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Effect of vertical imperfections on longitudinal railway track response

Wlodzimierz Czyczula[1*], and *Lukasz* Chudyba[1]

[1]Cracow University of Technology, Faculty of Civil Engineering, 31-155 Kraków, Poland

Abstract. The paper presents an analysis of linear models track response under longitudinal loads due to braking and accelerating of the train as well as constant traction forces. Longitudinal forces on the wheel-rail contacts are uniformly distributed on the whole length of train. Analysis is carried out under assumption that – in short time – train speed does not change significantly. Therefore stationary response of rail track is considered. The problem of critical speed is presented. Effect of vertical imperfections on the longitudinal rail displacement is considered under assumption that longitudinal forces varying in time are changed proportionally to the vertical forces. These forces are proportional to the amplitude of vertical imperfections. This analysis should be recognised as the main novelty of the paper. In the summary, certain practical conclusions are formulated as well as the further investigations are pointed out.

1 Introduction

Railway track dynamic response mainly concern vertical plane (comp. e.g. [1-3]). Horizontal plane (lateral and longitudinal directions) were analysed in the papers [4-7]. Effect of longitudinal forces in rails on the vertical track response was studies in the papers [2,8]. This paper should be considered as a significant improvement of the analyses presented in the previous papers of authors [4, 7]. Above mentioned papers concern only constant in time longitudinal forces. In this paper varying in time forces are also taken into account. Original approach for the analysis of the problem may be shortly formulated as follows: varying in time longitudinal loads are proportional to the varying in time vertical loads and these loads are proportional to the amplitude of vertical track imperfections. Last assumption is typically used in analyses of track vertical response (comp. [2, 9]).

2 Track and load model

The track is modelled by substitute beam resting on visco-elastic foundation. Beam is composed of two rails, described by longitudinal stiffness *EA* (where *E* rail steel modulus [N/m^2], *A* – cross section area of two rails [m^2]) and unit mass *m* [kg/m], which can include unit mass of two rail as well as uniformly distributed unit mass of sleepers. Rail foundation is modelled by unit stiffness *k* [N/m^2] and unit viscotic coefficient *c* [Ns/m^2]. Both foundation parameters describe longitudinal properties of fasteners and ballast (longitudinal resistance of sleepers in ballast). All rails and foundation parameters are constant along the track. Longitudinal load can be considered as traction forces for train acceleration, or for constant speed, as well as braking forces. In the small time period, train speed during braking or accelerating can be assumed as constant (comp. [4]). Under this assumption the stationary track response can be considered for both traction and train braking longitudinal action. In this paper the following track load model is introduced:

1. Longitudinal wheel/rail contact forces are distributed on the whole length of train;
2. Longitudinal load θ [N/m] depends on the vertical load q [N/m] and this relationship is described by a simple formula:

$$\theta = \mu \cdot q \tag{1}$$

   where μ – unit-less factor (less than friction coefficient);
3. Two parts of load are taken into account: θ_o – describes uniformly distributed load for constant in time forces (track without imperfections) and θ_v – load varying in time with circular frequency ω and amplitude $\Delta\theta$;
4. Circular frequency ω and amplitude $\Delta\theta$ of varying load are associated with vertical track imperfections, assumed as cosine shape, and train speed *v*. These relations are described by formulas:

$$\Delta\theta = \mu_v \cdot q_v \qquad \omega = \frac{2\pi v}{L_s} \tag{2}$$

*Corresponding author: czyczula@pk.edu.pl

where: μ_v and q_v – longitudinal factor and vertical unit amplitude of varying load (comp. Eq. (1), L_s – the length of vertical track imperfection.

For simplicity of the problem one can consider two types of imperfections (comp. e.g. [9,10]):

1. Short geometrical imperfections on the rail surface (corrugations) with the typical length in order 0,05 – 0,6 m and more and amplitude in the range 5-20 micrometers per one rail. In this case vertical vibrations occur on the wheel-rail contact and are associated with the change of length of wheel-rail contact spring;
2. Long geometrical imperfections related to the deformation of sleepers foundation (ballast and subgrade) with the length in the range of 3-15 and more meters and amplitude in the range of 10 – 25 mm. In this case, vibrations are related to the change of vehicle suspension spring length.

For assumed imperfection amplitude s_o per one rail and symmetry with respect to the track longitudinal axis, the vertical track load amplitude can be calculated from the simple formula:

$$q_v = 2 \cdot s_o \cdot k_c \tag{3}$$

where k_c – wheel/rail contact stiffness (short imperfections) or vehicle suspension stiffness (long imperfections).

In this paper linear track model is considered. Therefore the track response is calculated as the sum of two solutions: for a constant load and for a varying in time load.

3 Track without damping and under constant load

Basic equation of motion of track without damping under uniformly distributed load θ_o on the length of train $2l_t$ has the form (comp. [4]):

$$-EA\frac{\partial^2 u}{\partial x^2} + m\frac{\partial^2 u}{\partial t^2} + ku = \theta_o(x,t)\text{; } under\ train \tag{4a}$$

$$-EA\frac{\partial^2 u}{\partial x^2} + m\frac{\partial^2 u}{\partial t^2} + ku = 0\text{; } without\ train \tag{4b}$$

where:
$u(x,t)$ – longitudinal substitute beam (rails) displacements, other notation – see chapter 2.
In the moving coordinate system ($\eta = u$, $\xi = x - vt$), if load is constant in time, Eqs. (4) may be described as ordinary differential equations:

$$\frac{d^2u}{d\xi^2} - \beta^2 u = \frac{-\theta_o}{EA - mv^2};\ for|\xi| \le l_t \tag{5a}$$

$$\frac{d^2u}{d\xi^2} - \beta^2 u = 0;\ for|\xi| \rangle l_t \tag{5b}$$

where:

$$\beta = \sqrt{\frac{k}{EA - mv^2}} \tag{6}$$

As can be observed the solution of Eqs. (5) exists for the train speed v less than critical value v_{cr} , i.e.:

$$v \langle v_{cr} = \sqrt{\frac{EA}{m}} \tag{7}$$

General solution of inhomogeneous Eq. (5a), including particular part, can be described by the expression:

$$u(\xi) = C_1 \cdot ch(\beta\xi) + C_2 \cdot sh(\beta\xi) + \frac{\theta_o}{k};\ for|\xi| \le l_t \tag{8a}$$

and general solution of homogeneous Eq. (5b) obtains the form:

$$u(\xi) = D_1 \cdot e^{-\beta(\xi - l_t)} + D_2 \cdot e^{\beta(\xi - l_t)};\ for|\xi| \rangle l_t \tag{8b}$$

where: C_1, C_2, D_1, D_2 – constants.
Solution symmetrical with respect to the point $\xi = 0$, for only positive values ξ, one can obtain using the following boundary and matching conditions (comp. *[4])*:

$$\begin{aligned} &for\,\xi \to 0^+; \frac{du}{d\xi} \to 0; \\ &for\,\xi \to \infty; u \to 0; \\ &u(\xi = l_t^+) = u(\xi = l_t^-); \\ &\frac{du}{d\xi}(\xi = l_t^+) = \frac{du}{d\xi}(\xi = l_t^-) \end{aligned} \tag{9}$$

Under these conditions, the steady state solution of the problem, for positive ξ, can be expressed in the form:

$$u(\xi) = \frac{\theta_o}{k} \cdot \left(1 - \frac{ch(\beta\xi)}{ch(\beta l_t) + sh(\beta l_t)}\right);\ for\,\xi \le l_t \tag{10a}$$

$$u(\xi) = \frac{\theta_o}{k} \cdot \left(\frac{sh(\beta l_t)}{ch(\beta l_t) + sh(\beta l_t)}\right) \cdot e^{-\beta(\xi - l_t)};\ for\,\xi \rangle l_t \tag{10b}$$

and solution is symmetrical with respect to the point $\xi = 0$.
Thus the analysis of the track response without damping and under constant uniformly distributed load in longitudinal direction leads to certain remarks and conclusions formulated below:

1. Critical speed for longitudinal direction does not depend on foundation stiffness and it is relatively

high. For typical track structure used by railway companies it reaches the level of 4000 - 7000 km/h (comp. formula (7)). It means that critical speed for longitudinal direction is significantly higher in relation to the vertical plane and lateral direction (on the level of 1000-1500 km/h). Simple calculation shows that for train speed up to 300 km/h, the dynamic factor (related to the increase of rail longitudinal displacements with the increasing train speed) may be neglected for constant load and track without damping.
2. Steady state solution, described by Eqs. (10), is exact and has the closed form. The assumption on the symmetry of the solution for static case was proved by numerical experiments [4].

4 Steady state solution for varying load

Equation of motion in general case, for track with damping and any form of distributed longitudinal load, can be describes by formula:

$$-EA\frac{\partial^2 u}{\partial x^2}+m\frac{\partial^2 u}{\partial t^2}+c\frac{\partial u}{\partial t}+ku=\theta_v(x,t) \tag{11}$$

In the moving coordinate system ($\eta=u,\ \xi=x-vt$), for varying load, the Eq. (11) can be expressed as:

$$\begin{aligned}(mv^2-EA)\frac{\partial^2 u}{\partial \xi^2}-2mv\frac{\partial^2 u}{\partial\xi\partial t}+m\frac{\partial^2 u}{\partial t^2}+\\ +c\frac{\partial u}{\partial t}-cv\frac{\partial u}{\partial \xi}+ku=\theta_v(\xi,t)\end{aligned} \tag{12}$$

Under assumption that the varying load has the form (comp. chapter 2):

$$\theta_v(\xi,t)=\Delta\theta(\xi)\cdot\cos\omega t \tag{13}$$

the steady state solution, for linear case, can be described by the same expression, i.e.:

$$u(\xi,t)=U_c(\xi)\cdot\cos\omega t+U_s(\xi)\cdot\sin\omega t \tag{14}$$

One can obtain the set of ordinary equations, which represent cosine and sine part of solution:

$$\begin{aligned}&(mv^2-EA)\frac{d^2U_c}{d\xi^2}-2mv\omega\frac{dU_s}{d\xi}-m\omega^2U_c+\\ &+c\omega U_s-cv\frac{dU_c}{d\xi}+kU_c=\Delta\theta(\xi)\\ &(mv^2-EA)\frac{d^2U_s}{d\xi^2}+2mv\omega\frac{dU_c}{d\xi}-m\omega^2U_s-\\ &-c\omega U_c-cv\frac{dU_s}{d\xi}+kU_s=0\end{aligned} \tag{15}$$

Now the next assumption is introduced. The solution U_c and U_s as well as amplitude of the varying load $\Delta\theta$ can be described in terms of Fourier series in the finite interval *[0,λ]:*

$$\begin{aligned}\Delta\theta(\xi)&=\frac{a_0}{2}+\sum_{i=1}^{\infty}(a_i\cdot\cos\Omega_i\xi+b_i\cdot\sin\Omega_i\xi);\\ U_c(\xi)&=\frac{U_{10}}{2}+\sum_{i=1}^{\infty}(A_i\cdot\cos\Omega_i\xi+B_i\cdot\sin\Omega_i\xi);\\ U_s(\xi)&=\frac{U_{20}}{2}+\sum_{i=1}^{\infty}(C_i\cdot\cos\Omega_i\xi+D_i\cdot\sin\Omega_i\xi)\\ &\xi\in[0,\lambda];\Omega_i=\frac{2\pi\cdot i}{\lambda}\end{aligned} \tag{16}$$

After differentiation and rearrangement, one can obtain the solution in the following form:

$$\begin{aligned}A_i[P_1]+B_i[P_2]+C_i[P_3]+D_i[P_4]&=a_i\\ A_i[-P_2]+B_i[P_1]+C_i[-P_4]+D_i[P_3]&=b_i\\ A_i[-P_3]+B_i[-P_4]+C_i[P_1]+D_i[P_2]&=0\\ A_i[P_4]+B_i[-P_3]+C_i[-P_2]+D_i[P_1]&=0\end{aligned} \tag{17}$$

$$\begin{aligned}Y_{10}&=\frac{a_0[P_5]}{(P_5)^2+(P_3)^2}\\ Y_{20}&=\frac{a_0[P_3]}{(P_5)^2+(P_3)^2}\end{aligned} \tag{18}$$

where:

$$\begin{aligned}P_1&=(mv^2-EA)\Omega_i^2-m\omega^2+k\\ P_2&=-cv\Omega_i\\ P_3&=c\omega\\ P_4&=-2mv\omega\Omega_i\\ P_5&=k-m\omega^2\end{aligned} \tag{19}$$

5 Numerical examples

Calculations are carried out for the following track and load parameters (based on [2, 4])*:*

1. Track structure: 60E1 rails, $E = 2,1*10^{11}$ N/m^2, $A = 2*7687*10^{-6}$ m^2, *m = 2*60kg/m + 320kg/0,6m = 653,3* kg/m (also only rails, *m = 120* kg/m), foundation stiffness *k = 4000 – 6000* kN/m^2, damping properties *c = 9033-75000* Ns/m^2;
2. Track vertical imperfections: length $L_s =0,3 – 1,2$ m, amplitude *s = 2*10* μm;
3. Load: EMU-250 (Pendolino) train, with total length $2l_t = 185$ m, *28 axles *160 kN/axle*, unit vertical load $q_o = 24,22$ kN/m, unit constant longitudinal load $\theta_o = \mu q_o$, $\mu = 0,2$; $\theta_o = 4,84$ kN/m; unit amplitude of varying load $\Delta\theta = \mu_v q_v$; $\mu_v = 0,2$; wheel/rail contact stiffness $k_c = 1,2*10^6$ kN/m, then $\Delta\theta = 4,8$ kN/m, circular frequency of varying load ω – depending on the train speed (comp. formulas (1)-(3));

4. Other parameters: speed range v – up to *300* km/h; number of the Fourier series coefficients n – up to *1000*.

Introductory calculations show that non-zero track response occurs on the whole length of train and additional sections about 300 m ahead and behind the train. Therefore the total length of section λ in which approximation by Fourier series is introduced is equal: 2*300 m + 180 m = 780 m.

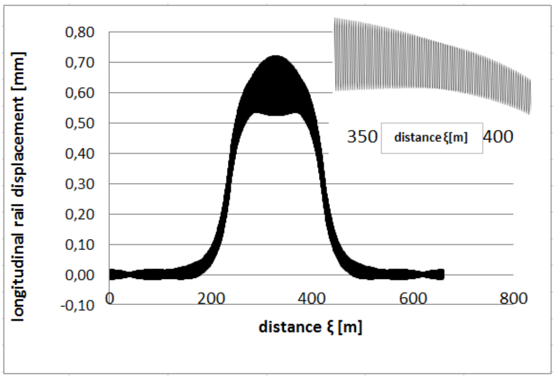

Fig. 1. Distribution of longitudinal rail displacements – train speed v = 200 km/h; longitudinal foundation stiffness k = 6000 kN/m^2

Fig. 1 shows distribution of the longitudinal rail displacements as a sum of constant longitudinal forces (without imperfections) and varying in time longitudinal forces as an effect of vertical track imperfections. The detailed longitudinal rail distrubution at the distance between 350 and 400 m is also presented. Other example of analysis is shown in Fig. 2.

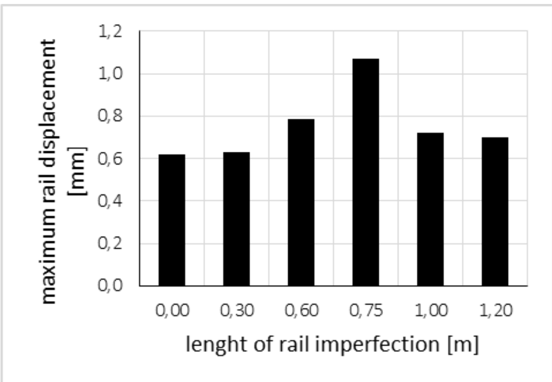

Fig. 2. Effect of rail imperfection length on maximum rail displacement (train speed v = 60 km/h; longitudinal foundation stiffness k = 6000 kN/m^2)

In this Figure, the maximum rail longitudinal displacement as a function of length of rail imperfection is presented. As can be seen the analysed effect is important – in this example the peak track response occurs at the length equal to 0,75 m.

6 Summary

The paper presents the examples of analysis of longitudinal track response by using analytical and semi-analytical models. Exact solution for constant longitudinal forces is shown. The problem of critical speed is also analysed. In contrast to the vertical and lateral track response, in the case longitudinal track response, critical speed for typical track parameters is a few time higher and reaches the level of 4000-7000 km/h. Original approach to study the effect of vertical track imperfection on the longitudinal track response is presented. Numerical examples show that effect of track imperfection parameters on longitudinal response is significant. Further investigations should be concerned with multilayer track structure including so called effect of head on web as well as nonlinear properties of fasteners and sleeper foundation.

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