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Frequency and power estimation of sinusoids using eigen-approach

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Abstract. Frequency and power estimation of sinusoidal signals, has captured the attention of researchers due to its important applications. This paper involves a frequency and power estimation method of unidentified number of source sinusoidal signals using Eigen technique. The eigenvector corresponding to the correlation matrix's minimum eigenvalue yields the minimum power of the FIR filter's output power. The corresponding roots are determined using the filter polynomial and root elimination technique is used to determine the pseudospectrum. Consequently, by deploying the pseudospectrum and filter polynomial, the real power spectrum is derived.

1 Introduction

Frequency and power evaluation of several sinusoids embedded in wide-band signals has been an important issue in engineering [1][2]. There has been extensive research regarding different estimation approaches ; as estimation methods have many uses in various domains such as biomedical engineering, image processing, radar applications, and communications [1][3]. Nevertheless, performance effectiveness of such approaches depends on the estimator's precision and complexity.

Eigen-approach methods in the literature grabbed the researchers' attention due to estimating with a high resolution; nevertheless, the majority of such techniques require the exact number of signals in order to distinct signal and noise subspace, which is a main drawback. ESPRIT, MUSIC, and Pisarenko harmonic decomposition (Phd) are examples of these techniques. [4][5]. These methods are considered poor estimation techniques for certain applications of which the number of source signals is unknown.

Additionally, the usage of random search optimization processes towards estimating frequencies and power of sinusoidal signals has been offered; the author in [6] uses a population based stochastic method, Particle Swarm Optimization (PSO) to estimate diverse sinusoids' frequencies along with their real power, without previous knowledge of the signals' number, amplitude, nor phase [7]. Furthermore, random search is employed to evaluate the direction-of-arrival and power estimation of source signals [8].

The current paper presents a non-parametric method that estimates the frequency and power of several sinusoids without knowing the number of signals in advance using an eigen-approach. The transfer function of the filter with weight vectors of the eigenvector

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conforming to the lowest eigenvalue of the correlation matrix produces the filter's minimum output power. Moreover, the transfer function's roots are placed on the unit circle [9]. Consequently, the angle of the roots reflects probable signal occurrences on the unit circle. On the basis of this criteria, the locations of the roots and the real signal power are calculated by excluding one root at a time and recalculating the output signal power.

2 Problem formulation

The received signal that arrives at the filter constitutes of sinusoidal signals alongside basic white gaussian noise and can be expressed as:

$$x(k) = s(k) + g(k) \tag{1}$$

where s(k) is the summation of M sinusoidal signals and g(k) denotes the noise signal. The input signal x(k) at sample (k) is :

$$x(k) = \sum_{m=1}^{M} A_m e^{j(2\pi \frac{fm}{f_s} k + \varphi m)} + g(k) \tag{2}$$

A_m,f_m and φ_m represents the sinusoidal signal's amplitude, frequency, and the phase. Sampling frequency is presented by fs and g(k) is additive white noise with mean of zero and power of σ^2.
The correlation matrix can be formulated as:

$$R_{xx} = VR_{ss}V^H + \sigma^2 I_{(N+1)} \tag{3}$$

where $\mathbf{R_{ss}} = \boldsymbol{E}[\boldsymbol{s}\boldsymbol{s}^{\boldsymbol{H}}]$ is the correlation matrix of the input **.** Identity matrix , I, consists of (N+1) * (N+1) elements. Symbol N is the FIR filter order and **V** is the $(\mathrm{N}+1) * \mathrm{M}$ delay matrix. Matrix **V** is expressed as:

$$\mathbf{V} = [v_1\ v_2 \ldots\ v_m \ldots v_M] \tag{4}$$

v_m is a column vector resembling the delay of the normalized angular frequency, $\widehat{\emptyset}_m$, i.e.,

$$v_m = \left[1\ \ e^{-j\emptyset_\mathrm{m}} \ldots e^{-jn\emptyset_\mathrm{m}} \ldots\ e^{-j\mathrm{N}\emptyset_\mathrm{m}}\right]^T \tag{5}$$

The FIR filter's output is made of the weighted linear arrangement of current input and previous values of the input. Using **a** for the coefficients of the filter **a**=$[a_0\ \ a_1 \ldots\ a_n \ldots a_N]$ of order (N+1), the filter's output ,y(k), is

$$y(k) = ax(k) \tag{6}$$

Using K snapshots (assuming ergodic random processes signals) and x(k) containing the sampled data, the output's power can be computed as

$$\begin{aligned} P_y &= \frac{1}{k}\sum_{k=1}^{k} y(k)y(k)^H \\ &= \mathrm{a}R_{XX}a^H \end{aligned} \tag{7}$$

where

$$R_{XX} = \frac{1}{K}\sum\nolimits_{\mathrm{k}=1}^{K} \mathrm{x(k)x(k)^H} \tag{8}$$

Considering the order of the filter, N, is more or equal to the signals' number M, then the output power of the noise signal, P_g, can be stated as

$$\mathrm{P_g} = \mathbf{a_0 R_{xx} a_0^H} \tag{9}$$

The filter coefficient vector, $\mathbf{a_0}$, resembles the roots on the unit circle that overlap with M signals' frequencies. The noise power is evaluated as follows:

$$\mathrm{P_g} = \mathbf{a_0 R_{xx} a_0^H} = \sigma^2 \mathbf{a_0 a_0^H} \tag{10}$$

Using the method discussed in [9][10] the problem can be expressed as follows:

$$\underset{a}{Minimize} \quad aR_{XX}a^H \tag{11}$$

subject to

$$\mathbf{aa^H} = 1.$$

The autocorrelation Matrix $\mathbf{R_{xx}}$ is a Toeplitz matrix [11] and the optimization problem is resolved using a Lagrange multiplier.

To minimalize the fitness function given in (11), the eigenvector $\mathbf{a_0}$ that corresponds to the minimum eigenvalue λ_{min} should be found; therefore the resultant power will be minimized to the noise power level as shown below:

$$P_{\mathrm{g}} = a_0 R_{XX} a_0^H = \lambda_{\min} = \sigma^2 \tag{12}$$

3 Frequency and power estimation

Eigen technique is utilized to minimize the output power function. The filter's transfer function is expressed as follows:

$$\sum_{n=0}^{N} a_n e^{-jn\phi} = \prod_{n=1}^{N} (e^{-j\phi} - e^{-j\phi_n}) \tag{13}$$

where $\emptyset_n$ is the n$^{\text{th}}$ root angular frequency. When N roots coincide with M sources, the fitness function is minimized.

By excluding one of the roots and recalculating the corresponding coefficient vector, the output power at the frequency of the excluded root is estimated. As a result, the estimated power indicates the presence of actual signal or noise.

The method for both frequency and power evaluation is clarified as follows:

1.Calculate eigenvalues along with eigenvectors of Rxx.

2.Pick the eigenvector, $\mathbf{a_0}$, matching to the minimum eigenvalue λ_{min} and output power ($\mathrm{P_g}$).

3.Determine the angular frequencies of the roots , $\emptyset_n$ (n=1,2,…,N), by using equation (14).

4.Calculate the new weight vector, $\mathbf{a_n}$, for n=(1,…,N), by excluding one root at a time from equation (13) and compute the corresponding power as:

$$\begin{aligned} P_{\mathrm{n}}^{'} &= a_n R_{XX} a_n^H \\ &= a_n V R_{ss} V^H a_n{}^H + \sigma^2 \end{aligned} \tag{14}$$

The output power at the frequency of the excluded root can be written as follows:

$$P_{\mathrm{n}}^{'} = \hat{P}_{\mathrm{n}}\, a_n v_m v_m{}^H a_n{}^H + \sigma^2 \tag{15}$$

where $\widehat{\mathrm{P}}_{\mathrm{n}}$ represents the estimated power at the frequency of the n$^{\text{th}}$ root.

5.Find the estimated power as

$$\hat{P}_n = \frac{P_n^{'} - \sigma^2}{a_n v_m v_m{}^H a_n{}^H} \tag{16}$$

Once the angular frequencies are found, the estimated real power spectrum is achieved by plotting $\widehat{\mathrm{P}}_{\mathrm{n}}$ versus $\emptyset_n/2\pi$. Therefore, the exact number of the incoming source signals can be determined from the estimated power spectrum.

4 Results

To determine the validity of the proposed approach, this involved analyzing an eighth order FIR filter. Consequently, we use eight roots to estimate the frequencies of utmost eight input signals. To find the output's minimum received power, Eigen technique is used. Equations (14) and (17) are used to find the roots and the source signals' power, respectively.

Two scenarios of input source signals are simulated with zero dBm noise level (σ^2) and 100 samples to show the effectiveness of the proposed method. The first scenario constitutes three input signals of 10 dB SNR with two closely spaced signals. As for the second scenario, eight input signals with 10 dB and 6 dB SNR levels are used.

Figure 1 shows the estimated frequencies at 0.1,0.125, and 0.6 cycles/sample with 10 dBm power levels. Eight power values at the estimated locations are calculated. Relating the output power values and the estimated noise power level; the real signal frequency is estimated. The estimated normalized frequencies are 0.1001, 0.1234 and 0.5998 cycles/sample. The proposed method estimates the power of the three input signals as shown in Figure 1 with accurate power levels of 9.8629 , 10.1505 , and 9.7772 dBm.
Figure 2 shows the pseudospectrum for the same three signals at the same frequencies of Figure 1, using the MUSIC algorithm with 100 snapshots. Comparing the results shown by Figures 1 and 2, the estimated frequencies using the proposed method are as accurate as the MUSIC algorithm. Nevertheless, MUSIC algorithm is a parametric approach; needs the number of input signals in advance.

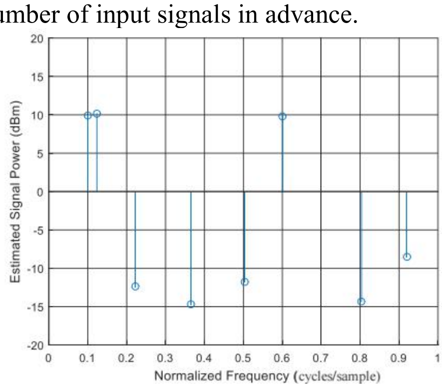

Fig. 1. The estimated real power spectrum, when three signals are of 0.1, 0.125, and 0.6 cycles/sample frequencies with 10 dBm power levels. (No. of roots = 8, $\sigma^2 = 0$ dBm, and 100 samples.).

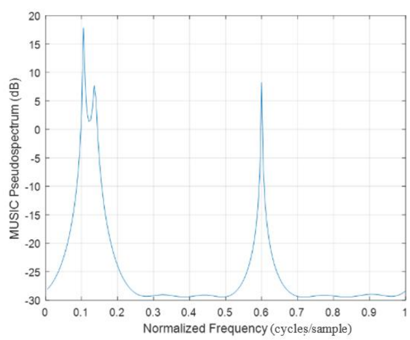

Fig. 2. The power pseudospectrum of the MUSIC algorithm when three signals are of 0.1, 0.125, and 0.6 cycles/sample frequencies with 10 dBm power levels. (No. of roots = 8, $\sigma^2 =$ 0 dBm, and 100 samples.).

For the second scenario, eight source signals having frequencies of 0.1, 0.125, 0.25, 0.35, 0.45, 0.65, 0.80, and 0.90 cycles/sample respectively are simulated. Figure 3 shows that the estimated frequencies and the real power of the eight signals are estimated accurately.

For comparison reasons, Figure 4 shows the pseudospectrum using the MUSIC algorithm when the same eight signals are assumed. All signal frequencies are resolved, yet the closely spaced ones are not as accurate.

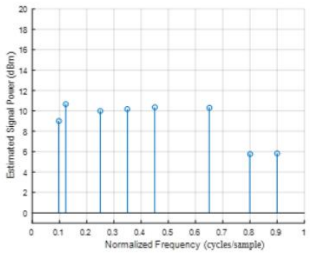

Fig. 3. The estimated real power spectrum when eight signals are of 0.1, 0.125.0.25, 0.35, 0.45, 0.65, 0.80, and 0.90 cycles/sample frequencies and different SNR levels (No. of roots = 8, $\sigma^2 = 0$ dBm, and 100 samples).

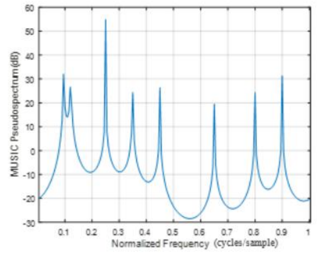

Fig. 4. The power pseudospectrum of the MUSIC algorithm when eight signals are of 0.1, 0.125.0.25, 0.35, 0.45, 0.65, 0.80, and 0.90 cycles/sample frequencies and different SNR levels (No. of roots = 8, $\sigma^2 = 0$ dBm, and 100 samples).

Table 1 shows the estimated frequencies for the proposed approach and for the MUSIC Algorithm. Both are accurate in estimating the frequency; however, the proposed approach estimates the exact power of these signals as well.

Table 1. The estimated frequency ,$\widehat{\emptyset}_m$,and the estimated power level, $\widehat{P}_m$, for eight input signals. (no. of roots = 8, $\sigma^2 = 0$ dBm, and 100 samples).

		Proposed Approach		MUSIC Algorithm
$\emptyset_m\ /2\pi$	P_m(dBm)	$\widehat{\emptyset}_m/2\pi$	$\widehat{P}_m$ (dBm)	$\widehat{\emptyset}_m/2\pi$
0.10	10	0.0970	9.0083	0.095
0.125	10	0.1229	10.6635	0.12
0.25	10	0.2503	9.9943	0.25
0.35	10	0.3499	10.1442	0.35
0.45	10	0.4497	10.3266	0.45
0.65	10	0.6503	10.3257	0.65
0.80	6	0.7997	5.7881	0.8
0.90	6	0.8999	5.8252	0.9

To determine the influence of different signal-to-noise ratios on both frequency and power estimation, an input sinusoidal signal of 0.15 cycles/sample frequency with different SNR levels is considered. Table 2 shows the estimated frequency, $\widehat{\emptyset}_m/2\pi$, and the estimated power,$\widehat{P}_m$, for different SNR and 100 samples. The results show a good level of accuracy.

Table 2. The estimated frequency,$\widehat{\emptyset}_m/2\pi$, and the estimated power level, $\widehat{P}_m$, against different SNR levels, when a 0.15 cycles/sample frequency signal enters the filter (no. of roots = 1 , $\sigma^2 = 0$ dBm, and 100 samples).

SNR(dB)	$\widehat{\emptyset}_m/2\pi$	$\widehat{P}_m$ (dBm)
10	0.1511	9.8227
8	0.1447	7.7288
6	0.1488	6.1155
3	0.1580	3.3227
1	0.1568	0.9497
-2	0.1508	-1.8694

5 Conclusions

Due to the importance of frequency and power estimation of sinusoidal signals, a new technique has been proposed in this paper using the eigen-approach. The frequencies of unknown input sinusoidal signals along with their real power values are estimated. The Transfer function of the FIR filter with the eigenvector corresponding to the minimum eigenvalue of the correlation matrix produces the minimum output power of the FIR filer. From the roots' locations of the transfer function, the estimated power is calculated by removing the roots one at a time and recalculating the output power. The new technique estimates the signals' frequencies and power accurately without any need for prior estimate of the number of input source signals.

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