

Equivalent stabilizing force of members parabolically compressed by longitudinally variable axial force

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Abstract. The EN 1993-1-1 model of equivalent stabilizing force q_d and R_d of bracings conservatively assumes that the braced member is compressed with a force constant along its length. This assumption is incorrect since the axial force distribution varies along the length of the braced member. As a result, the braced member generates equivalent stabilizing forces different from equivalent force q_d and R_d acc. to EN 1993-1-1. This paper presents parametric studies of the equivalent stabilizing forces of the braced, compression top chord of roof trusses. The girder's top chord is compressed parabolically by a variable axial force. The values of the axial compressive forces is: N_{supp} - in the support zone of truss and N_{span} - in the central zone of truss. Parametric analyses of the equivalent stabilizing force and the stress of the purlins and the bracings depending on axial forces N_{supp} and N_{span} in the braced member were carried out. The investigated problem is illustrated with exemplary calculations of the equivalent force in trusses.

1 Introduction

According to EN 1993-1-1 [1], an evaluation of the loading capacity of frame systems and bracings should take into account the forces due to random initial bow imperfections with amplitude e_0 (Fig. 1a). In calculations the forces are replaced with equivalent stabilizing force q_{d1} and R_{d1} (Fig. 1c) determined from the formulas

$$q_{d1} = 8N_{Ed,max} \frac{e_0}{L^2}, \quad (1)$$

$$R_{d1} = -4N_{Ed,max} \frac{e_0}{L}, \quad (2)$$

where:

$N_{Ed,max}$ – the axial force in the braced member,

e_0 – the maximum amplitude of the braced member imperfection,

L – the span of the braced member.

Uniformly distributed imperfect span equivalent stabilizing force $q_{d1} = \text{const.}$ acc. to (1) and support reactions R_{d1} acc. to (2) were determined assuming that the braced member is compressed by axial force $N_1(x) = N_{Ed,max}$, constant along its length (Fig. 1b). Frame columns satisfy this assumption since they are compressed by a longitudinally invariable force [2-4].

In the EN 1993-1-1 [1] computational model also (1) and (2) are used to analyse the stress of purlins and bracings caused by the actions of the bow curved laterally stiffened top flanges of roof girders. In this standard model, even though axial force $N_2(x)$ varies along the braced member (e.g. as in Fig. 2b), it was

assumed that the latter is compressed by axial force $N_1(x)$ constant along its length (Fig. 1b).

It should be noted that in the case of the equivalent stabilizing force of the braced top flanges of roof girders the assumption $N_1(x) = \text{const.}$ is incorrect since it does not correspond to the actual longitudinally variable distribution of the axial force in the braced member (Fig. 1d). The axial force usually changes parabolically (e.g. as in Fig. 1e, 2c) or stepwise parabolically (as in Fig. 2b), and also its sign can change (compression and tension occur).

In the case of distribution of axial force as in Fig. 1e, the equivalent stabilizing force $q_{d2}(x)$ is also variable over the length of member – it changes longitudinally (parabolically) and changes sign (Fig. 1f). The equivalent stabilizing force $q_{d2}(x)$ differs fundamentally from the $q_{d1}(x)$ (compare Fig. 1c and Fig. 1f) which results in significant differences in the assessment of purlin safety and bracings in comparison to calculated according to EN 1993-1-1 [1].

As demonstrated in [5-8], this seemingly safe assumption about constant axial force $N_1(x) = \text{const.}$ can lead to the underrating of the stress of purlins and bracings.

Figure 2a shows the loading diagrams of the transverse systems of steel industrial sheds in which the roof girder is pin jointed with the columns. Due to the overhead crane load (V, H) and wind loads (w_p, w_n) axial force N_{supp} is transferred to the braced top flange of the roof girder. This force and the parabolically longitudinally variable axial force in the top flange generated by the roof girder's equivalent stabilizing force (p) add up. In the considered case, axial force $N_3(x)$

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changes stepwise parabolically along the length of the braced top flange of the roof girder, as shown in Fig. 2b. In the support zone compressive force $N_3(0) = N_{supp}$, whereas in the central zone $N_3(0.5L) = N_{span} = N_{Ed,max}$.

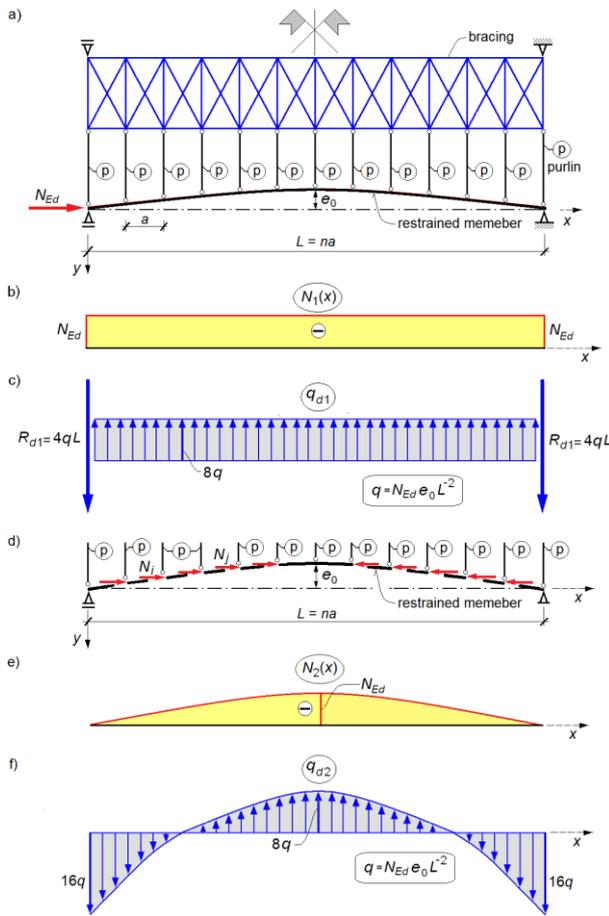


Fig. 1. Scheme of: a) member to be restrained, b) uniform distribution of the axial force, c) equivalent stabilizing force of the uniformly compressed member, d) member to be restrained with variable axial force, e) parabolically variable axial force in the member, f) equivalent stabilizing force of the compression member under parabolically variable axial force.

The consequence of the initial bow imperfection e_0 of the braced upper chord of truss, in the roof plane, is the twist of the girder's principal plane (because the upper chord is curvilinear, and the lower chord is, for example, rectilinear). Hence not only the bow imperfection e_0 of compression flange, but also an imperfection consisting in a twist of the truss's principal plane by angle $\phi_0(x)$ occurs. As a result of the action of vertical loads P_{Ed} in the upper joints on the truss twisted by angle $\phi_0(x)$ additional equivalent horizontal forces $H_i(x)$ arise [5]. The forces are transmitted to the purlins and the transverse bracing whereby their stress increases. The bracing calculation model adopted in EN 1993-1-1 [1] does not take into account equivalent horizontal forces $H_i(x)$ generated by the twist of the girder.

The aim of this paper is to identify the equivalent stabilizing forces of the braced top chord of the roof truss. The study takes into account the initial bow imperfection e_0 of the braced top chord and the

imperfection consisting in a twist of the roof girder's principal plane by angle $\phi_0(x)$. Moreover, compressive axial force $N_3(x)$ in the top chord of the truss is assumed to be longitudinally parabolically variable and to have a distribution as shown in Fig. 2b. The values of the axial forces in the support zone ($N_{supp} = \alpha N_{Ed,max}$) and in the central zone ($N_{span} = N_{Ed,max}$) of the braced top flange of the girder depend on the load parameters (p, w_p, w_n, V, H) and the stiffness parameters of the transverse system (moment of inertia I_g and span L of the roof girder, moment of inertia I_c and height h of the column). As part of this study parametric analyses of the equivalent stabilizing forces and the stress of the purlins and the bracings depending on axial forces N_{supp} and N_{span} in the braced member were carried out.

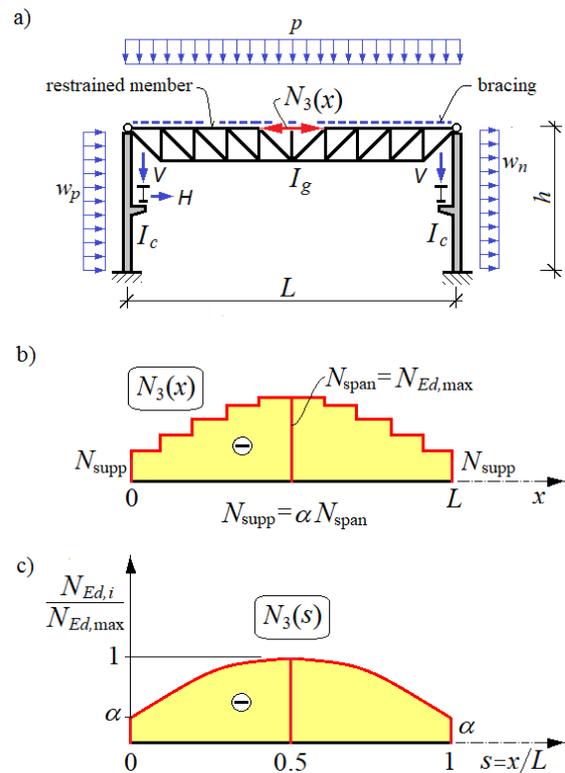


Fig. 2. Schemes of: a) roof girder to columns pin jointed, b), c) distribution of the axial force in the restrained upper chord of the roof girder.

2 Analytical determination of equivalent stabilizing force due to initial bow imperfection e_0 of braced member parabolically compressed by longitudinally variable axial force

The longitudinally stepwise parabolically variable distribution of the axial force in the braced top chord of the truss (Fig. 2b) can be approximated with the continuous distribution shown in Fig. 2c. The equivalent stabilizing force of the braced member compressed with axial force distributed according to Fig. 2c was analysed. Extreme axial force $N_3(x)$ in the middle of its span

$N_{span} = N_{Ed,max}$ and axial force in the support zone is $N_{supp} = \alpha N_{Ed,max}$.

The longitudinal variation of axial force $N_3(s)$ of the braced member was defined using a parametric parabolic function (Fig. 2c) described by the relation

$$N_3(s) = N_{Ed,max} [\alpha(1-2s)^2 - 4s(s-1)], \quad (3)$$

where:

$N_{Ed,max}$ – the value of the extreme axial force at the midspan of the braced member,

α – a dimensionless coefficient of the axial force at respectively the left and right end of the braced member, assuming values from the $\langle 0,1 \rangle$ interval (a “+” value – compression),

$s = x/L$ – the relative location (Fig. 2c) of the considered cross section along the length of the braced member, assuming values from the $\langle 0,1 \rangle$ interval.

Using the adopted function of the variation of axial force $N_3(s)$ acc. to (3) one can analyse the equivalent stabilizing forces of braced members compressed by force $N_1(x)$ constant along the length (acc. to EC 1993-1-1 [1]) and by longitudinally variable axial forces $N_2(x)$ and $N_3(x)$ whose distributions are shown in Figs 1e, 2c.

The initial geometric bow imperfection of the braced member was assumed to have the form of a parabola [1], [4] described by the relation

$$y(s) = 4e_0s(1-s), \quad (4)$$

where:

e_0 – the maximum amplitude of imperfection of the braced member (acc. to [1], $e_0 = L/500$).

General formulas for span equivalent stabilizing force $q(s)$ and support reactions R_d of a member loaded by longitudinally variable axial force $N(s)$ with any distribution $N_{Ed}(0) \neq N_{Ed}(0.5) \neq N_{Ed}(1)$ were derived in [8].

If axial force $N_3(s)$ varies longitudinally as in Fig. 2c and its extreme value occurs at the midspan of the braced member: $N_3(0.5) = N_{span} = N_{Ed,max}$, span equivalent stabilizing force $q_{d3}(s)$ and support reactions R_{d3} are calculated from the formulas

$$q_{d3}(s) = 8 \frac{e_0}{L^2} N_{Ed,max} [3\alpha(1-2s)^2 - 2(6s^2 - 6s + 1)], \quad (5)$$

$$R_{d3} = -4\alpha \frac{e_0}{L} N_{Ed,max}. \quad (6)$$

Table 1. Parameter α of considered schemes of distribution of the axial force $N_3(s)$

Scheme no	α	Scheme no	α	Scheme no	α
1	1.0	5	0.6	9	0.2
2	0.9	6	0.5	10	0.1
3	0.8	7	0.4	11	0.0
4	0.7	8	0.3		

Parametric analyses of equivalent stabilizing forces $q_{d3}(s)$, $R_{d3}(s)$ of braced members longitudinally stressed

by variable distributions of axial force $N_3(s)$ were carried out. The values of coefficient α for the analysed schemes of the longitudinal variation of axial force $N_3(s)$ in the braced members are given in Table 1.

Scheme 1 ($\alpha = 1$) applies to a braced member compressed by longitudinally constant axial force $N_1(x) = \text{const.}$ (acc. to the model in EN 1993-1-1 [1]).

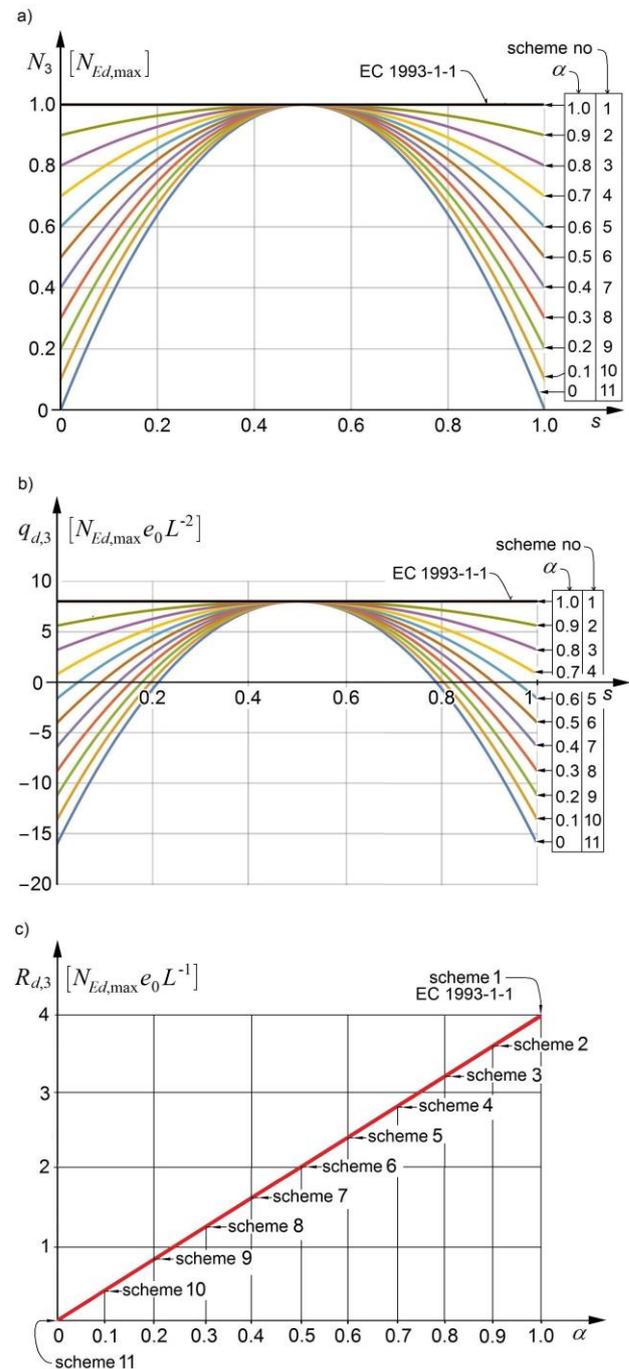


Fig. 3. Distributions of: a) axial force $N_3(s)$ in restrained members as a function of parameter α , b) equivalent stabilizing force $q_{d3}(s)$ in restrained members as a function of parameter α , c) support reactions R_{d3} in restrained members as a function of parameter α .

Schemes 2-11 concern the parabolically variable distributions of axial force $N_3(s)$ in the braced members, shown in Fig. 2c. They take into account the variation in

the values of the compressive forces at the support: $N_3(0) = N_{supp}$ (variable parameter α) and the constant value of the compressive force at the midspan of the braced member: $N_3(0.5) = N_{span} = N_{Ed,max}$. Scheme 11 ($\alpha = 0$) corresponds to the distribution of axial force $N_3(s)$ in the braced top flange of a pin-supported roof girder. The latter is uniformly loaded and the axial force at the support in the braced top flange is $N_3(0) = N_{supp} = 0$ (Fig. 1d, e).

The considered schemes of the longitudinal variation of axial force $N_3(s)$ in the braced members as a function of parameter α are shown in Fig. 3a. The distributions of span equivalent stabilizing force $q_{d3}(s)$ resulting from the stress of the braced member consistently with the considered schemes of axial force $N_3(s)$ are shown in Fig. 3b. Figure 3c shows support reactions R_{d3} of the compressed members versus α , corresponding to the considered diagrams of axial forces $N_3(s)$.

An analysis of the parametric calculations indicates that the parabolic distribution of span equivalent stabilizing force $q_{d3}(s)$ corresponds to the parabolic distribution of axial force $N_3(s)$ in the braced member (compare Fig. 3a and 3b). Span equivalent stabilizing force $q_{d3}(s)$ and support reactions R_{d3} in the analysed diagrams of the longitudinal stress of the brace members are a linear function of dimensionless axial force parameter α .

The equivalent stabilizing forces of the braced member consists of span force $q_{d3}(s)$ (Fig. 3b) and support reactions R_{d3} (Fig. 3c). Equivalent stabilizing force $q_{d3}(s)$ and support reactions R_{d3} together form a self-equilibrating system. Whereas in the case of the equivalent force of the member acc. to scheme 11 ($\alpha = 0$), the braced flange of the pin-supported roof girder – Figs 2 a, c) there is no support reaction ($R_{d3} = 0$, see Fig. 3c). Then span equivalent stabilizing force $q_{d3}(s)$ is self-equilibrating along the length of the braced member.

Span equivalent stabilizing force $q_{d3}(s)$ (Fig. 3b) is variable (nonuniform) along the length of the braced member and when $\alpha < 0.65$, it is also longitudinally sign-variable.

If $\alpha < 0.3$, then the highest equivalent stabilizing force $q_{d3}(0) = q_{d3}(1)$ occur in the support zones of the braced members (Fig. 3b). Moreover, when $\alpha < 0.3$, equivalent force $q_{d3}(0.5)$ in the central zone have opposite senses and are lower than $q_{d3}(0)$ and $q_{d3}(1)$ (Fig. 3b). For example, in the case of scheme 11 ($\alpha = 0$), the equivalent stabilizing force in the support zone of the braced member: $q_{d3}(0) = q_{d3}(1) = 16q$ (where $q = N_{Ed,max}e_0L^{-2}$) is twice as high as the equivalent force at midspan: $q_{d3}(0.5) = 8q$ (Fig. 3b).

It should also be noted that support equivalent stabilizing force $q_{d3}(0) = q_{d3}(1) = 16q$ is twice as high as equivalent stabilizing force $q_{d1} = 8q$ calculated acc. to EC 1993-1-1 [1].

A comparison of equivalent stabilizing forces q_{d1} and $q_{d3}(s)$ clearly indicates that their estimates acc. to EN 1993-1-1 [1] differ fundamentally from the ones acc. to the adopted model. The differences are both qualitative and quantitative. This has a significant bearing on the stress of both the purlins and the bracings.

Total span equivalent stabilizing force $q_{d3,m}(s)$ and support reactions $R_{d3,m}$ from m braced members, with the bracing deformation taken into account, is calculated from the formulas

$$q_{d3,m}(s) = 8 \left[3\alpha(1-2s)^2 - 2(6s^2 - 6s + 1) \right] \frac{e + \delta_{q,w}}{L^2} \sum_{j=1}^m N_{Ed,max,j} \quad (7)$$

$$R_{d3,m} = -4\alpha \frac{e + \delta_{q,w}}{L} \sum_{j=1}^m N_{Ed,max,j} \quad (8)$$

where:

$\delta_{q,w}$ – the midspan deflection of the bracing under equivalent stabilizing force q_{d3} and all the other external loads, obtained from the 1st order analysis (when the 2nd order theory is used to analyse the structure, one can assume $\delta_{q,w} = 0$),

m – the number of braced members,

e – the total amplitude of m braced members imperfection, amounting to

$$e = \frac{L}{500} \sqrt{0,5 \left(1 + \frac{1}{m} \right)} \quad (9)$$

Span load $q_{d3}(s)$ is transferred to the principal purlins and the eaves purlins while support reactions R_{d3} stress the eaves purlins.

The equivalent axial forces in the eaves purlins ($F_{d3,m}$) and the ones in the principal purlins ($F_{d3,i,m}$), originating from m braced roof girder flanges are calculated from the formulas

$$F_{d3,m} = R_{d3,m} + \int_0^{0,5a/L} q_{d3}(s) ds = 4 \frac{e + \delta_{q,w}}{L} \left(\frac{a}{L} - 1 \right) \times \left[\alpha \left(\frac{a}{L} - 1 \right)^2 - \left(\frac{a}{L} - 2 \right) \frac{a}{L} \right] \sum_{j=1}^m N_{Ed,j} \quad (10)$$

$$F_{d3,i,m} = \int_{s_i - 0,5a/L}^{s_i + 0,5a/L} q_{d3}(s) ds = 8a \frac{e + \delta_{q,w}}{L^2} \times \left[\alpha \left(\frac{a^2}{L^2} + 3(1 - 2s_i)^2 \right) - \left(2 + \frac{a^2}{L^2} - 12s_i + 12s_i^2 \right) \right] \sum_{j=1}^m N_{Ed,j} \quad (11)$$

where: a – the spacing of the purlins.

The equivalent axial forces in the principal purlins ($F_{d1,i,m}$), determined using the computational model contained in EN 1993-1-1 [1] (Fig. 1b, when $N_1 = \text{const.}$), have identical values and the same sense (since $q_{d1} = \text{const.}$, Fig. 1c).

In the case of schemes 2-11 with parabolic distributions of axial force $N_3(s)$, the values of all the equivalent axial forces in the principal purlins ($F_{d3,i,m}$) vary along the length of the braced member. Moreover, if parameter $\alpha < 0.5$, also the senses of axial forces

purlins ($F_{d3,i,m}$) change along the length of the braced member.

3 Parametric analysis of stress of purlins and bracings due to initial bow imperfection e_0 of braced top chord of truss

A scheme of the analysed bracing is shown in Fig. 4a. It stiffens the compression top flange of the truss roof girder shown in Fig. 4b. Truss cross sections: top chord HEA 120, bottom chord RHS 100×100×5, diagonals RHS 60×60×5. The purlins (I-section IPE 180) are spaced at every 2.0 m. The span of the girder is $L = 12 \times 2 \text{ m} = 24 \text{ m}$. Its transverse concentrated load is $P_{Ed} = 10 \text{ kN}$.

The axial force at the midspan of the girder's braced top chord is $N_{\text{span}} = N_{Ed,\text{max}} = 163.64 \text{ kN}$, while the axial force at its support is $N_{\text{supp}} = \alpha N_{Ed,\text{max}}$ (Fig. 4c).

The initial bow imperfection of the girder's braced flange is $e_0 = L/500 = 24000/500 = 48 \text{ mm}$.

The diagonals of the bracing are assumed to be thin beams which do not transfer compressive forces (tie rods with diameter 20mm).

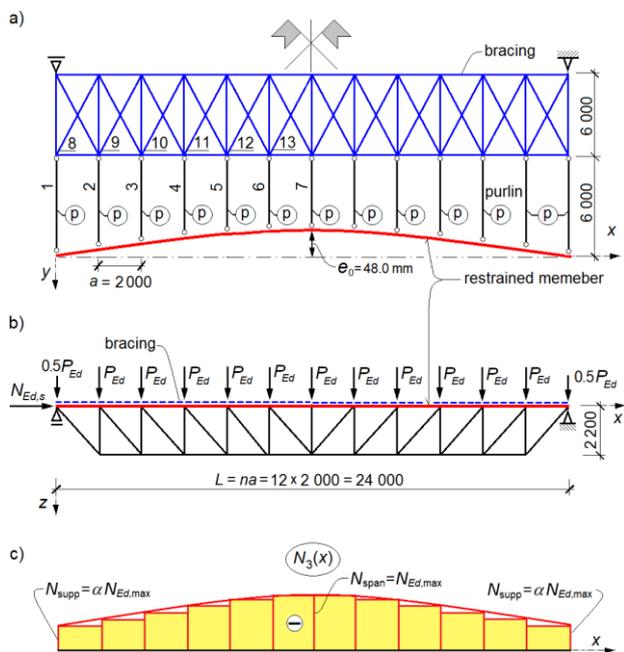


Fig. 4. Schemes of: a) roof bracing, b) truss, c) distribution of axial force in braced chord of truss

The axial forces in the bracing's purlins and diagonals caused by the equivalent stabilizing force generated by one stabilized top flange of the girder were analysed. Eleven schemes of the variation of force $N_3(s)$ in the braced top flange of the girder were considered. Parameters α of the variability of axial forces $N_3(s)$ are given in Table 1. The values of the axial forces in the purlins ($F_{d,i}$) and in the bracing's diagonals ($S_{d,i}$) in schemes 1-11 of the stress in the girder's braced flange are presented in Table 2. The top part of Table 2 shows parameter α for schemes 1-11. In Table 2 the numbers of the roof structure members are given in column 1 (the

numbering of the members is shown in Fig. 4a) while the numbers of the columns are given in row 4.

The following conclusions can be drawn from the parametric analyses:

1. If the real parabolic distribution of the axial force in the braced member is adopted in the computational model, the equivalent stabilizing force and the axial forces in the bracing's purlins and members differ both qualitatively and quantitatively from the ones yielded by the model recommended by EN 1993-1-1 [1].
2. In comparison with the evaluation of the stress acc. to the model recommended by EN 1993-1-1 [1] (column 2 in Table 2, equivalent stabilizing force q_{d1} as in scheme 1 at $\alpha = 1$), if the longitudinally parabolically variable axial force in the braced member of the uniformly loaded pin-supported girder (column 12 in Table 2, equivalent stabilizing force q_{d3} for scheme 11 at $\alpha = 0$) is taken into account in the analysis, this follows:
 - an 84% reduction in the axial force in the eaves purlin (in element 1),
 - a 9.2% increase in the axial force in the before-eaves purlin (in element 2),
 - in the model acc. to scheme 1 (q_{d1}) the strongest axial force occurs in the before-eaves purlin (in element 1, $S_1 = 1.200 \text{ kN}$),
 - in the model acc. to scheme 11 (q_{d3}) the strongest axial force occurs in the before-eaves purlin (in element 2, $S_2 = 0.238 \text{ kN}$),
 - the axial force in the purlin at the midspan of the bracing (in element 7) is the same for both the computational models,
 - an 8.7% (element 6) 75.7% (element 4) reduction in the axial forces in the other purlins (in elements 3-6),
 - a 0.9% (element 13) 84% (element 8) reduction in the axial forces in all the diagonals of the bracing,
 - when equivalent stabilizing force q_{d1} is assumed, the maximum axial force reliable for dimensioning the bracing occurs in eaves diagonal 8 ($S_8 = 1.265 \text{ kN}$),
 - assuming q_{d3} , the maximum axial force reliable for dimensioning the bracing occurs in diagonal 10 ($S_{10} = 0.531 \text{ kN}$),
 - the axial force in diagonal S_{10} (generated by equivalent force q_{d3}) is by 58% weaker than the axial force in diagonal S_8 (generated by equivalent force q_{d1}).
3. For all the schemes 2-11 as the support axial force decreases S_{supp} ($\alpha < 1$), the axial forces in all the purlins decrease (in comparison with the ones calculated acc. to [1]). In the central zone this decrease is only slight. Also the axial force values reliable for dimensioning purlins decrease (relative to the ones calculated acc. to [1]). As a result, the axial forces in the diagonals decrease (relative to the ones calculated acc. to [1]) and the axial force values reliable for dimensioning the bracing also decrease.

Table 2. Axial forces in purlins and bracing diagonals for different schemes of equivalent stabilizing force in the restrained member

Scheme no	1	2	3	4	5	6	7	8	9	10	11
α	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
Member no	Forces in purlins bracing diagonals in analyzed schemes of equivalent stabilizing force										
1	2	3	4	5	6	7	8	9	10	11	12
Forces in purlins $F_{d,i}$ [kN]											
1	-1.200	-1.099	-0.998	-0.898	-0.797	-0.696	-0.595	-0.494	-0.393	-0.293	-0.192
2	0.218	0.173	0.127	0.081	0.036	-0.010	-0.055	-0.101	-0.147	-0.192	-0.238
3	0.218	0.189	0.160	0.131	0.101	0.072	0.043	0.013	-0.016	-0.045	-0.074
4	0.218	0.202	0.185	0.169	0.52	0.136	0.119	0.103	0.086	0.070	0.053
5	0.218	0.211	0.203	0.196	0.189	0.181	0.174	0.166	0.159	0.151	0.144
6	0.218	0.216	0.214	0.212	0.210	0.208	0.206	0.204	0.202	0.201	0.199
7	0.218	0.218	0.218	0.218	0.218	0.217	0.217	0.217	0.217	0.217	0.218
Forces in diagonals $S_{d,i}$ [kN]											
8	-1.265	-1.159	-1.052	-0.946	-0.840	-0.734	-0.627	-0.521	-0.415	-0.308	-0.202
9	-1.035	-0.977	-0.918	-0.860	-0.802	-0.744	-0.686	-0.627	-0.569	-0.511	-0.453
10	-0.805	-0.778	-0.750	-0.723	-0.695	-0.668	-0.641	-0.613	-0.586	-0.559	-0.531
11	-0.575	-0.565	-0.555	-0.545	-0.535	-0.525	-0.515	-0.505	-0.495	-0.485	-0.475
12	-0.345	-0.343	-0.341	-0.339	-0.336	-0.334	-0.332	-0.330	-0.328	-0.326	-0.323
13	-0.115	-0.115	-0.115	-0.115	-0.115	-0.115	-0.115	-0.114	-0.114	-0.114	-0.114
Notation of axial forces: „+” – compression, „-” – tensile											

4 Equivalent horizontal forces due to twist of truss's principal plane by angle ϕ

In the calculations acc. to EN 1993-1-1 [1] braced top chords are treated as initial bow curved members “isolated” from the truss (not connected via lattice work with the bottom chords).

This computational model does not reflect the behaviour and stress of a real truss, i.e. the twist of its principal plane by angle ϕ_0 due to initial bow imperfection e_0 of the top chord (Fig. 5) and also the stiffness parameters of its members and joints.

Figure 5a shows initial bow imperfection e_0 of the truss's top chord in the plane of the roof. Since the top chord is curvilinear while bottom chord is rectilinear (Fig. 5c) the bow imperfection results in the twist of the principal plane of the truss. Hence besides the initial bow imperfection $y(s)$ of top chord, there is an imperfection consisting in the twist of the truss's principal plane by angle $\phi_0(s)$. As a result of the action of vertical loads $P_{Ed,i}$ in top joints i on the truss twisted by angle $\phi_0(s_i)$ equivalent horizontal forces H_i arise. They are transferred to the purlins and to the bracing (Fig. 5b), increasing their stress.

Assuming that vertical forces P_{Ed} are identical and that only the top chord is bow curved (Fig. 5c) acc. to (4), equivalent horizontal forces $H_{i,m}$ in joint i originating

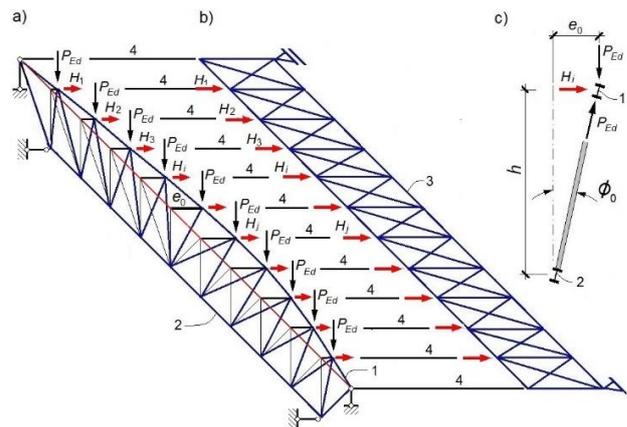


Fig. 5. Schemes of: a) twist of truss principal plane, b) bracing load, c) top chord initial bow imperfection; 1 – top chord of truss, 2 – bottom chord of truss, 3 – bracing system, 4 – purlin.

from braced m trusses can be estimated using the formula [6, 8]

$$H_{i,m} = 4 \sum_{j=1}^m P_{Ed,j} (e + \delta_{q,H,w}) \frac{s_j(1-s_j)}{h_i}, \quad (12)$$

where:

$\delta_{q,H,w}$ – the midspan deflection of the bracing under equivalent stabilizing forces q_d and H_i and all

the external loads (e.g. wind load W), obtained from the 1st order analysis (if the 2nd order theory is used one can assume $\delta_{q,H,w} = 0$),

h_i – the construction depth of the truss in joint i .

The distribution of equivalent horizontal forces H_i due to the twist of the principal plane of the truss varies along the length of the latter. In the considered case, the distribution is parabolic consistently with the adopted bow imperfection of the axis of the braced member ($y(s)$). The strongest equivalent horizontal force H_i occur at the midspan of the truss when $y(0.5) = e_0$.

5 Analysis of total equivalent stabilizing forces due to initial bow imperfection e_0 of braced flange of girder and twist of its principal plane by angle ϕ

Initial bow imperfection e_0 of the braced flange and the twist of the principal plane of the girder by angle ϕ together generate equivalent forces transferred to the purlins and the bracing. Therefore when evaluating the equivalent stabilizing forces one should add up the force due to the braced flange bow imperfection (q_{d3} and R_{d3}) and to the twist of the girder (H_i).

In order to quantitatively evaluate the total imperfect effect due to initial bow imperfection e_0 and to the twist of the principal plane of the truss by angle ϕ_0 on the stress of the purlins calculations were done for the truss presented in Fig. 4b. The truss loaded only with $P_{Ed} = 10$ kN and $N_{Ed,s} = 0$, was analysed.

The axial force in the central zone of the span of the braced top chord of truss is $N_{span} = N_{Ed,max} = 163.64$ kN, and axial force in its support zone is $N_{supp} = 49.42$ kN; factor $\alpha = 0.151$. The results of the calculations are presented in Table 3. The latter contains the values of the axial forces in the purlins due to the equivalent stabilizing forces generated by one braced truss. The forces in the purlins due to respectively equivalent force q_{d3} (for $\alpha = 0.151$) and the equivalent horizontal forces H_i (determined only for initial bow imperfection of top chord – Fig. 5c) are specified in columns 3 and 4 in Table 3.

Table 3. Axial forces in purlins due to equivalent stabilizing force q_{d3} (for $\alpha = 0.151$) and forces H_i [kN]

Member no	Axial forces from loads [kN]				
	q_{d1}	q_{d3}	H_i	$q_{d3}+H_i$	$(q_{d3} + H_i)/q_{d1}$
1	-1.200	-0.346	0	-0.346	0.288
2	0.218	-0.168	0.067	-0.101	0.463
3	0.218	-0.030	0.121	0.091	0.417
4	0.218	0.078	0.164	0.242	1.100
5	0.218	0.155	0.194	0.349	1.601
6	0.218	0.201	0.212	0.413	1.894
7	0.218	0.217	0.218	0.436	2.000

Notation of axial forces: „+” – compression

In comparison with equivalent stabilizing force q_{d1} (column 1 in Table 3) acc. to EN 1993-1-1 [1], the total equivalent forces: $q_{d3} + H_i$ (column 5 in Table 3) causes:

- an increase in the axial forces in the midspan purlins of the braced truss chord, i.e. by 11% in purlin 4, 60.1% in purlin 5, 89.4% in purlin 6 and 100% in purlin 7 (the middle one);
- a reduction in the axial forces in the support zone of the braced top chord of the truss, i.e. by 71.2% in purlin 1 (the eaves purlin), 53.7% in purlin 2 and 58.3% in purlin 3.

Additional calculations of the bracings loaded with forces $q_{d3} + H_i$ showed a reduction in the forces in the diagonals in the support zone and an increase in the forces in the diagonals in the central zone of the bracing (from 18.4% to 48.7%) in comparison to EN 1993-1-1 [1] model.

6 Conclusions

The analyses have clearly shown that the computational model of equivalent stabilizing force q_{d1} and R_{d1} acc. to EN 1993-1-1 [1] does not reflect the behaviour and stress of the real structure. It incorrectly estimates the axial forces in the purlins and the bracing members, which may lead to a wrong assessment of their reliability. This is due to the fact that the model does not take into account either the real longitudinally variable (and often sign-variable) [5-8] parabolic distribution of the axial force in the bow curved (e_0) braced flange of the girder or the twist of the girder's principal plane by angle ϕ .

If the distributions of the axial force in the braced members are longitudinally variable parabolic (as in schemes 2-11, sections 2 and 3; $\alpha < 1.0$), their span equivalent stabilizing force q_{d3} are nonuniform and sign-variable at $\alpha < 0.65$. They can be much higher than equivalent force q_{d1} acc. to [1]. For example, in the case of scheme 11 ($\alpha=0$) the equivalent stabilizing force in the support zone of the braced member: $q_{d3}(0) = q_{d3}(1) = 16q$ (where $q = N_{Ed,max}e_0L^{-2}$) is twice as high as the equivalent force at midspan: $q_{d3}(0.5) = 8q$ (Fig. 3b).

It should also be noted that support equivalent force $q_{d3}(0) = q_{d3}(1) = 16q$ is twice as high as equivalent force $q_{d1} = 8q$ calculated acc. to EC 1993-1-1 [1].

If the actual longitudinally variable parabolic distribution of the axial force in the braced member is assumed, this results in major changes, in comparison with the model acc. to EC 1993-1-1 [1], in the schemes and parameters of the equivalent stabilizing forces. A comparison of equivalent stabilizing forces q_{d1} and $q_{d3}(s)$ and support reactions R_{d1} and R_{d3} clearly shows fundamental differences between the estimates of the equivalent forces. As a result, the purlins and the bracings are stressed differently. As compared with the equivalent forces calculated acc. to EC 1993-1-1 [1], equivalent forces $q_{d3}(s)$ and R_{d3} can cause not only an increase in the axial forces, but also a different distribution of the latter in the principal purlins. The parametric analysis of the stress of the purlins and the

bracing diagonals carried out in sect. 3 clearly shows that by disregarding the real distribution of the axial force in the braced member one will incorrectly assess of their reliability.

The “imperfect” model in EN 1993-1-1 [1] is limited to an analysis of the effects of initial bow imperfection e_0 of the braced top flange of the girder. However, the consequence of the top flange bow imperfection is the twist of girder’s principal plane by angle ϕ_0 , which generates equivalent horizontal forces H_i . The latter are transferred to the purlins and to the bracing, increasing their stress. The computational model in EN 1993-1-1 [1] does not take into account equivalent horizontal forces H_i caused by the twist of the truss.

When determining the total equivalent stabilizing force (resulting of e_0 i ϕ_0) one can use the nonlinear geometric analysis of the 3D computational beam model, taking into account the real stiffnesses of the members and joints of the considered structure.

The proposed methods of evaluating equivalent stabilizing forces which take into account the real distribution of the axial force in the braced member with arch initial bow imperfection e_0 and the twist of girder’s principal plane by angle ϕ_0 enable a more precise analysis of the stress of purlins and bracings. The quantitative and qualitative differences between the proposed computational models and the evaluation acc. to [1] are considerable. Therefore one should consider introducing (after additional numerical analyses) proper corrections pertaining to the investigated problem in the revised EN 1993-1-1 [1].

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