damage plasticity model for passively confined concrete

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Abstract. This paper presents a modified concrete damage plasticity model (CDPM) for passively confined concrete within the concrete damage plasticity theory frame in ABAQUS. The modified CDPM can be used to simulate concrete under non-uniform passive confinement, for example, Fiber-reinforced polymer (FRP)-confined square concrete columns. The modification of CDPM includes a flow rule and a strain hardening/softening criterion in which dilation angle and yield stress are important parameters. Based on the true-triaxial experiment results of passively confined concrete, the dilation angle and yield stress were determined considering different confinement stiffness and non-uniform confinement stiffness ratio. Finally, the modified CDPM were incorporated in the ABAQUS model. The prediction of the finite element model of FRP-confined square concrete columns shows good prediction accuracy.

1 Introduction

Fiber-reinforced polymer (FRP) has been widely used in retrofit of concrete columns. Since FRP is a linear elastic material, when it is used to wrap concrete columns, the confinement provided by FRP jacket is passive confinement. Passive confinement means the confinement is not a constant, but changes with the deformation of concrete. The confinement in circular sections is uniform, whereas, the confinement is non-uniform in many other situations, such as in square and rectangular sections, partially wrapped columns, and columns under eccentric loading. The constitutive model for passively confined concrete is the basic for modelling FRP-confined concrete columns with finite element method. The existing constitutive models mainly focus on concrete under uniform passive confinement [1, 2]. For concrete under non-uniform passive confinement, the equivalent uniform confinement concept is adopted in the constitutive relationship [3, 4]. However, as reported by Zeng et al.[5], the prediction accuracy needs to be improved in simulation of columns under non-uniform passive confinement with constitutive model based on equivalent uniform concept. The constitutive model considering the influence of non-uniform confinement ratio is urgent to be established to give precise prediction of concrete columns under non-uniform passive confinement.

The constitutive model for non-uniformly passively confined concrete was established in this paper based on the concrete damage plasticity model (CDPM) frame in ABAQUS. First, the shear strength ratio in Lubliner and Lee yield criterion [6, 7] was determined based on the distribution characteristic of experiment data in stress space. Then, the yield stress was calculated according to the Lubliner and Lee yield criterion [6, 7]. The relationship between yield stress and concrete strength, axial plastic strain, confinement stiffness, and non-uniform confinement stiffness ratio was established. Afterwards, the dilation angle was calculated according to the flow rule and the lateral strain-axial strain curve. The relationship between dilation angle and axial plastic strain, confinement stiffness and non-uniform confinement stiffness ratio was determined. Finally, the modified CDPM was

incorporated with ABAQUS, and was used to simulate six groups of FRP-confined square concrete columns with different corner radius.

2 Experimental database

The constitutive model is established on the experiment data of true-triaxial compression experiment on passively confined concrete[8]. In the experiment, GFRP bars were used to generate passive confinement which was transferred to concrete cube by steel platen. Totally, 117 concrete cubes (Table 1) were casted, which were divided into 3 groups according to concrete strength (f_{c0} =25.4MPa, 36MPa and 44MPa). Thirteen confinement conditions were designed for each group, considering lateral confinement stiffness ρ (Eq. (1)) and non-uniform confinement stiffness ratio η (Eq. (2)).

$$\rho_1 = \frac{\Delta\sigma_1}{\Delta\varepsilon_1 f_{c0}}; \rho_2 = \frac{\Delta\sigma_2}{\Delta\varepsilon_2 f_{c0}} \quad (1)$$

$$\eta = \frac{\min(\rho_1, \rho_2)}{\max(\rho_1, \rho_2)} \quad (2)$$

where ρ_1, ρ_2 is lateral confinement stiffness in two lateral directions, respectively; σ_1, σ_2 is lateral confinement; $\varepsilon_1, \varepsilon_2$ is lateral strain; f_{c0} is unconfined concrete strength.

Table 1. Experimental database

Spe. ID	f_{c0}	D_1	D_2	ρ_1	ρ_2	η
N-A-20	20	0	8	0	20.8	0
N-B-20	20	0	10	0	27	0
N-C-20	20	0	14	0	40.8	0
N-D-20	20	0	16	0	42.2	0
N-A-30	36	0	8	0	15.4	0

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Continued Table 1

Spe. ID	f_{c0}	D_1	D_2	ρ_1	ρ_2	η
N-B-30	36	0	10	0	18.9	0
N-C-30	36	0	14	0	20.3	0
N-D-30	36	0	16	0	24.4	0
N-A-40	43	0	8	0	4.7	0
N-B-40	43	0	10	0	13.8	0
N-C-40	43	0	14	0	23.7	0
N-D-40	43	0	16	0	26.8	0
A-A-20	25.4	8	8	16.3	16.3	1
B-B-20	25.4	10	10	22.9	22.9	1
C-C-20	25.4	14	14	33.2	33.2	1
D-D-20	25.4	16	16	37.7	37.7	1
A-A-30	36	8	8	11.3	11.3	1
B-B-30	36	10	10	16.1	16.1	1
C-C-30	36	14	14	23.5	23.5	1
D-D-30	36	16	16	25.9	25.9	1
A-A-40	44	8	8	9.4	9.4	1
B-B-40	44	10	10	13.3	13.3	1
C-C-40	44	14	14	19.2	19.2	1
D-D-40	44	16	16	21.2	21.2	1
A-C-20	20	8	14	19.6	41.9	0.47
B-C-20	20	10	14	28.9	39.9	0.72
A-D-20	20	8	16	18.6	40.7	0.46
B-D-20	20	10	16	28.9	39.8	0.72
C-D-20	20	14	16	38.4	47.0	0.82
A-C-30	36	8	14	11.1	24.5	0.45
B-C-30	36	10	14	17.0	22.5	0.76
A-D-30	36	8	16	10.2	23.9	0.42
B-D-30	32	10	16	17.8	26.1	0.68
C-D-30	32	14	16	26.8	28.6	0.94
A-C-40	43	8	14	9.4	18.9	0.50
B-C-40	43	10	14	13.7	19.5	0.70
A-D-40	43	8	16	9.6	21.0	0.46
B-D-40	43	10	16	13.2	20.7	0.64
C-D-40	44	14	16	19.6	20.9	0.94

Note: D_1 and D_2 are diameters of GFRP bars in two lateral directions.

3 Yield criterion

The yield function F of concrete damage plasticity model (CDPM) proposed by Lubliner et al.[6] and modified by Lee and Fenves[7] was adopted in this paper.

$$F = \frac{(\bar{q} - 3A\bar{p} + B(\tilde{\epsilon}^{pl})\langle\bar{\sigma}_{max}\rangle - C\langle-\bar{\sigma}_{max}\rangle)}{1-A} - \bar{\sigma}_{cn}(\tilde{\epsilon}_c^{pl}) = 0 \quad (3)$$

$$\langle x \rangle = \frac{1}{2}(|x| + x) \quad (4)$$

where $\bar{\sigma}_i = \sigma_i / (1 - d_c)$, \bar{q} , \bar{p} and $\bar{\sigma}_{cn}$ are effective deviatoric stress, average stress and yield stress, respectively; d_c is damage parameter. For FRP-confined concrete is under triaxial compression. $\bar{\sigma}_{max} < 0$, and $\langle\bar{\sigma}_{max}\rangle = 0$, $\langle-\bar{\sigma}_{max}\rangle = -\bar{\sigma}_{max}$, Eq.(3) is then simplified into Eq.(5).

$$F = \frac{1}{1-A}(\bar{q} - 3A\bar{p} + C\bar{\sigma}_{max}) - \bar{\sigma}_{cn}(\tilde{\epsilon}_c^{pl}) = 0 \quad (5)$$

where $A = \frac{f_{b0} / f_{c0} - 1}{2f_{b0} / f_{c0} - 1} \quad (6)$

$$C = \frac{3(1-K)}{2K-1} \quad (7)$$

where f_{b0} is concrete strength under equal biaxial compression; f_{c0} is unconfined concrete strength; K is shear strength ratio.

3.1 Shear strength ratio

Shear strength ratio is the ratio of deviatoric stress of concrete under equal biaxial compression ($\sigma_1=0$, $\sigma_2=\sigma_3<0$) to triaxial compression where two lateral stresses are equal and larger than axial stress ($0>\sigma_1=\sigma_2>\sigma_3$). The shear strength ratio K is taken as 0.725, which was first proposed by Yu et al.[2]. The distribution of stress point in deviatoric plane is shown in Fig.1. The Lode angle of the experimental data is between 56° and 60° . As can be seen, $K=0.725$ can well simulate the distribution of stress point in this range.

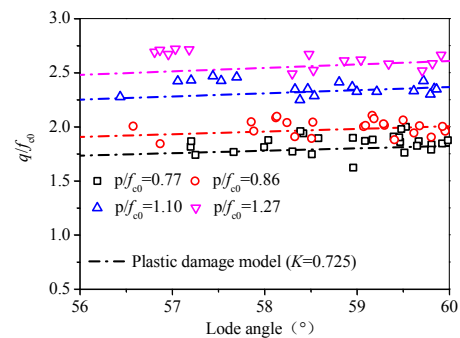


Fig. 1. Experimental and theoretical equivalent pressure

3.2 Damage parameter

For concrete with strain hardening, the damage parameter is defined as zero. For concrete with strain softening, the damage parameter is defined with Eq. (8).

$$d_c = 1 - \frac{\sigma_c}{f_{cc}} \quad (8)$$

Where d_c is compression damage parameter, σ_c is axial stress in strain softening phase, f_{cc} is the peak stress.

3.3 yield stress

According to Eq. (5), the yield stress can be determined as follows

$$\bar{\sigma}_{cn}(\bar{\varepsilon}_c^{pl}) = \frac{1}{1-A}(\bar{q}-3A\bar{p}+C\bar{\sigma}_{max}) \quad (9)$$

The curve that yield stress varies with axial plastic strain is shown in Fig.2 and Fig.3. It is noted that in Fig.2 and Fig.3, nominal yield stress is the result of yield stress divided by concrete strength. As can be seen in Fig.2, when concrete is under uniform passive confinement, the nominal yield stress increases with the increase of confinement stiffness. It is shown in Fig.3, For concrete with unidirectional passive confinement($\eta=0$), the nominal yield stress exhibits strain softening. With the increase of non-uniform confinement stiffness ratio, the nominal yield stress changes from strain softening to strain hardening gradually. Eq. (10) was used to describe the relationship between nominal yield stress and axial plastic strain, confinement stiffness and non-uniform confinement stiffness ratio. The behavior of Eq. (10) is shown in Fig.2 and Fig.3, which shows good coincidence with the experimental results.

$$\frac{\bar{\sigma}_{cn}}{f_{c0}} = a' + \frac{b'\varepsilon_c^p}{1+c'\varepsilon_c^p} + d'\varepsilon_c^p \quad (10)$$

$$a' = 0.142; \quad b' = 8755; \quad c' = 9949 \quad (11)$$

$$d' = \frac{13.66(\eta-1)}{\eta+0.26} + \frac{-31.85+2.76\rho_{max}-0.023\rho_{max}^2}{1+0.032\rho_{max}} \quad (12)$$

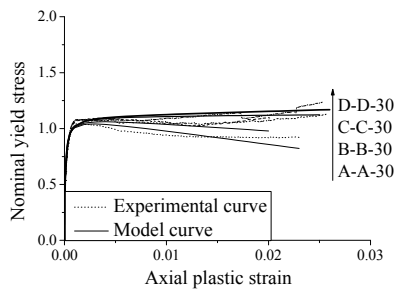


Fig. 2. Nominal yield stress of concrete under non-uniform passive confinement

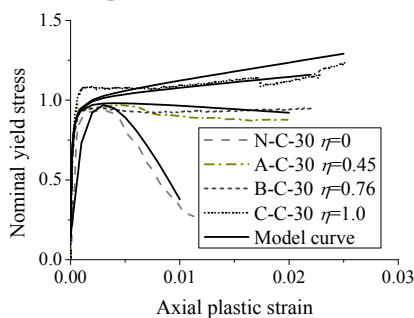


Fig. 3. Nominal yield stress of concrete under non-uniform passive confinement

4 Dilation angle

The flow rule of potential function of CDPM is described in hyperbolic function (Eq. (13)).

$$G = \sqrt{(\varepsilon \bar{\sigma}|_0 \tan \psi)^2 + \bar{q}^2} - \bar{p} \tan \psi \quad (13)$$

where ε referred to the eccentricity; $\bar{\sigma}|_0$ referred to uniaxial tensile stress; ψ referred to dilation angle.

According to the normality flow rule of plastic strain, the plastic strain of each direction can be determined by

$$d\varepsilon_i^p = \lambda \frac{\partial G}{\partial \sigma_i} (i=1,2,3) \quad (14)$$

The relationship between dilation angle and the axial strain increment can be obtained :

$$\tan \psi = \frac{\sqrt{3}}{2} \frac{dI_1^p}{d\sqrt{J_2^p}} \quad (15)$$

$$dI_1^p = d\varepsilon_{i1}^p + d\varepsilon_{i2}^p + d\varepsilon_c^p \quad (16)$$

$$d\sqrt{J_2^p} = \sqrt{\frac{1}{6}[(d\varepsilon_{i1}^p - d\varepsilon_{i2}^p)^2 + (d\varepsilon_{i2}^p - d\varepsilon_c^p)^2 + (d\varepsilon_{i1}^p - d\varepsilon_c^p)^2]} \quad (17)$$

where ε_{i1}^p , ε_{i2}^p , ε_c^p referred to lateral plastic strain in two directions and axial plastic strain, respectively.

The dilation angle of unidirectional passive confinement is shown in Fig. 4. For concrete under unidirectional passive confinement, the effective confinement is not formed. The dilation angle stays around 45° . Concrete keeps in dilation, and the stress-strain curve shows strain softening.

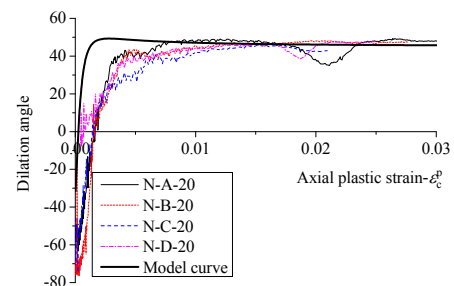


Fig. 4. Dilation angle of concrete under unidirectional passive confinement

The dilation angle of concrete under uniform passive confinement is shown in Fig.5. As shown in Fig.5, dilation angle increases first and then decreases gradually. Small dilation angle means concrete exhibits large compaction, rather than dilation. When the confinement ratio is larger, concrete is under more effective confinement and shows smaller dilation angle.

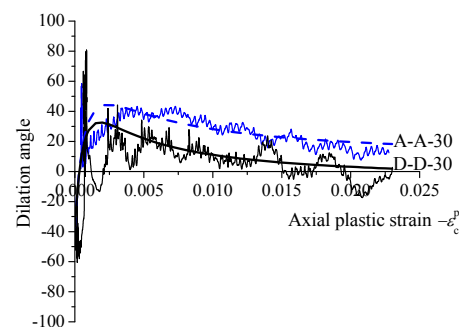


Fig. 5. Dilation angle of concrete under uniform passive confinement

For concrete under non-uniform passive confinement (Fig.6), when the non-uniform confinement ratio is larger, the difference between confinements in two lateral directions is smaller, which leads to more uniform confinement field, more effective confinement for concrete and smaller dilation angle.

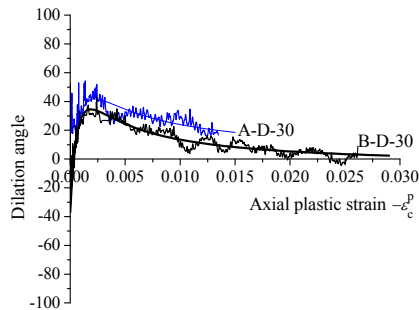


Fig. 6. Dilation angle of concrete under non-uniform passive confinement

Eq. (18) was used to describe the relationship between dilation angle and axial plastic strain. The result of function is shown in Fig.4, Fig.5 and Fig.6, which shows good coincidence.

$$\psi = \frac{\psi_0 + (M_0 + \lambda_1 \psi_0) \left(\frac{\epsilon_c^p}{0.01} \right) + \lambda_2 \psi_u \left(\frac{\epsilon_c^p}{0.01} \right)^2}{1 + \lambda_1 \left(\frac{\epsilon_c^p}{0.01} \right) + \lambda_2 \left(\frac{\epsilon_c^p}{0.01} \right)^2} \quad (18)$$

$$\psi_0 = -37^\circ; \quad M_0 = 1570 \quad (19)$$

$$\lambda_1 = 0.0011 \rho_\eta^2 - 0.0277 \rho_\eta + 11.022 \quad (20)$$

$$\lambda_2 = \frac{253.6 + 29.86 \rho_\eta + 2.24 \rho_\eta^2}{1 + 2.24 \rho_\eta} \quad (21)$$

$$\psi_u = \frac{-59.35 \rho_\eta + 1699}{\rho_\eta + 37.87} \quad (22)$$

$$\rho_\eta = (1.9\eta - 0.9\eta^2) 25.77 f_{c0}^{-0.55} \rho_{max}^{0.72} \quad (23)$$

5 Simulation of FRP-confined square columns

The proposed concrete damage plasticity model, Eq. (10)-Eq. (12) for yield stress and Eq. (18)-Eq. (23) for dilation angle, was adopted to simulate concrete columns under non-uniform passive confinement in finite element software ABAQUS. The experimental results of Wang and Wu [9] were adopted. In their study, the specimens were 150mm×150mm concrete square columns with different corner radius r ($r=0,15\text{mm},30\text{mm},45\text{mm},60\text{mm}$ and 75mm). The concrete strength was 31.7MPa, the elastic modulus, tensile strength and thickness of FRP were 219GPa, 4364MPa and 0.165mm, respectively.

In the finite element model, concrete was simulated with solid element C3D8R, FRP was simulated with membrane element M3D4R. They were tied together to ensure collaborative work. The lateral strain-axial stress curve and axial strain-axial stress curves are shown in Fig.7. The good agreement proves the feasibility of the proposed model.

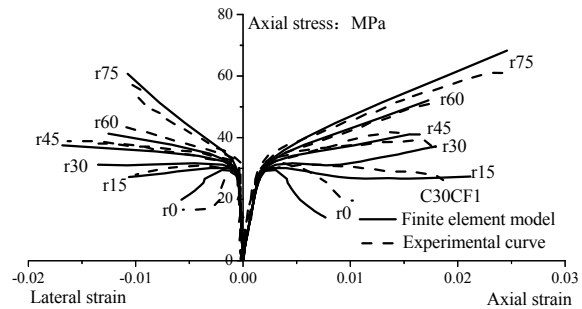


Fig. 7. Overall performance of the proposed model

6 Conclusions

A concrete damage plasticity model for passively confined concrete is presented in this paper. Based on the experimental results, the shear strength ratio, damage parameter, and yield stress in the yield criterion and dilation angle in flow rule were analysed and determined. The main findings of the paper are as follows.

- 1) The shear strength ratio of 0.725 can well describe the deviatoric stress distribution of non-uniformly passively confined concrete.
- 2) The yield stress of concrete under unidirectional passive confinement shows strain softening. With the increase of lateral confinement stiffness and non-uniform confinement stiffness ratio, yield stress curve gradually develops from strain softening to strain hardening.
- 3) For concrete under unidirectional passive confinement, the dilation angle stays stable around 45° . For concrete under uniform and non-uniform passive confinement, the dilation angle decreases with higher confinement stiffness and non-uniform confinement stiffness ratio.
- 4) The proposed model can well simulate the mechanical behaviour of FRP-confined square concrete columns with different corner radius.

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