

# Lane-based optimization for signalized network configuration designs

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**Abstract.** Lane-based traffic signal design has been developed for designing signal-controlled intersections. Conventional designs take on fixed configurations as exogenous inputs to design traffic signal settings. The proposed study will be an extension to merge geometrical junction arrangement and signal controls together for network configuration designs. Design methodology would be directly extended from existing lane-based design method. New Path flows variables and new flow conservation constraints are required to ensure the users' input OD flows could be assigned onto network paths through different signal-controlled intersections. This problem is new that involves binary variables and related linear constraints which is formulated as a BMILP. Standard technique could solve the optimum solution. A four-intersection network with two approach lane settings is optimized for demonstration purposes.

## 1 Introduction

In design signalized networks, users' routes are compiled by path flows. Flow patterns in a network could be affected by signal settings and the connections of network links. Connections of network links should be based on lane marking arrows. To design networks, different researchers developed different mathematical formulations and solution algorithms with different objective functions [1-4]. Literature review in the network design is found [5]. Coordination of traffic signal settings and path-based traffic flow assignment algorithm was combined [6]. Network configurations including lane marking arrows are generally fixed by users excluding from the optimization process. With given lane marking arrows, traffic lanes could be grouped to form network links. Signal timings can be optimized using traditional stage- or phase-(group-) based methods [7]. Stage-based method was applied [8-11]. Group-based method is used [6,12,13]. Users' given network configuration may be suboptimal to serve the travel demands. Lane markings are defined as control variables in the proposed lane-based design framework to optimize the entire network link connections.

## 2 Constraints for link connections in signal-controlled networks

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## 2.1 Distributing OD flows onto available network paths

$$\mu Q_{o,d} = \sum_{p_{o,d}=1}^{\bar{P}_{o,d}} f_{o,d,p_{o,d}}, \quad \forall o \in O, d \in D \quad (1)$$

where  $o$  and  $d$  are respectively the origin node and destination node in a network,  $O$  and  $D$  are total numbers of origins and destinations in a study network,  $\mu$  is a common flow multiplier to rate the users' OD flows,  $p_{o,d}$  is the path number for origin  $o$  and destination  $d$ ,  $\bar{P}_{a,b}$  is the total paths to connect the OD,  $Q_{o,d}$  is users' input flows from origin  $a$  to destination  $b$ ,  $f_{o,d,p_{o,d}}$  is the path flow along the path  $p_{o,d}$  for OD pairs  $o$  and  $d$ .

## 2.2 Existence of OD paths

$$5 \cdot N f_{o,d,p_{o,d}} \geq N \alpha_{o,d,p_{o,d}} \geq f_{o,d,p_{o,d}}, \quad \forall o \in O, d \in D, p_{o,d} \in \left| \bar{P}_{o,d} \right| \quad (2)$$

where  $\alpha_{o,d,p_{o,d}}$  is a binary-type integer variable to model the  $p_{o,d}$  path for OD  $o$  and  $d$ .  $N$  is an arbitrary large integer figure used in the formulation.

## 2.3 Flows turning at intersection level

Users' given OD demand flows are input data. Inside a network, numerous users' traveling paths could be allowed. For intersection  $n$ , flows turning at intersections may be from different OD locations. Thus, turning flows equal the total of the path flows.

$$q_{n,i,j} = \sum_{o=1}^O \sum_{d=1}^D \sum_{p_{o,d}=1}^{\bar{P}_{o,d}} f_{o,d,p_{o,d}}, \quad \forall (o,d,p_{o,d}) \in F(n,i,j) \quad (3)$$

where  $F(n,i,j)$  is a mathematical function to search the path  $p_{o,d}$  from origin to destination ( $o-d$ ) when  $(n,i,j)$  are inputs for the function.

## 2.4 Availability of demands in OD pairs

$$N \beta_{o,d} \geq Q_{o,d} \geq \beta_{o,d}, \quad \forall o \in O; d \in D \quad (4)$$

$\beta_{o,d}$  is a variable to model the existence of OD demand flows. If  $Q_{o,d}$  is greater than zero, then the binary variable  $\beta_{o,d}$  should be forced to be "1".  $N$  is an arbitrary large number.

## 2.5 Path flows to serve OD flows

When users' given demand flow  $Q_{o,d}$  is not zero, then  $\beta_{o,d}$  should equal to "1". If  $\beta_{o,d} = 1$ , a path should be available to connect the OD demand. Eq. (5) is developed to ensure this minimum requirement and also put forward the multiple path situation ( $\alpha_{o,d,p_{o,d}} > 1$ ) when  $\beta_{o,d} = 1$ .

$$\sum_{p_{o,d}=1}^{\bar{P}_{o,d}} \alpha_{o,d,p_{o,d}} \geq \beta_{o,d}, \quad \forall o \in O; d \in D \tag{5}$$

If there is zero demand flow,  $\beta_{o,d}$  is given to be zero. And it then ensures  $\alpha_{o,d,p_{o,d}}$  to be zero. No path is generated in the network.

$$\beta_{o,d} \geq \alpha_{o,d,p_{o,d}}, \quad \forall o \in O; \forall d \in D; \forall p_{o,d} = 1, \dots, \bar{P}_{o,d} \tag{6}$$

### 2.6 Lane markings to generate a path for OD

$\delta_{n,i,j,k}$  in Eq. (7) is a 0 or 1 variable to model a lane marking arrow that allows a turn from arm  $i$  to arm  $j$  on lane  $k$  at intersection  $n$  and  $\alpha_{o,d,p_{o,d}}$  is an auxiliary variable to represent the path  $p_{o,d}$  to serve OD  $o$  and  $d$ . If path is existed,  $\alpha_{o,d,p_{o,d}} = 1$ , the lane marking arrows for movement turns at the intersection must be painted on approach lane  $k$  so that path  $p_{o,d}$  is available for OD flow pair  $o$  and  $d$ .

$$\sum_k \delta_{n,i,j,k} \geq \alpha_{o,d,p_{o,d}}, \quad \forall (n,i,j,k) \in F'(o,d,p_{o,d}) \tag{7}$$

### 2.7 No lane marking for zero demand flow entering destination node

If OD flow inputs are zero, destination nodes may not be served and respective lane markings could be removed from the optimization process. Thus, respective movement turns entering these may be unnecessary. The lane marking arrows should not exist by Eq. (8).

$$\sum_{o=1}^O \sum_{p_{o,d} \in P'_{o,d}} \alpha_{o,d,p_{o,d}} \geq \sum_{k=1}^{L_{n,i}} \delta_{n,i,j,k}, \quad \forall d \in D \tag{8}$$

### 2.8 Removing redundant lane marking arrows

In Section 2.7, the formulation may restrict lane marking arrows near destinations if modeled path flows are zero. However, some redundant lane markings in the network may exist where the intersections do not serve and connect destination nodes. Such lane marking arrows should be removed. Eq. (9) and Eq. (10) are required.

$$\Delta_{n,i,j,o,d,p_{o,d}} = \alpha_{o,d,p_{o,d}}, \quad \forall (n,i,j) = F'(o,d,p_{o,d}) \tag{9}$$

$$5 \cdot N \sum_{k=1}^{L_{n,i}} \delta_{n,i,j,k} \geq N \sum_{o=1}^O \sum_{d=1}^D \sum_{p_{o,d}=1}^{\bar{P}_{o,d}} \Delta_{n,i,j,o,d,p_{o,d}} \geq \sum_{k=1}^{L_{n,i}} \delta_{n,i,j,k}, \quad \forall n \in M, \forall i \in I_n, \forall j \in I_n \tag{10}$$

## 3 Constraint sets for governing signal-controlled intersections

In lane-based design framework, we may have some 13 sets of well-established linear constraints to ensure safe traffic signal settings in designs. They include (1) Conservation of assigned lane flows to demand turning flows at intersections, (2) Minimum lane marking on

approach lanes, (3) Maximum lane markings on approach lanes without exceeding downstream exit lane numbers, (4) Consistency among lane markings and assigned lane flows, (5) Compatible lane markings across adjacent lanes, (6) Operating ranges of cycle length, (7) Synchronization of traffic signal settings for approach lanes and turns, (8) Operating ranges of starts of green times, (9) Operating ranges of duration of green times, (10) Displaying sequence of traffic signal timings, (11) Minimum clearance times separating conflicting movements, (12) Identical flow factors across adjacent approach lanes with identical lane marking sets, (13) Maximum acceptable degree of saturation along approach lanes. Mathematical constraints are all available in [14,15].

## 4 Optimization for network link connections

For optimizing the network designs, we apply the capacity maximization that has been optimized for intersections. Given turning flows are directly multiplied by the common flow multiplier  $\mu$  [14,15]. The optimization process maximizes the common flow multiplier so that the largest possible demand flows to enter a signal-controlled system are determined. The system's maximum degree of saturation can be attained. Using the same concept, the OD demand flows (matrix) are inputs for network study. The common flow multiplier is applied in the present study to multiply the OD demand flows until the largest OD flows can enter the system without exceeding the maximum allowable degree of saturation. Then, lane marking arrows on approach traffic lanes at intersection level could be optimized. Network link connections can then be established. Respective lane flows and path flows could be optimized as well. It is expected that all path flows can be compatible with the given OD demands satisfying the flow conservation purposes. The capacity maximization problem is a BMILP: Max  $\mu$ , subject to Eq. (1) - Eq. (10) and the thirteen sets of lane-based constraints. Standard B&B solution technique is able to optimize this standard mathematical programming problem to achieve optimum solutions.

## 5 Case study

A four-intersection network is modeled,  $n=4$  with 2 approach lanes,  $L_{n,i} = 2$ , and two exit lanes,  $E_{n,j} = 2$  throughout the network. Saturation flows for nearside lane and non-nearside lane are  $\bar{s}_{n,i,k=1} = 1,965$  and  $\bar{s}_{n,i,k=2} = 2,080$  pcu/h (for straight-ahead movements). Lane marking arrows to be designed on approach traffic lanes. Four origins and four destinations are modeled at the four corners of the network. Demand flow pattern is assumed to be given by users as given in table 1.

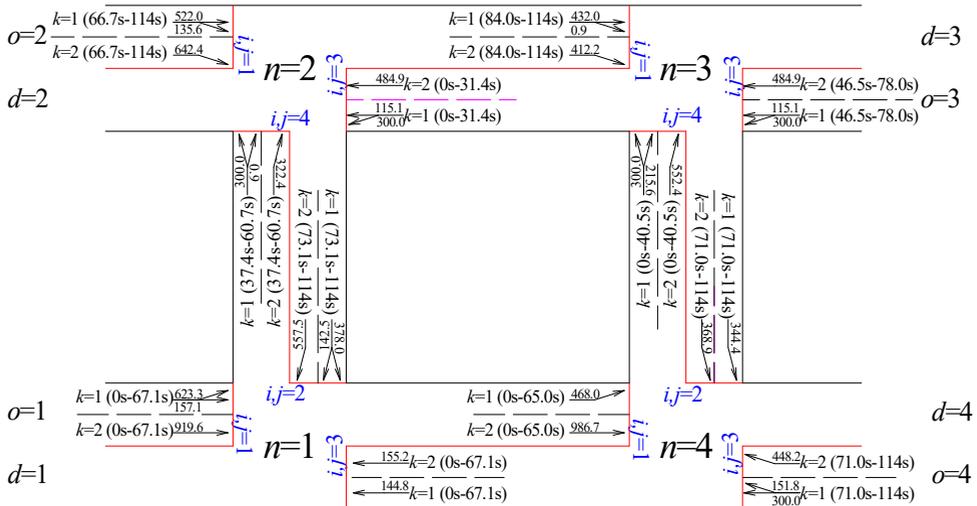
**Table 1.** Demand flow inputs across OD pairs.

$Q_{o,d}$ (pcu/h)		Destination, $d$			
		1	2	3	4
Origin, $o$	1	-	300	600	800
	2	400	-	300	600
	3	300	300	-	300
	4	300	300	300	-

To optimize lane marking arrows at various intersections for configuring the network link connections, specific OD pairs  $(o = 1, d = 3)$ ,  $(o = 1, d = 4)$ , and  $(o = 2, d = 4)$  are modeled with two path choices and other OD pairs are modeled with the shortest connections in the numerical demonstrations. Optimization results will be able to show the

optimized network configurations through using optimized set of lane marking arrows. The large numerical figure  $N$  will be set to equal 10,000. Clearance times are 6.0 and minimum green duration times are 5.0 seconds. Effective green time is +1.0 second longer than the actual green time (i.e.  $e = 1.0$  second). Maximum allowable degree of saturation for traffic lanes is assumed to be 90%. With these inputs, the network can be optimized by maximizing the common flow multiplier. And the optimized common flow multiplier  $\mu = 1.0557$  with 5.57% reserve capacity (for the same sets of demand flow inputs and problem settings, if the signal-controlled network is connected by a set of suboptimal lane markings, the common flow multiplier would be as low as 0.852). Figure 1 provides the optimization results on the network connection details including lane marking arrows, lane flows, and traffic signal timings.

At intersection  $n=2$ , from arm  $i=2$ , a shared left-turn and right-turn lane marking is optimized to enable both left-turn and right-turn movements to enter arms  $j=1$  and 3 on the nearside lane  $k=1$ . For the next non-nearside lane  $k=2$ , a right-turn lane marking arrow to permit right-turn traffic from arm  $i=2$  to arm  $j=3$  is optimized. Respective lane turning flows are given next to the lane marking arrows. Signal times (in seconds) are given inside the brackets. Differences of the two signal timings (ends of green times – starts of green times) are the actual green duration times. Effective green times should be the actual green time plus one second. In the case study, nine shared lane markings are optimized on different approach lanes permitting two turning movements. The lane flow details are optimized fulfilling the OD flow conservation constraints and also satisfying the constraints for intersection (safe) operations. A typical result table for lane flows is tabulated in table 2.



**Fig. 1.** Optimized network configurations.

**Table 2.** Optimized lane flow results at intersection  $n=1$ .

From arm, $i$	Lane, $k$	To arm, $j$			Total flow on lane $k$ (pcu/h)	Sat. flow (pcu/h)	Flow factor, $y_{n,i,k}$	Green start time (s)	Effective green time (s)	Degree of saturation
		1	2	3						
1	1	-	623.3	157.1	780.4	1786.63	0.4369	0.0	68.1	0.7699
	2	-	0.0	919.6	919.6	2105.00	0.4369	0.0	68.1	0.7699
	Total	-	623.3	1076.7	<b>1700.0</b>					
2	1	142.5	-	378.0	520.5	1746.67	0.2980	73.1	41.9	0.8535

	2	557.5	-	0.0	557.5	1871.11	0.2980	73.1	41.9	0.8535
	Total	<u>700.0</u>	-	378.0	1078.0					
3	1	144.8	0.0	-	144.8	1965.00	0.0737	0.0	68.1	0.1299
	2	155.2	0.0	-	155.2	2105.00	0.0737	0.0	68.1	0.1299
	Total	<u>300.0</u>	0.0	-	300.0					

Remarks: Lane marking does not exist when assigned lane flow = 0.0 pcu/h; **1700.0** pcu/h is the total demand flow from origin  $o=1$ ; 700.0 and 300.0 pcu/h are the total 1000.0 pcu/h flows entering destination  $d=1$ .

## 6 Conclusions

In the proposed study, a lane-based formulation is developed to design the network settings. Lane marking arrows are optimized together with lane flows and signal timings to connect network links. Users' OD demand flows are key model inputs. The OD flows are then distributed onto different network links. In the case study network consisting of four intersections, the network performance could be overloaded by 14.83% if suboptimal lane marking arrows without shared lane markings are used. By using the proposed lane-based optimization model, the entire network could be improved to have +5.57% reserve capacity (with reserve capacity instead of being congested). The key contribution of the proposed optimization framework is to optimize lane marking arrows to better connect the network links for practical implementations. The proposed problem is a BMILP problem and a branch-and-bound routine has been used to solve the problem.

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## References

1. M. Du, X. Jiang, and L. Cheng, "Estimating the Capacity of Urban Transportation Networks with an Improved Sensitivity Based Method", *Discrete Dynamics in Nature and Society*, vol. 2012, Article ID 490810, 13 pages, 2012.
2. S.W. Chiou, "An efficient algorithm for optimal design of area traffic control with network flows," *Applied Mathematical Modelling*, 33(6), 2710-2722, 2009.
3. Q. Meng, H. Yang, and M.G.H. Bell, "An equivalent continuously differentiable model and a locally convergent algorithm for the continuous network design problem," *Transportation Research Part B-Methodological*, 31(5), 83-105, 2001.
4. S.C. Wong and H. Yang, H., "Reserve capacity for signal-controlled network," *Transportation Research Part B-Methodological*, 31(5), 397-402, 1997.
5. H. Yang and M.G.H. Bell, "Models and algorithms for road network design: a review and some new developments," *Transport Review*, 18(3), 257-278, 1998.
6. S.C. Wong, C. Yang, and H.K. Lo, "A path-based traffic assignment algorithm using the TRANSYT traffic model," *Transportation Research Part B-Methodological*, 35(2), 163-181.

7. R.E. Allsop, "Evolving application of mathematical optimisation in design and operation of individual signal-controlled road junctions," J.D. Griffiths (ed.) *Mathematics in Transport and Planning and Control*, 1-24, Clarendon Press, Oxford, 1992.
8. R.A. Vincent, A.I. Mitchell and D.I. Robertson, "User guide to TRANSYT version 8," *Transport and Road Research Laboratory Report LR888*. TRL, Crowthorne, 1980.
9. M.R. Crabtree, "TRANSYT/9 Users' manual," *Transport and Road Research Laboratory Report AG8*, TRL, Crowthorne, 1988.
10. N.H. Gartner, J.D.C. Little and H. Gabbay, "Optimisation of traffic signal settings by mixed-integer linear programming. Part I: the network coordination problem," *Transpn Sci.*, 9, 321-343, 1975a.
11. J.D.C. Little, M.D. Kelman and N.H. Gartner, "MAXBAND: a program for setting signals on arterials and triangular networks," *Transpn Res. Rec.*, 795,40-46, 1981.
12. S.C. Wong, "Group-based optimisation of signal timings using the TRANSYT traffic model," *Transportation Research Part B-Methodological*, 30(3), 217-244, 1996.
13. S.C. Wong and C. Yang, "An iterative group-based signal optimization scheme for traffic equilibrium networks," *Journal of Advanced Transportation*, 33(2), 201-217, 1999.
14. C.K. Wong and S.C. Wong, "Lane-based optimization of signal timings for isolated junctions," *Transportation Research Part B-Methodological*, 37(1), 63-84, 2003.
15. C.K. Wong and B.G. Heydecker, "Optimal allocation of turns to lanes at an isolated signal controlled junction," *Transportation Research Part B-Methodological*, 45(4), 667-681, 2011.