Runners on footbridges – a new *VGRF* model for heel strike running technique

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Abstract. In the paper the models of dynamic forces generated by people running referred to two running techniques i.e. forefoot strike running and heel strike (rearfoot strike) running are presented. The model of forces generated during forefoot strike running was refined taking into account proposals of various authors and the results of author's own laboratory tests of running people. The model of the forces generated during heel strike running is a new author's proposal elaborated on the basis of the results of laboratory tests. Presented results of analyses and the proposal of a new dynamic load model correct and complement the shortcomings of the existing models and complement the shortage of recommendations in the area of dynamic forces generated during heel strike running technique.

1 Introduction

The dynamic susceptibility of the footbridges can be a serious problem in the case of the footbridges located along important pedestrian routes exposed to intense pedestrian traffic flow, footbridges in city centres, footbridges located in the vicinity of public utility buildings (large public transport stations, sports facilities, cultural centres, etc.).

In dynamic analyses of the footbridges the issues of dynamic action of the group of pedestrians and crowd come to the foreground. Impacts of the pedestrian groups and crowd become especially important when the natural vibration frequency of the footbridge is within the range 1.7 – 2.0 Hz for vertical vibrations or 0.9 – 1.1 Hz for horizontal vibrations. When the natural vibration frequency of the footbridge is within the range 2.2 – 3.4 Hz (footbridges with medium span of 35 – 50 m) a dynamic impact of people running can be even more important than action of people walking (group of pedestrians or crowd).

The dynamic action of runners can be especially important in the case of footbridges located in the vicinity of large public transport stations (train or metro stations, bus or tram stops etc.) as well as in the case of footbridges located in the recreational areas (city parks, walking boulevard or promenade etc.). In these places the risk of occurrence of people running (hurrying or exercising (jogging)) is high. The footbridges with natural vibration frequency in the range of frequency of running (2.2–3.4 Hz) will be exposed to excessive vibration of the deck.

The results of the researches of numerous footbridges of a medium span carried out by the author (steel, concrete, composite and wood footbridges, beam, truss, arch and cable-stayed structure) indicate the that fundamental vibration frequency of the structures with span 35 – 50 m are often in the range of the step frequency during running with medium speed (jogging) 2.2–3.0 Hz (Fig. 1).

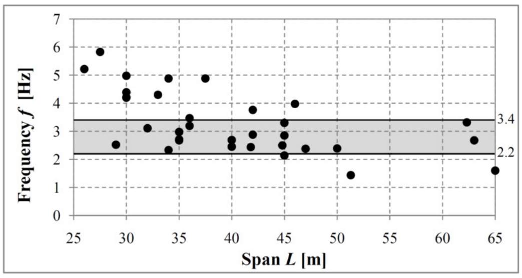

Fig. 1. Fundamental vertical vibration frequencies of the footbridges with a medium span in relation to the step frequency during running (shaded area) – the author's own research results.

The dynamic load generated by people running (ground reaction forces *GRF*, in particular their vertical component *VGRF*) reaches a value of 2.0*G* – 2.5*G* (*G* – body weight of running person). Due to the large amplitudes of dynamic forces, in the case of excitation of resonant vibrations, the vibration acceleration of the footbridges can reach values of about 1.5 – 4.0 m/s^2, particularly in the case of steel footbridges with low damping. These large amplitudes of vibrations can be induced only by one running person. The large vibration amplitudes strongly disturb walking of other pedestrians passing through the footbridge. The feet of pedestrians are bounced by vibrating footbridge deck, walking is difficult, vibrations are unpleasant. This level of vibrations is unacceptable especially in the case of frequent occurrence [1, 2]. At the design stage of the footbridges the modal analysis, calculations of the dynamic response of the structure, evaluation of the risk

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of excitation of excessive vibration of the structure as well as evaluation of the comfort of use of the structure are necessary. It should be remembered that the footbridge location plays an important role in assessing the probability of occurrence and importance of dynamic load case in the form of people running on the analysed/designed footbridge.

2 The *VGRF* generated during running

Running, i.e. one of the ways of human locomotion, can be divided into two basic phases: support phase in which one leg has a contact with ground and flight phase in which neither feet are in contact with the ground (Fig. 2).

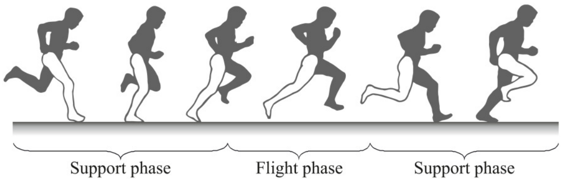


Fig. 2. Basic phases of running [3].

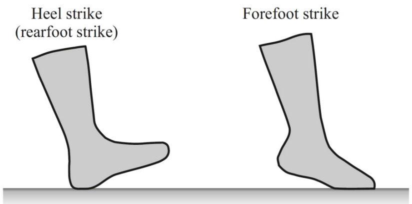
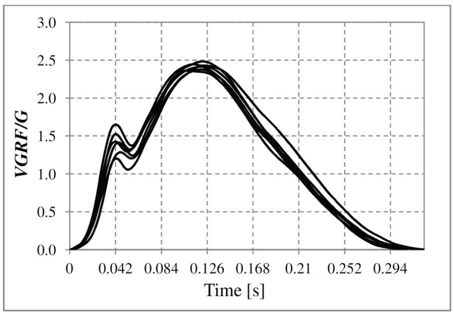


Fig. 3. Foot strike patterns during running: heel strike (rearfoot strike) and forefoot strike.

a)



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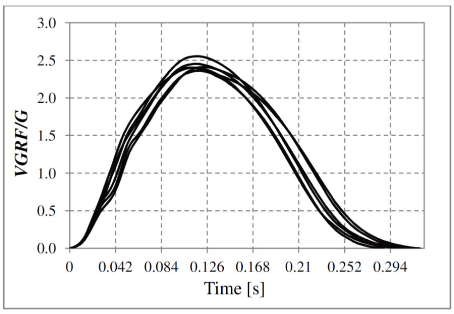


Fig. 4. Normalized vertical component of the ground reaction forces *VGRF/G* generated during running with frequency $f_r = 2.6$ Hz by a) heel strike (rearfoot strike) runner, b) forefoot strike runner (curves on the graphs are an examples of scatter of the *VGRF*s generated by one running person).

During the support phase of running, the weight of the body is transferred to the ground via one foot. Moreover, human body falls to the surface of the ground from a certain height. With this reasons the vertical component of the ground reaction force *VGRF* reach a value of $2.0G$–$2.5G$ (G – body weight of running person). During running displacement of the body centre of mass is about 8.5 cm in the vertical direction and about 2.0 cm in the horizontal direction for running speed V_r = 2.5 m/s ($f_r \approx 2.75$ Hz, f_r – frequency of running).

The time course of the *VGRF* depends on the foot strike patterns during running. The two most common strike patterns are: heel strike (rearfoot strike) and forefoot strike (Fig. 3). The heel strike runner initiates the contact with the ground throughout the heel (the rear part of the foot) whereas the forefoot strike runner contacts with the ground throughout the fore part of the foot. The normalized ground reaction forces (*VGRF/G*) for these two strike patterns are presented in Fig. 4.

The *VGRF* generated by a heel strike runner is characterised by two peaks: an impact peak (the first small peak) and a propulsive peak (the second big peak also known as the active peak). During the forefoot strike running only one active peak occurs. The active peak occurs when the body centre of mass is directly over the foot.

The time of occurrence of impact and active peaks as well as value of both peaks in relation to body weight of running person G were determined by the author on the basis of laboratory tests carried out with the participation of 14 people (women and men, age 22–40 years, height 160–187 cm, weight 52–108 kg, running in frequency range $f_r = 2.4$–3.4 Hz i.e. slow and moderate pace of running, data acquisition by two AMTI force plates, sampling rate 1.0 kHz).

In the case of heel strike running the impact peak reached the values of F_{ip} = $1.2G$–$2.1G$ (mean value $F_{ip,\mathrm{m}} = 1.6G$). The impact peak occurs at the time of t_{ip} = 32–48 ms after contact of the foot with the ground (mean value $t_{ip,m}$ = 40 ms). The active peak occurs at the time of t_{ap} = 80–150 ms after contact of the foot with the ground respectively for moderate pace of running ($f_r = 3.4$ Hz) and slow pace of running ($f_r = 2.4$ Hz) (mean value $t_{ip,m}$ = 115 ms) and reached the values $F_{ap} = 1.7G$–$2.5G$ (mean value $F_{ap,\mathrm{m}} = 2.1G$).

3 The *VGRF* models

3.1 Forefoot strike running

The *VGRF* generated by forefoot strike runner has a shape similar to half-cycle of sine wave and can be determined using equations (1) or (2) presented respectively in [4] and [5, 6]. In [4] the *VGRF* is described by Fourier series. The recommendation of [5, 6] describes the *VGRF* using half-cycle of sine wave.

$$F(t) = G + \sum_{i=1}^{n} A_i \sin\left(2\pi \cdot i \cdot f_r \cdot t - \varphi_i\right) \qquad (1)$$

where: G – the body weight of the running person, $A_i = \alpha_i \cdot G$ – dynamic amplification factor, α_i – dynamic load factors, f_r – running frequency, φ_i – the phase shifts of the i^{th} harmonics with respect to the 1st harmonic, i – the order number of the harmonic i = 1, 2, 3 ..., n – the total number of included harmonics (in the case of running the first three harmonics are usually considered), t – time step.

$$F(t)=\begin{cases} A_r G \sin\left(\dfrac{\pi \cdot f_r}{k}\cdot t\right) & \text{for} \quad i\cdot T_r < t \le (i+k)\cdot T_r \\ 0 & \text{for} \quad (i+k)\cdot T_r < t \le (i+1)\cdot T_r \end{cases} \tag{2}$$

where:
k – the contact time factor

$$k = \frac{t_{cr}}{T_r} \tag{3}$$

A_r – the dynamic amplification factor

$$A_r = \frac{\pi}{2 \cdot k} \tag{4}$$

G – the body weight of the running person, t_{cr} – contact time of the foot with the ground for running ($t_{cr} = 0.5T_r$ is recommended in [5, 6] for ordinary running activity), f_r – running frequency, i = 0, 1, 2, 3,…, $T_r = 1/f_r$ – running period, t – time step.

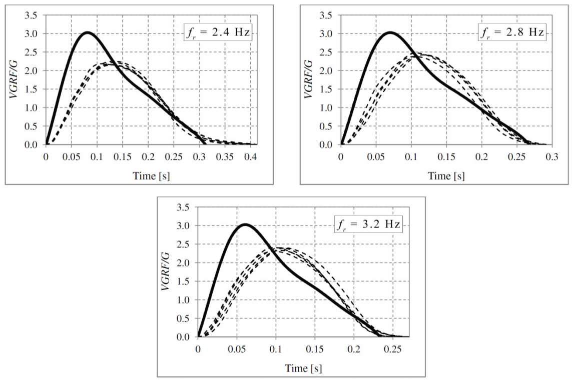
It is worth remembering that the recommendations presented in [4] and [5, 6] should be used with caution for several reasons:

- the *VGRF* estimated using the values of the dynamic load factors α_i and the phase shifts φ_i recommended in [4] i.e. $\alpha_1 = 1.6$, $\alpha_2 = 0.7$, $\alpha_3 = 0.2$, φ_i – lack of recommendations (assumed values $\varphi_2 = \varphi_3 = 0$), represents the maximum value of the *VGRF* generated by running person ($VGRF/G \approx 3.0$) which occurs sporadically. Usually the value of $VGRF/G \approx$ 2.1 – 2.5,
- the *VGRF* estimated using equation (1) is negative in the time interval corresponding approximately to the flight phase during the running. The negative *VGRF* values must be set to zero. Only positive *VGRF* values should be used in dynamic analyses.
- the value of the dynamic amplification factor A_r in equation (2) is very sensitive to changes of the value of the contact time t_{cr}. For the $t_{cr} = 0.5T_r$ (i.e. $k = 0.5$) suggested in [5, 6] for ordinary running activity the *VGRF* reaches the high maximum value which occurs sporadically ($VGRF/G = A_r = 3.14$).

The *VGRF*s determined by means of equations (1) and (2) using the recommendations given in [4] and [5, 6] with reference to the *VGRF*s acquired during laboratory tests for running frequency 2.4 Hz, 2.8 Hz and 3.2 Hz are presented in Fig. 5.

A large inexactness can be observed in the estimation of the *VGRF* values. The *VGRF*s courses are estimated inaccurately. The maximum *VGRF* values are overestimated (consequently the vibration amplitudes of the structure will be overestimated). Moreover, assuming $t_{cr} = 0.5T_r$ according to [5, 6] the contact time value t_{cr} is estimated incorrectly. Consequently the *VGRF*s course is inaccurate and incorrect. The parameters in equations (1) and (2) should be adjusted to improve the estimation of the *VGRF* values.

a)



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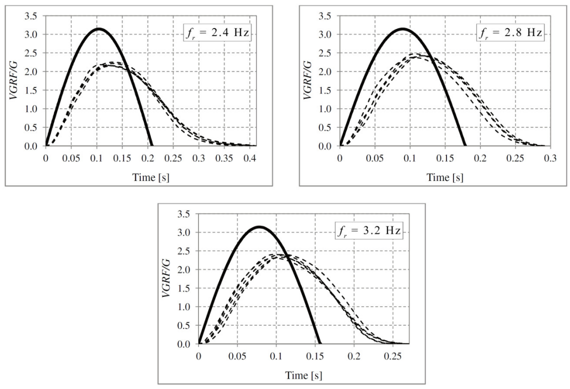


Fig. 5. Calculated *VGRF/G* (bold line) with reference to the *VGRF/G* determined during laboratory tests (dashed lines) for running frequency 2.4 Hz, 2.8 Hz and 3.2 Hz a) the *VGRF/G* determined by means of equations (1) for $\alpha_1 = 1.6$, $\alpha_2 = 0.7$, $\alpha_3 = 0.2$, $\varphi_2 = \varphi_3 = 0$, b) the *VGRF/G* determined by means of equations (2) for $t_{cr} = 0.5T_r$.

Recommendations presented in [7, 8], regarding the parameters of equation (1), allow to increase the accuracy of estimation of the *VGRF* values. The dynamic load factors and the phase shifts equal respectively $\alpha_1 = 1.3$, $\alpha_2 = 0.3$, $\alpha_3 = 0.1$, $\varphi_2 = \varphi_3 = 0$ are recommended in [7, 8]. The *VGRF*s determined using these recommendations are presented in Fig. 6.

Analyses carried out by author showed the possibility of further correction of the dynamic load factor α_i to better match the calculated *VGRF* to the real *VGRF* acquired during the tests. The α_i values can be assumed in the range $\alpha_1 = 1.0 – 1.7$, $\alpha_2 = 0.0 – 0.3$, $\alpha_3 = 0.0 – 0.1$. The phase shifts values remain unchanged $\varphi_2 = \varphi_3 = 0$. The *VGRF* determined using adjusted dynamic load factors α_i are shown in Fig. 7.

It is worth noting that the lower values of $\alpha_1 = 1.0 –$ 1.3 and higher values of $\alpha_2 = 0.2 – 0.3$ are appropriate to determine the *VGRF* generated by forefoot runners during slow running, whereas the $\alpha_1 = 1.4 – 1.7$ and $\alpha_2 = 0.0 – 0.15$ are appropriate to determine the *VGRF* generated during normal and fast running. The changes in the value of α_3 allow for a slight correction of the *VGRF* values but have no significant impact on the *VGRF*.

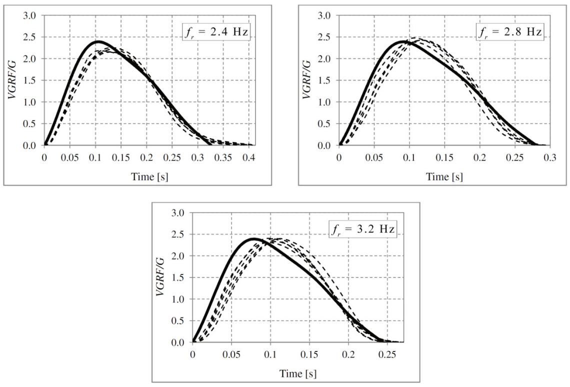


Fig. 6. The *VGRF/G* determined by means of equation (1) for $\alpha_1 = 1.3$, $\alpha_2 = 0.3$, $\alpha_3 = 0.1$, $\varphi_2 = \varphi_3 = 0$ (bold line) with reference to the *VGRF/G* determined during laboratory tests (dashed lines) for running frequency 2.4 Hz, 2.8 Hz and 3.2 Hz

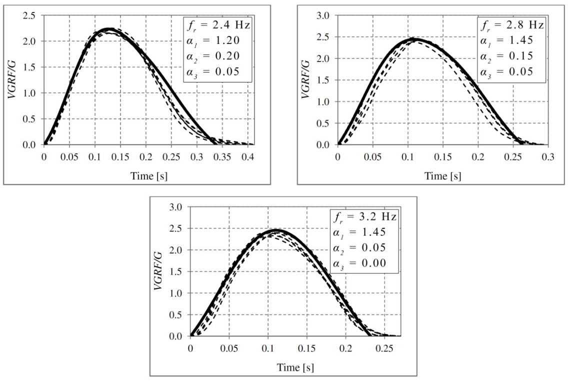


Fig. 7. The *VGRF/G* determined by means of equation (1) using adjusted dynamic load factors α_i.

It is also worth noting that the maximum value of the normalized ground reaction force (*VGRF*/*G*) is typically in the range of about 1.4 – 1.7 for slow running ($f_r = 2.0$ – 2.3 Hz), 1.7 – 2.3 for moderate pace of running (also known as "normal running", $f_r = 2.3$ – 2.7 Hz) and 2.3 – 2.8 for fast running ($f_r \geq 2.7$ Hz). Although the minimum and maximum *VGRF* values can reach the limits presented in Fig. 9a.

The equation (1) allow to determine the minimum value of the *VGRF/G* = 1.9. To determine the *VGRF/G* < 1.9 further analyses and adjustments of the parameters of the equation (1) are necessary.

The *VGRF* values can be efficiently and simply determined using equation (2) provided that the correct input parameters are used. The two most important parameters in equation (2) are t_{cr} and A_r.

The results of the laboratory test carried out by author showed that in the case of normal and fast running (for running frequency $f_r > 2.4$ Hz) the t_{cr} mean value is in the range $(0.75 - 0.85)\cdot T_r$, i.e. $k = 0.75 - 0.85$ ($t_{cr} = 0.75T_r$ for moderate (normal) pace of running and $t_{cr} = 0.85T_{\mathrm{r}}$ for fast running). Furthermore, in the case of slow pace of running (for running frequency $f_r = 2.0$ – 2.3 Hz) the contact time value is $t_{cr} \approx (1.0 - 1.1)\cdot T_r$, i.e. $k = 1.0 - 1.1$. It means that in the case of slow running for a very short period of time (a few millisecond) both feet have a simultaneous contact with the ground.

The recommendation concerning the t_{cr} and A_r values can be found in [9, 10] (Fig 8). Moreover in Fig. 9 the author's own research results are presented.

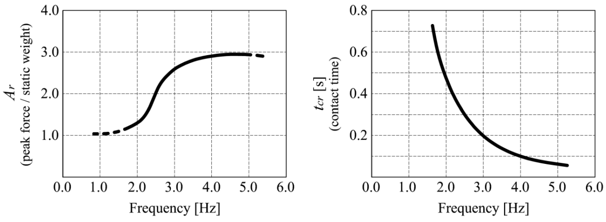
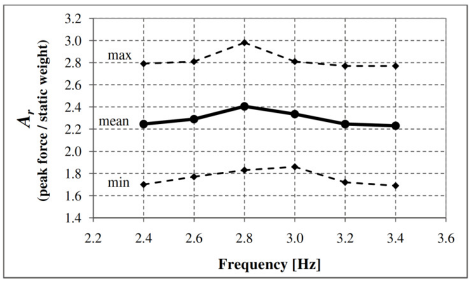


Fig. 8. The dynamic amplification factor A_r (peak force/static weight) and contact time t_{cr} for running [9, 10].

a)



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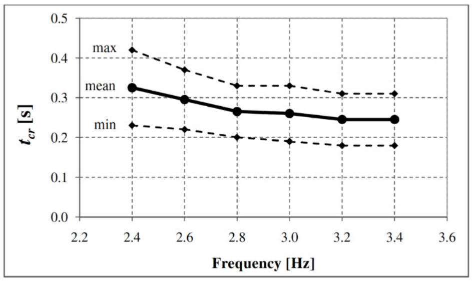


Fig. 9. Parameters of the running a) the dynamic amplification factor A_r (peak force/static weight), b) contact time t_{cr} (the author's own research results).

The use of the t_{cr} value presented in Fig. 9b (mean value or appropriate value between the minimum and maximum limits) allow for better adjustment of the *VGRF* graph to the real *VGRF* values acquired during the tests (Fig. 10).

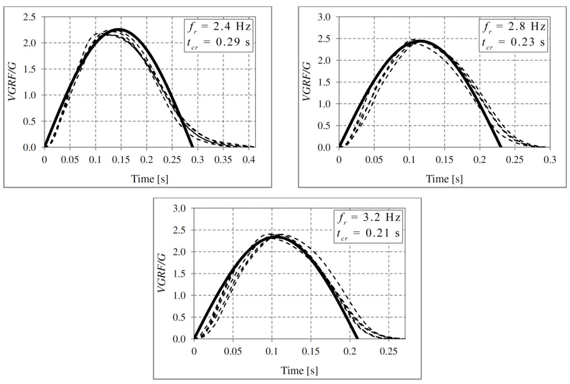


Fig. 10. The *VGRF/G* determined by means of equation (2) using adjusted t_{cr} values.

Further adjustment of the *VGRF* values calculated using equation (2) is possible by replacing the A_r value determined in accordance with equation (4) with the appropriate value read from Fig. 9a between the minimum and maximum limits.

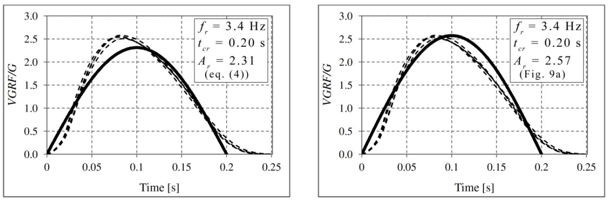


Fig. 11. The *VGRF/G* determined by means of equation (2) using adjusted t_{cr} values.

3.1 Heel strike running

The second techniques of running – the heel strike running (rearfoot strike running) requires the correction or supplementation of previous *VGRF* models due to the impact peak occurrence a few millisecond after contact of the foot of running person with the ground.

The author's proposal of the *VGRF* model generated by heel strike runners are the general equations (5) supplemented with equation (6) – the impact peak function.

$$\Phi(t)=\begin{cases} F(t)+\varphi(t) & \text{for} \quad 0\le t\le \lambda\cdot T_r \quad \varphi(t)\ge 0 \\ F(t) & \text{for} \quad t\ge \lambda\cdot T_r \end{cases} \tag{5}$$

$$\varphi(t)=A_{ip}G\cdot\sin(\alpha\cdot\pi\cdot f_r\cdot t)^{\beta} \tag{6}$$

where:
F(t) – the *VGRF* values determined by means of equation (1) or (2),
φ(t) – the impact peak function, $\varphi(t) \geq 0$ (only the first cycle of sine wave should be used),
G – the body weight of the running person,
f_r – running frequency,
A_{ip} – amplitude of the impact peak e.g. $A_{ip} \approx 0.70$. The A_{ip} value can be adjusted accordingly. Typical A_{ip} value can be assumed in the range 0.5-1.3 (the $A_{ip} \geq 1.0$ applies to the *VGRF* calculated using equation (1)),
α – the impact peak location coefficient, $\alpha = 4 – 8$ (the α can be a decimal number). For $\alpha = 5 – 8$ the impact peak on *VGRF* graph will occur at the time of $t_{ip} \approx 30 – 50$ ms after contact of the foot with the ground in frequency range $f_r = 2.4 – 2.8$ Hz. For frequency range $f_r = 2.8 – 3.4$ the $\alpha = 4, 5, 6$ can be assumed. The coefficient should be adequately adjusted to the running frequency to achieve the location of the impact peak at the time of $t_{ip} = 30–45$ ms after contact of the foot with the ground,
β – the impact peak slenderness coefficient (exponent). The $\beta = 2, 3, 4, 5, ...$ can be assumed. Recommended value $\beta = 4$ (the β can be a decimal number),
λ – the impact peak time range coefficient, $\lambda \approx 0.17{\cdot}T_r$ (only the first cycle of the impact peak function should be used),
t – time step.

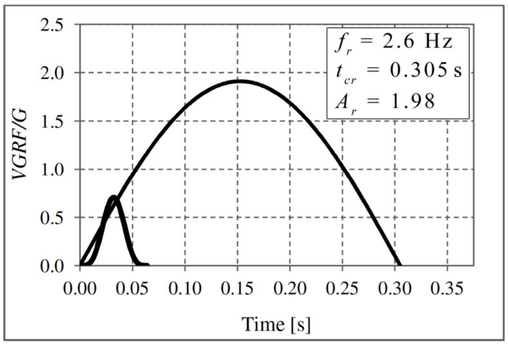
Examples of the impact peak function for running are equations (7) and (8) (equation (7) applies to *VGRF* calculated by means of (2), equation (8) applies to *VGRF* calculated by means of (1)).

$$\varphi(t)=0.85\cdot G\cdot\sin(6\cdot\pi\cdot f_r\cdot t)^4 \tag{7}$$

$$\varphi(t)=1.15\cdot G\cdot\sin(6\cdot\pi\cdot f_r\cdot t)^3 \tag{8}$$

In Fig. 12 the visualization of the *VGRF* graph generated using equation (1) and (2) and the impact peak graph determined by means of equation (7) and (8) are presented.

a)



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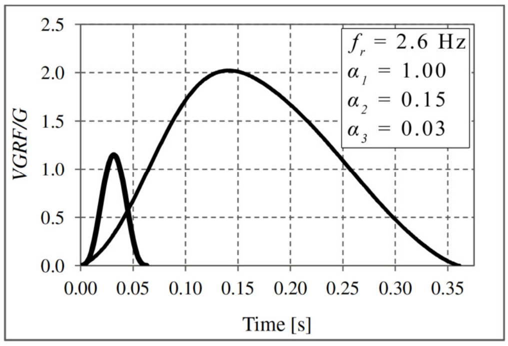
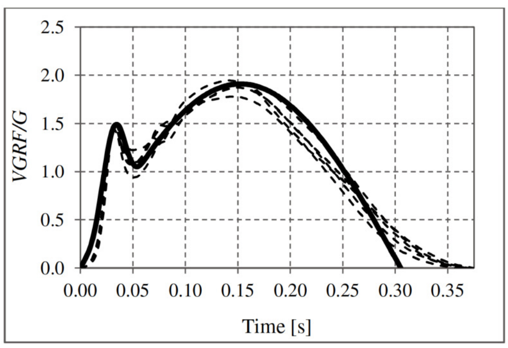


Fig. 12. The *VGRF/G* and the *φ(t)/G* visualisation a) graphs determined by means of equations (2) and (7), b) graphs determined by means of equations (1) and (8).

In Fig 13 the Φ(t)/*G* graphs determined using equations (2) and (7) as well as (1) and (8) in relation to the *VGRF/G* obtained using the results of laboratory tests are presented.

a)



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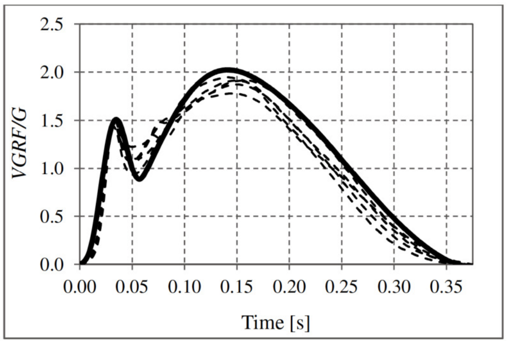

Fig. 13. The $\Phi(t)/G$ graphs in relation to the *VGRF/G* from laboratory tests a) $\Phi(t)/G$ graph determined by means of equations (2) and (7), b) $\Phi(t)/G$ graph determined by means of equations (1) and (8).

4 Summary and conclusions

The vertical component of the dynamic force generated by running man (*VGRF*) reaches amplitudes from two to two and half times higher than the runner's body weight. This dynamic action can cause the large vibration of the structure which can strongly disturb walking of other pedestrians crossing the footbridge. The structure can be exposed to excessive vibrations especially when its natural vibration frequency is within the frequency range of running. An important task at the designing phase can be reliable prediction of amplitudes of vibration caused by people running. Accurate determination of the dynamic forces (*VGRF*) can be crucial.

The paper presents the analyses of two models of the *VGRF* proposed by various authors (equations (1) and (2) [4, 5, 6, 7, 8]) and the possibility of its adjustment to the *VGRF* measured during laboratory tests as well as a new author's proposal of the *VGRF* model describing the *VGRF* generated by heel strike runners (equations (5) and (6)). It has been shown that proper selection of the parameters of each models allows for very accurate determination of *VGRF* values.

In the case of model described by equation (1) [4] the dynamic load factors α_i can be assumed in indicated range and do not have to be constant. The changes in α_i allow to generate the minor random changes of the *VGRF* values occurred during running.

The *VGRF* model described by equation (2) [5, 6] have to be used with caution. The *VGRF* values determined using equation (2) strongly depend on the contact time value t_{cr} used in the calculation. In the paper the values of the t_{cr} determined on the basis of laboratory tests in frequency range 2.4 – 3.4 Hz were presented. Moreover the possibility of adjustment of the dynamic amplification factor A_r in equation (2) was indicated and the range of A_r values was presented.

Equations (5) and (6) describe a new model developed by author for *VGRF* generated by heel strike runners. In this *VGRF* model the additional impact peak occurring during heel strike running technique is included. This proposal complements the shortage of recommendations in this area.

Presented proposals for the development of existing *VGRF* models describing the forefoot strike running technique as well as proposed new *VGRF* model for heel strike running technique allow to increase the accuracy of prediction of dynamic loads and consequently increase the accuracy of dynamic analyses of the structures exposed to dynamical loads generated by runners.

References

1. A. Flaga, M. Pańtak, *Vibration comfort criteria for pedestrians on footbridges,* Proc. 3rd Int. Conf. “Footbridge 2008” (2008)
2. M. Pańtak, *Analysis of vibration comfort criteria for steel footbridges susceptible to dynamic actions*, PhD Thesis, (Cracow University of Technology, Cracow, 2007)
3. R.S. Adelaar, Am J Sports Med **14(6)** (1986)
4. H. Bachmann, W. Ammann, F. Deischl et al., *Vibration Problems in Structures: Practical Guidelines* (Birkhäuser Verlag, Basel, 1995)
5. A. Occhiuzzi, M. Spizzuoco, *Running pedestrian dynamic load model for footbridges*, Proc. 3rd Int. Conf. “Footbridge 2008” (2008)
6. A. Occhiuzzi, M. Spizzuoco, F. Ricciardelli, Struct Control & Health Monit **15(3)** (2008)
7. J.H. Rainer, G. Pernica, Canad Acoustic **14**(**2**) (1986)
8. J.H. Rainer, G. Pernica, D. E. Allen, Canad J Civ Eng **15**(**1**) (1988)
9. J.E. Wheeler, ASCE J. Str Div **108**(**9**) (1982)
10. S. Živanović, A. Pavić, P. Reynolds, J. Sound Vib **279** (**1-2**) (2005)