# Bandwidth optimization for lane-based traffic signal settings with improved stopping criteria in non-linear TRANSYT delay

*C K* Wong*, *Yi* Liu, and *Yixin* Shen

Department of Architecture and Civil Engineering, City University of Hong Kong, Kowloon Tong, HKSAR, Hong Kong

**Abstract.** While assigning given OD demand flows onto network links, travel times along different routes could be varied depending on respective traffic volumes. To achieve equilibrium, all used routes should have the minimum and identical travel times. Such route travel times are composed by link travel times and end of link delay. Upstream and downstream traffic signals are coordinated through bandwidth maximization. Path flows and path travel times are modeled to be responsive to traffic signal settings to enable attractive path choices. Approximated linear function will be established to linearize the end of link delay in TRANSYT model. The problem is formulated as a Binary-Mixed-Integer-Linear-Program and could be solved by standard branch-and-bound method.

## 1 Introduction

A conventional approach for designing linked traffic signal settings is the MAXBAND method in which bandwidths of green signals are maximized to ensure smooth arterial progression across junctions [1]. In the lane-based framework, upstream and downstream junctions are connected through proper lane markings and traffic signal settings. Green duration times for turns at junctions are optimized by bandwidth maximization along selected origins, destinations, paths across upstream and downstream junctions. Along a selected main stream of flow, a maximum green bandwidth is designed to promote efficient traffic flowing. To integrate this design concept to the latest lane-based method to design signalized networks, we would like to maximize green bandwidths between upstream and downstream junctions so as to provide proper offsets among consecutive upstream and downstream junctions to have smooth traffic flowing. For this, green bandwidths are defined as objective function and to be maximized.

## 2 Constraints for signalized junction controls and network link connections

### 2.1 Linear constraint sets for controlling individual junctions

* Corresponding author: wongck@cityu.edu.hk

13 sets of linear mathematical constraints were well developed to put forward proper traffic signal designs. Constraint sets include (1) Flow conservation for assigned lane flows and demand turning flows, (2) Minimum number of lane markings, (3) Maximum number of lane markings without exceeding exit lane numbers, (4) Compatibility of lane markings and lane flows, (5) Non-conflicting lane markings across adjacent lanes, (6) Lengths of cycle time, (7) Synchronizing traffic signal settings on approach lanes for turns, (8) Feasible ranges of green starts, (9) Maximum green duration times, (10) Displaying sequence of signal settings, (11) Clearance times, (12) Flow factors across adjacent approach lanes with identical lane markings, (13) Maximum acceptable degree of saturation for approach lanes. Detailed mathematical constraint sets are found in [2, 3].

### 2.2 Linear constraint sets for generating flow paths connecting OD in a network

To put forward proper connections between lane marking arrows at upstream junctions and those at downstream junctions so as to allow feasible paths for OD flows from different origin and destination nodes, another 8 sets of linear constraints are required. They include (I) Conservation of OD flows with sum of path flows, (II) Existence of paths connecting OD pairs, (III) Generating turning flows at junctions from path flow patterns, (IV) Existence of OD demand flows, (V) Minimum path flow for connecting OD flows, (VI) Existence of lane marking arrows at junctions along paths, (VII) Restricting redundant lane markings at destination nodes for non-existence OD, and (VIII) Eliminating unnecessary lane markings at junctions for zero path flows.

## 3 Developing proper green bands for signal timing coordination

In the present formulation, we proposed a binary-mix-integer-linear-programming approach for designing a signalized network design problem. To optimize green signal timings to generate efficient green bands. A transformation process is applied to align green durations from upstream junctions along a horizontal axis by adding travel times to downstream junctions. After transformation, linear constraints may be formulated to regulate signals at consecutive pairs of junctions.

### 3.1 Green duration times for pairs of adjacent junctions

Four working conditions (I-IV) are required. Details of the linear constraint sets are given as follows.

Condition I.

$$M\Lambda_{1,n,i,k,m,i',k'} \geq \Theta_{m,i',k'} + x_{n,m} - \Theta_{n,i,k} - \tau_{m,i',k'} \geq M(\Lambda_{1,n,i,k,m,i',k'} - 1),$$
$$\forall(m,i',k') = D(n,i,k); n = 1,\ldots,N; i = 1,\ldots,I_n; k = 1,\ldots,L_{n,i} \quad (1)$$

where $\Lambda_{1,n,i,k,m,i',k'}$ is binary variable (=1 if transformed upstream green starts earlier, or = 0 if not).

For Conditions II-IV, we have similar comparisons in Eq. (2)-Eq. (4).

$$M\Lambda_{2,n,i,k,m,i',k'} \geq (\Theta_{m,i',k'} + \Phi_{m,i',k'}) + x_{n,m} - (\Theta_{n,i,k} + \Phi_{n,i,k}) - \tau_{m,i',k'} \geq M(\Lambda_{2,n,i,k,m,i',k'} - 1),$$
$$\forall(m,i',k') = D(n,i,k); n = 1,\ldots,N; i = 1,\ldots,I_n; k = 1,\ldots,L_{n,i} \quad (2)$$

$$M\Lambda_{3,n,i,k,m,i',k'} \geq \Theta_{n,i,k} + \Phi_{n,i,k} + \tau_{m,i',k'} - \Theta_{m,i',k'} - x_{n,m} \geq M(\Lambda_{3,n,i,k,m,i',k'} - 1),$$
$$\forall(m,i',k') = D(n,i,k); n = 1,\ldots,N; i = 1,\ldots,I_n; k = 1,\ldots,L_{n,i} \quad (3)$$

$$M\Lambda_{4,n,i,k,m,i',k'} \geq \Theta_{m,i',k'} + \Phi_{m,i',k'} + x_{n,m} - \Theta_{n,i,k} - \tau_{m,i',k'} \geq M(\Lambda_{4,n,i,k,m,i',k'} - 1),$$
$$\forall(m,i',k') = D(n,i,k); n = 1,\ldots,N; i = 1,\ldots,I_n; k = 1,\ldots,L_{n,i} \quad (4)$$

### 3.2 Bandwidth evaluations for 6 different signal patterns

Traffic signal settings are free to be designed in the optimization process. With 4 different conditions identified in Section 3.2, different combinations in the optimized signal settings exist. To evaluate their bandwidths, we evaluate 6 general cases by considering different condition combinations.

Case 1: Satisfying all Conditions (I-IV)

$$M(4 - \Lambda_{1,n,i,k,m,i',k'} - \Lambda_{2,n,i,k,m,i',k'} - \Lambda_{3,n,i,k,m,i',k'} - \Lambda_{4,n,i,k,m,i',k'}) \geq W_{n,m} - (\Theta_{n,i,k} + \Phi_{n,i,k} + \tau_{m,i',k'} - \Theta_{m,i',k'} - x_{n,m}) \geq -M(4 - \Lambda_{1,n,i,k,m,i',k'} - \Lambda_{2,n,i,k,m,i',k'} - \Lambda_{3,n,i,k,m,i',k'} - \Lambda_{4,n,i,k,m,i',k'}),$$

Case 2: Satisfying Conditions II, III, and IV

$$M(3 + \Lambda_{1,n,i,k,m,i',k'} - \Lambda_{2,n,i,k,m,i',k'} - \Lambda_{3,n,i,k,m,i',k'} - \Lambda_{4,n,i,k,m,i',k'}) \geq W_{n,m} - \Phi_{n,i,k} \geq -M(3 + \Lambda_{1,n,i,k,m,i',k'} - \Lambda_{2,n,i,k,m,i',k'} - \Lambda_{3,n,i,k,m,i',k'} - \Lambda_{4,n,i,k,m,i',k'})$$

Case 3: Satisfying Conditions I, III, and IV

$$M(3 - \Lambda_{1,n,i,k,m,i',k'} + \Lambda_{2,n,i,k,m,i',k'} - \Lambda_{3,n,i,k,m,i',k'} - \Lambda_{4,n,i,k,m,i',k'}) \geq W_{n,m} - \Phi_{m,i',k'} \geq -M(3 - \Lambda_{1,n,i,k,m,i',k'} + \Lambda_{2,n,i,k,m,i',k'} - \Lambda_{3,n,i,k,m,i',k'} - \Lambda_{4,n,i,k,m,i',k'})$$

Case 4: Satisfying Conditions I, II, and IV

$$M(3 - \Lambda_{1,n,i,k,m,i',k'} - \Lambda_{2,n,i,k,m,i',k'} + \Lambda_{3,n,i,k,m,i',k'} - \Lambda_{4,n,i,k,m,i',k'}) \geq W_{n,m} \geq -M(3 - \Lambda_{1,n,i,k,m,i',k'} - \Lambda_{2,n,i,k,m,i',k'} + \Lambda_{3,n,i,k,m,i',k'} - \Lambda_{4,n,i,k,m,i',k'})$$

Case 5: Satisfying Conditions III and IV

$$M(2 + \Lambda_{1,n,i,k,m,i',k'} + \Lambda_{2,n,i,k,m,i',k'} - \Lambda_{3,n,i,k,m,i',k'} - \Lambda_{4,n,i,k,m,i',k'}) \geq W_{n,m} - (\Theta_{m,i',k'} + \Phi_{m,i',k'} + x_{n,m} - \Theta_{n,i,k} - \tau_{m,i',k'}) \geq -M(2 + \Lambda_{1,n,i,k,m,i',k'} + \Lambda_{2,n,i,k,m,i',k'} - \Lambda_{3,n,i,k,m,i',k'} - \Lambda_{4,n,i,k,m,i',k'}),$$

Case 6: Satisfying Condition III only

$$M(1 + \Lambda_{1,n,i,k,m,i',k'} + \Lambda_{2,n,i,k,m,i',k'} - \Lambda_{3,n,i,k,m,i',k'} + \Lambda_{4,n,i,k,m,i',k'}) \geq W_{n,m} \geq -M(1 + \Lambda_{1,n,i,k,m,i',k'} + \Lambda_{2,n,i,k,m,i',k'} - \Lambda_{3,n,i,k,m,i',k'} + \Lambda_{4,n,i,k,m,i',k'})$$
$$\forall(m,i',k') = D(n,i,k); n = 1,\ldots,N; i = 1,\ldots,I_n; k = 1,\ldots,L_{n,i} \quad (5\text{-}10)$$

### 3.3 Maximum bandwidth

It is expected that actual (green) bandwidth should always be shorter than or equal to the green duration times found in all respective junctions for the traffic along selected major routes and OD pairs. Eq. (11) and Eq. (12) are given to restrict the maximum (green) bandwidth in the formulation.

$$0 \leq W_{n,m} \leq \Phi_{n,i,k},$$
$$\forall(m,i',k') = D(n,i,k); n = 1,\ldots,N; i = 1,\ldots,I_n; k = 1,\ldots,L_{n,i}, \quad (11)$$

$$0 \leq W_{n,m} \leq \Phi_{m,i',k'},$$
$$\forall(m,i',k') = D(n,i,k); n = 1,\ldots,N; i = 1,\ldots,I_n; k = 1,\ldots,L_{n,i}, \quad (12)$$

### 3.4 Bandwidth requirements for co-ordination

To put forward effective coordination, maximizing green bandwidth is set as objective function. Before bandwidth can be optimized, we must ensure that bandwidth exists by Eq. (13) and Eq. (14).

$$1-\Lambda_{1,n,i,k,m,i',k'} \geq \Lambda_{3,n,i,k,m,i',k'} - 1 \geq \Lambda_{1,n,i,k,m,i',k'} - 1\,,$$
$$\forall(m,i',k') = D(n,i,k); n=1,\ldots,N; i=1,\ldots,I_n; k=1,\ldots,L_{n,i} \tag{13}$$

$$\Lambda_{1,n,i,k,m,i',k'} \geq \Lambda_{4,n.i,k,m,i',k'} - 1 \geq -\Lambda_{1,n,i,k,m,i',k'}\,,$$
$$\forall m = D(n,i,k); n=1,\ldots,N; i=1,\ldots,I_n; k=1,\ldots,L_{n,i} \tag{14}$$

Constraint set in Eq. (15) is required to fix the common bandwidth from all consecutive junctions.

$$\overline{W}_{a,b,h_{a,b}} \leq W_{n,m}\,,\ \forall(n,m)\in F(a,b,h_{a,b}) \tag{15}$$

## 4 Delay time

According to the TRANSYT traffic model [4-7], link travel times include two components: cruise times on network links and delays at the ends of links. Delay at the end of link consists of uniform delay due to alternations of green and red traffic signals and random and oversaturation delay [8] that is obviously non-linear. Incorporating delay formula directly into the proposed formulation may turn the entire design problem into a binary-mixed-integer-nonlinear-program which is very difficult to be solved [9-10]. Linear approximation technique is applied to predict delays at the ends of links [11]. For slowly varying non-linear function, it is possible to decouple the original non-linear terms into individual linear terms for approximations. In the present formulation, lane (turning) flow, ratio of lane flow to saturation flow, effective green duration time, and cycle length are extracted to be the linearized function variables for predicting the delays at ends of lanes. Eq. (16) is used in the present formulation.

$$\varphi_{n,i,k} = \eta_c\zeta + \eta_e(\Phi_{n,i,k} + e\zeta) + \eta_y y_{n,i,k} + \eta_q \sum_j q_{n,i,j,k}$$
$$\forall n=1,\ldots,N; i,j=1,\ldots,I_n; k=1,\ldots,L_{n,i} \tag{16}$$

where $\varphi_{n,i,k}$ is delay time along lane *k* on arm *i* at junction *n*. $\eta_c$, $\eta_e$, $\eta_y$, and $\eta_q$ are coefficients.

## 5 Identical path travel times for used paths

In a traffic network, users tend to travel the shortest path between OD pair. In the formulation, paths exist to go through series of junctions on different lanes. To assign path flows to different paths, path travel time and associated delays at junctions are considered. In Eq. (17), path travel time equals to the sum of all involved lane travel times $\tau_{n,i,k}$ and all delays at ends of lanes $\varphi_{n,i,k}$.

$$\chi_{a,b,h_{a,b}} = \sum_{(n,i,k)\in \mathrm{K}(a,b,h_{a,b})} \tau_{n,i,k} + \sum_{(n,i,k)\in \mathrm{K}(a,b,h_{a,b})} \varphi_{n,i,k} \tag{17}$$
$$\forall(n,i,k)\in \mathrm{K}(a,b,h_{a,b}); n=1,\ldots,N; i=1,\ldots,I_{n,i}; k=1,\ldots,L_{n,i}; a=1,\ldots A; b=1,\ldots,B; h_{a,b}=1,\ldots; H_{a,b}$$

where $\chi_{a,b,h_{a,b}}$ is path travel time from origin *a* to destination *b* using path $h_{a,b}$. As multiple paths may exist, Eq. (18) is to equalize $\chi_{a,b,h_{a,b}} = \chi_{a,b,h'_{a,b}}$ when those paths exist.

$$M(\alpha_{a,b,h'_{a,b}} - 1) \leq \chi_{a,b,h_{a,b}} - \chi_{a,b,h'_{a,b}} \leq M(1 - \alpha_{a,b,h_{a,b}}),$$
$$\forall a = 1, \ldots A; \forall b = 1, \ldots, B; \forall \left(h_{a,b} \neq h'_{a,b}\right) \in H_{a,b} \quad (18)$$

## 6 Objective function for bandwidth optimization

Maximizing green bandwidth is a well-known approach in coordinating traffic signal settings in a signalized network system. Offsets for all involved signal settings along selected major paths could be optimized. In the present study, lane markings are key variables for network configuration designs. To take full advantages of existing lane-based methods, a linear platform in the optimization framework is required. Maximizing green bandwidth is set as the objective for optimization. The problem is formulated as a Binary-Mixed-Integer-Linear-Programming (BMILP) problem. The bandwidth maximization problem can therefore be formulated as equation (19), subject to linear constraints in (1-18) and constraint sets in Section 2 that is solved by standard routine.

$$\text{Maximize} \sum_{\text{selected } a} \sum_{\text{selected } b} \sum_{\text{selected } h_{a,b}} \left(z_{a,b,h_{a,b}} W_{a,b,h_{a,b}}\right) \quad (19)$$

## 7 Case study

A 4-junction example setting is used for demonstrations, *n*=4. 2 approach lanes, $L_{n,i} = 2$, and 2 exit lanes, $E_{n,j} = 2$, are modeled for all arms. Saturation flows are $\bar{s}_{n,i,k=1} = 1{,}965$ and $\bar{s}_{n,i,k=2} = 2{,}080$ pcu/h (for straight-ahead movements) for respectively nearside and non-nearside lanes. 4 corner nodes are regarded as origins and destinations. Input demand flows are given in table 1.

**Table 1.** Input demand flows.

| $T_{a,b}$ (pcu/h) | | Destination, *b* | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| Origin, *a* | 1 | - | 200 | 500 | 400 |
| | 2 | 300 | - | 200 | 600 |
| | 3 | 500 | 300 | - | 300 |
| | 4 | 400 | 400 | 300 | - |

To optimize two (in opposite direction) green bandwidths in figures 1 and 2, traffic flows from origin *a*=2 to destination *b*=4 through junctions *n*=2, *n*=3, and *n*=4 as one major path and from origin *a*=4 to destination *b*=2 through the junctions *n*=4, *n*=3, and *n*=2 as another are modeled with equal weightings in Eq. (19). Lane travel times are depending on link performance function. Distances are the same with lane travel times of 24 seconds. A linearized delay function is used to maintain a linear platform. Coefficients are fixed by least square optimization taking previous TRANSYT model outputs as training datasets. Using a genetic algorithm with improved stopping criteria, linearized and approximated delay function for computing the delays at ends of lanes in the present optimization framework, is obtained. Approximated delay is verified to match the sheared delay formula in table 2.

**Table 2.** Comparisons of the sheared delay and the linear approximated delay.

| Junction, $n$ | Arm, $i$ | Lane, $k$ | Linear approximated delay (s) by Eq. (16), $\varphi_{n,i,k}/\zeta$ | Sheared delay (s)*, $\varphi^{s}{}_{n,i,k}/\zeta$ | Error $\frac{\left\vert \varphi^{s}{}_{n,i,k} - \varphi_{n,i,k} \right\vert}{\varphi^{s}{}_{n,i,k}} \times 100\%$ |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 27.0910 | 26.6475 | 1.7% |
| | | 2 | 26.4027 | 26.4988 | 0.4% |
| | 2 | 1 | 38.6795 | 37.7261 | 2.5% |
| | | 2 | 28.5278 | 28.8905 | 1.3% |
| | 3 | 1 | 52.1418 | 52.8134 | 1.3% |
| | | 2 | 52.0730 | 52.8125 | 1.4% |
| 2 | 1 | 1 | 28.6109 | 28.6787 | 0.2% |
| | | 2 | 43.4717 | 43.9575 | 1.1% |
| | 3 | 1 | 38.8587 | 39.1040 | 0.6% |
| | | 2 | 34.9660 | 35.7371 | 2.2% |
| | 4 | 1 | 49.7780 | 50.2797 | 1.0% |
| | | 2 | 49.6953 | 50.2789 | 1.1% |
| 3 | 1 | 1 | 40.8652 | 41.4103 | 1.3% |
| | | 2 | 40.9748 | 41.4106 | 1.1% |
| | 3 | 1 | 39.2237 | 39.5928 | 0.9% |
| | | 2 | 38.8406 | 39.5919 | 1.9% |
| | 4 | 1 | 41.1561 | 41.4450 | 0.7% |
| | | 2 | 40.9980 | 41.4445 | 1.1% |
| 4 | 1 | 1 | 50.3208 | 50.8413 | 1.0% |
| | | 2 | 28.8532 | 28.7485 | 0.4% |
| | 2 | 1 | 39.7651 | 39.5387 | 0.6% |
| | | 2 | 39.6294 | 39.5384 | 0.2% |
| | 3 | 1 | 44.5272 | 45.1253 | 1.3% |
| | | 2 | 22.1402 | 22.1774 | 0.2% |

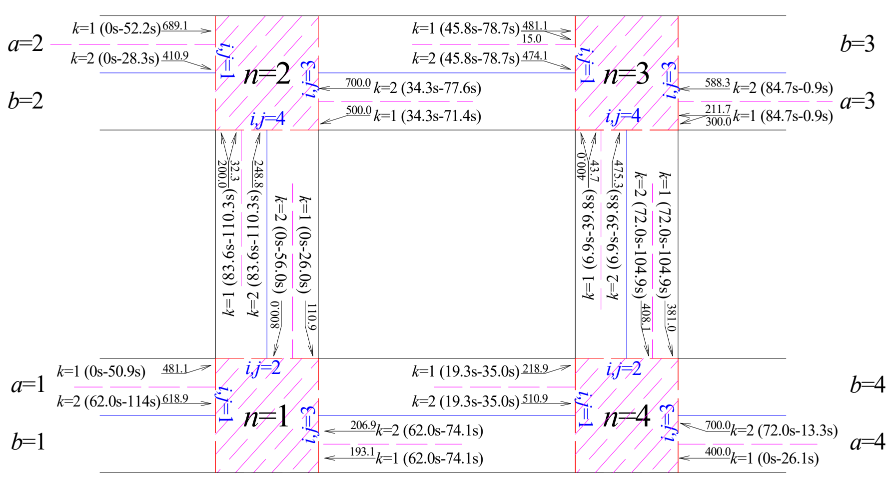

**Fig. 1.** Optimized lane marking patterns for network connections.

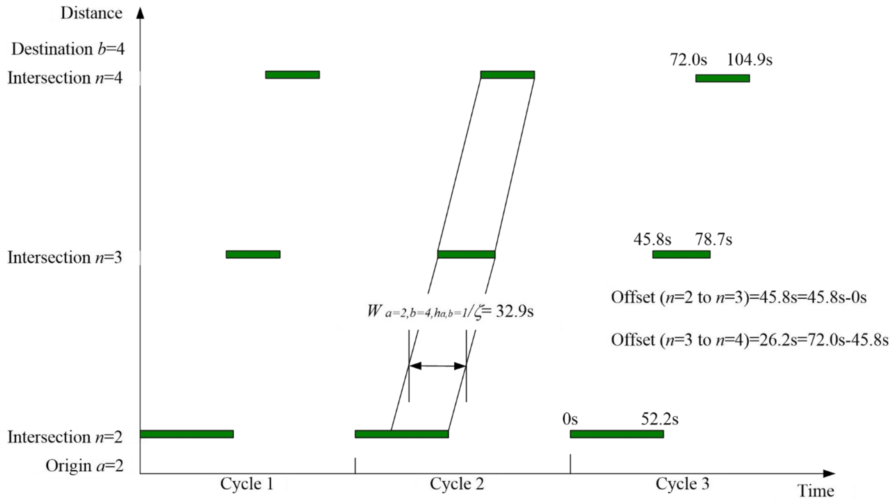


**Fig. 2.** Optimized bandwidth for the case study.

## 8 Conclusions

In the proposed study, the lane-based design concept is extended to incorporate with bandwidth maximization to optimize traffic signal settings and lane markings. Approximated linearized delay function is employed to estimate delays at ends of links to maintain a linear optimization framework. Genetic algorithms with improved stopping criteria are used to optimize the least square errors between TRANSYT and approximated delays.

## Acknowledgments

The work described in this paper was supported by the GRF funding scheme from the Research Grants Council of the Hong Kong Special Administrative Region (Project no. CityU11213514).

## References

1. Little, J.D.C., Kelman, M.D. and Gartner, N.H. (1981) MAXBAND: a program for setting signals on arterials and triangular networks, *Transportation Research Record*, **795**, 40-46.
2. C.K. Wong and S.C. Wong, (2003) Lane-based optimization of signal timings for isolated junctions, *Transportation Research Part B-Methodological*, 37(1), 63-84.
3. C.K. Wong and B.G. Heydecker, (2011) Optimal allocation of turns to lanes at an isolated signal controlled junction, *Transportation Research Part B-Methodological*, 45(4), 667-681.
4. Robertson, D.I. (1969) TRANSYT: a traffic network study tool, *Transport and Road Research Laboratory Report*, LR253, Crowthorne.
5. Vincent, R.A., Mitchell, A.I. and Robertson, D.I. (1980) User guide to TRANSYT version 8, *Transport and Road Research Laboratory Report LR888. TRL, Crowthorne*.

6. Wong, S.C., Yang, C., and Lo, H.K. (2001) A path-based traffic assignment algorithm using the TRANSYT traffic model, *Transportation Research Part B-Methodological*, **35**(2), 163-181.
7. Wong, C.K. and Wong, S.C. (2002) Lane-based optimization of traffic equilibrium settings for area traffic control, *Journal of Advanced Transportation*, **36**(3), 349-386.
8. Han, B. (1996) A new comprehensive sheared delay formula for traffic signal optimization, *Transportation Research Part A- Policy*, **30**(2), 155-171.
9. Wong, C.K. and Wong, S.C. (2003) A lane-based optimization method for minimizing delay at isolated signal-controlled junctions. *Journal of Mathematical Modelling and Algorithms*, **2**(4), 379-406.
10. Wong, C.K. and Lee, Y.Y. (2012) Convergence study of minimizing the nonconvex total delay using the lane-based optimization method for signal-controlled junctions, *Discrete Dynamics in Nature and Society*, vol. 2012, Article Number: 858731.
11. Cai, C., Wong, C.K., and Heydecker, B.G. (2009) Adaptive traffic signal control using approximate dynamic programming, *Transportation Research Part C-Emerging Technologies*, **17**(5), 456-474.